LAPLACE HOMOTOPY PERTURBATION METHOD FOR SOLVING COUPLED SYSTEM OF LINEAR AND NONLINEAR PARTIAL DIFFERENTIAL EQUATION

by

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Abstract

Laplace Homotopy Perturbation Method (LHPM) is a combination of Laplace transformation method and homotopy perturbation method. It is an approximate analytical method, which can be adopted in solving system of linear and non linear differential equations. In this research, Boussineq-Burger equation and Diffusion-reaction equation has been solved by LHPM which gives an approximate analytical solution that converges faster to the exact solution by using only few iteration of the recursive relation.

Keywords*: Boussineq-Burger equation, Diffusion-reaction equation, Homotopy perturbation transformation method, Lapalce transforms method, System of partial differential equatons, He's polynomial.*

Introduction

The linear and nonlinear phenomena which appears in areas of scientific fields which includes solid state, physics, fluid dynamics, plasma physics and chemical kinetics can be modeled by partial differential equation. Analytical method and numerical methods are used in handling these problems (Abdou and Wakil, 2007).

In this research, three (3) examples of coupled system of nonlinear partial differential physical equations including diffusion reaction equation and Boussinesq-Burger equation are investigated by means of Laplace homotopy perturbation transformation method (LHPTM). Diffusion reaction equation have been investigated by Xu, (2007); HE and Zhang, 2007; Barari et al, 2008; Momani and Abuasad, 2005; and Ghotbi et al., 2009),_using variational iteration method, Boussinesq-Burger equation also has been investigated by Patel and Kanta (2005) using Laplace Adomian Decomposition method. The application of LHPTM to the mentioned example is to compute an approximate solution to the equations in order to verify the result; a comparison will be made to the exact solution. These equations (reaction diffusion equation) describe a wide variety of nonlinear system in biology, ecology and engineering, It can be analysed by the means of some methods from the theory of partial differential equation. Boussinsq-Burger equation normally arises in the study of fluid flow and describes the propagation of water waves.

Basic Idea Of Laplace Homotopy Perturbation Transportation Method

Consider the system of partial differential equations in operator form;

$$D_t u + R_1(u,v) + N_1(u,v) = g_1 \qquad (1)$$

$$D_t v + R_2(u,v) + N_2(u,v) = g_2 \qquad (2)$$

with initial conditions

$$u(x,0) = f_1(x) \qquad (3)$$

$$v(x,0) = f_2(x) \qquad (4)$$

where D_t is considered here as the first order partial differential operators, R_1 and R_2 are linear operators, N_1 and N_2 are nonlinear operators, g_1 and g_2 are inhomogeneous terms. By applying Laplace transform to both sides of equation (1) and (2) and using initial conditions (3) and (4) to obtain

$$L[D_t u]+L[R_1(u,v)]+L[N_1(u,v)]=L[g_1] \tag{5}$$

$$L[D_t v]+L[R_2(u,v)]+L[N_2(u,v)]=L[g_2] \tag{6}$$

Using the differentiation property of Laplace transform, it gives

$$L[u]=\frac{f_1(x)}{s}+\frac{1}{s}L[g_1]-\frac{1}{s}L[R_1(u,v)]-\frac{1}{s}L[N_1(u,v)] \tag{7}$$

$$L[v]=\frac{f_2(x)}{s}+\frac{1}{s}L[g_2]-\frac{1}{s}L[R_2(u,v)]-\frac{1}{s}L[N_2(u,v)] \tag{8}$$

The Laplace transform method decomposes the unknown functions $u(x,t)$ and $v(x,t)$ by an infinite series of component as

$$u(x,t)=\sum_{n=0}^{\infty}p^n u_n(x,t) \tag{9}$$

$$v(x,t)=\sum_{n=0}^{\infty}p^n v_n(x,t) \tag{10}$$

Substituting equation (9) and (10) into equation (7) and (8), it gives

$$L\left[\sum_{n=0}^{\infty}p^n u_n(x,t)\right]=\frac{f_1(x)}{s}+\frac{1}{s}L[g_1]-\frac{1}{s}L\left[R_1\sum_{n=0}^{\infty}p^n u_n(x,t)\sum_{n=0}^{\infty}p^n v_n(x,t)\right] -\frac{1}{s}L\left[N_1\sum_{n=0}^{\infty}p^n u_n(x,t)\sum_{n=0}^{\infty}p^n v_n(x,t)\right] \tag{11}$$

$$L\left[\sum_{n=0}^{\infty}p^n v_n(x,t)\right]=\frac{f_2(x)}{s}+\frac{1}{s}L[g_2]-\frac{1}{s}L\left[R_2\sum_{n=0}^{\infty}p^n u_n(x,t)\sum_{n=0}^{\infty}p^n v_n(x,t)\right] -\frac{1}{s}L\left[N_2\sum_{n=0}^{\infty}p^n u_n(x,t)\sum_{n=0}^{\infty}p^n v_n(x,t)\right] \tag{12}$$

Applying the linearity of Laplace transform in equation (11) and (12), to obtain the following formula

$$L\left[\sum_{n=0}^{\infty}p^n u_n(x,t)\right]=\frac{f_1(x)}{s}+\frac{1}{s}L[g_1]-p\left[\begin{array}{l}\frac{1}{s}L\left[R_1\sum_{n=0}^{\infty}p^n u_n(x,t)\sum_{n=0}^{\infty}p^n v_n(x,t)\right]\\ -\frac{1}{s}L\left[N_1\sum_{n=0}^{\infty}p^n u_n(x,t)\sum_{n=0}^{\infty}p^n v_n(x,t)\right]\end{array}\right] \tag{13}$$

$$L\left[\sum_{n=0}^{\infty}p^n v_n(x,t)\right]=\frac{f_2(x)}{s}+\frac{1}{s}L[g_2]-p\left[\begin{array}{l}\frac{1}{s}L\left[R_2\sum_{n=0}^{\infty}p^n u_n(x,t)\sum_{n=0}^{\infty}p^n v_n(x,t)\right]\\ -\frac{1}{s}L\left[N_2\sum_{n=0}^{\infty}p^n u_n(x,t)\sum_{n=0}^{\infty}p^n v_n(x,t)\right]\end{array}\right] \tag{14}$$

Matching both sides of equation (13) and (14) and find the inverse Laplace, it gives the following transformative relation

$$u_0=L^{-1}\left[\frac{f_1(x)}{s}\right]+L^{-1}\left[\frac{1}{s}L[g_1]\right] \tag{15}$$

$$v_0 = L^{-1}\left[\frac{f_2(x)}{s}\right] + L^{-1}\left[\frac{1}{s}L[g_2]\right] \quad (16)$$

$$u_1 = -L^{-1}\left[\frac{1}{s}L[(u_0, v_0)]\right] - L^{-1}\left[\frac{1}{s}L[N_1(u_0, v_0)]\right] \quad (17)$$

$$v_1 = -L^{-1}\left[\frac{1}{s}L[(u_0, v_0)]\right] - L^{-1}\left[\frac{1}{s}L[N_2(u_0, v_0)]\right] \quad (18)$$

for $n \geq 1$, the recursive relation are given by

$$u_{k+1} = -L^{-1}\left[\frac{1}{s}L[(u_k, v_k)]\right] - L^{-1}\left[\frac{1}{s}L[N_1(u_k, v_k)]\right] \quad (19)$$

$$v_{k+1} = -L^{-1}\left[\frac{1}{s}L[(u_k, v_k)]\right] - L^{-1}\left[\frac{1}{s}L[N_2(u_k, v_k)]\right] \quad (20)$$

Numerical Example

In this section, we apply the LHPTM for solving Boussinesq-Burger and diffusion-reaction equations

Example 1.

Consider the Boussinesq-Burger equations

$$u_t = \frac{1}{2}v_x - 2uu_x, \qquad 0 \leq x \leq 1, \qquad t > 0 \quad (21)$$

$$v_t = \frac{1}{2}u_{xxx} - 2u_x v_x \quad (22)$$

with initial conditions

$$u(x,0) = \frac{ck}{2} + \frac{ck}{2}\tanh\left(\frac{-kx - \ln b}{2}\right) \quad (23)$$

$$v(x,0) = \frac{k^2}{8} + \sec h^2\left(\frac{kx + \ln b}{2}\right) \quad (24)$$

Applying Laplace transform to both sides of equation (21) and (22) to obtain

$$L[u] = \frac{u(x,0)}{s} + \frac{1}{2s}L[v_x] - 2\frac{L}{s}[uu_x] \quad (25)$$

$$L[v] = \frac{v(x,0)}{s} + \frac{1}{2s}L[u_{xxx}] - 2\frac{L}{s}[u_x v_x] \quad (26)$$

The inverse Laplace transform of equation (25) and (26) implies that;

$$u = L^{-1}\left[\frac{u(x,0)}{s}\right] + \frac{1}{2}L^{-1}\left[\frac{L}{s}[v_x]\right] - 2L^{-1}\left[\frac{L}{s}[uu_x]\right] \quad (27)$$

$$v = L^{-1}\left[\frac{v(x,0)}{s}\right] + \frac{1}{2}L^{-1}\left[\frac{L}{s}[u_{xxx}]\right] - 2L^{-1}\left[\frac{L}{s}[u_x v_x]\right] \quad (28)$$

Substituting (9) and (10) with initial conditions (23) and (24) into equation (27) and (28) and applying equation (15)-(18), we obtained

$$u_0 = \frac{1}{e^{-kx} + 1} \quad (29)$$

$$v_0 = \frac{1}{4}\frac{4 + 3e^{kx}}{\left(1 + e^{kx}\right)^2} \quad (30)$$

$$u_1 = \frac{1}{4}t\left[\begin{array}{l} \dfrac{4}{\left(-\cosh(kx)+\sinh(kx)-1\right)^3} + \dfrac{5\cosh(kx)+5\sinh(kx)+4e^{2kx}}{\left(\cosh(kx)-\sinh(kx)+1\right)\left(\cosh(kx)+\sinh(kx)+1\right)^2} \\ + \dfrac{k^2\left(-\left(e^{-\frac{1}{3}kx}+e^{\frac{2}{3}kx}\right)^3 + 8\cosh\left(\frac{1}{2}kx\right)^2\right)}{\cosh\left(\frac{1}{2}kx\right)^2\left(e^{-\frac{1}{3}kx}+e^{\frac{2}{3}kx}\right)^3} \end{array}\right] \quad (40)$$

$$v_1 = \frac{1}{16}t\left[\begin{array}{l} \dfrac{-16-24e^{kx}+72e^{3kx}k^2+3e^{2kx}(-3+32k^2)}{\left(1+e^{kx}\right)^4} + \dfrac{4(4+3e^{kx})}{\left(e^{\frac{1}{2}kx}+e^{\frac{3}{2}kx}\right)^2\left(e^{-kx}+1\right)} \\ + \dfrac{\left(3\left(1+e^{kx}\right)^3\left(e^{-\frac{2}{3}kx}+e^{\frac{1}{3}kx}\right)^3\left(6\left(1+e^{kx}\right)^3+\left(e^{-\frac{2}{3}kx}+e^{\frac{1}{3}kx}\right)^3(4e^{kx}+3e^{2kx})\right)k^2\right)}{\cosh\left(\frac{1}{2}kx\right)^2\left(e^{-\frac{2}{3}kx}+e^{\frac{1}{3}kx}\right)^3\left(1+e^{kx}\right)^3} \end{array}\right] \quad (41)$$

Therefore, the solution is given by

$$u(x,t) = \sum_{n=0}^{1} p^n u_n(x,t) \quad (42)$$

$$= \frac{1}{e^{-kx}+1} + \frac{1}{4}t\left[\begin{array}{l} \dfrac{4}{\left(-\cosh(kx)+\sinh(kx)-1\right)^3} + \dfrac{5\cosh(kx)+5\sinh(kx)+4e^{2kx}}{\left(\cosh(kx)-\sinh(kx)+1\right)\left(\cosh(kx)+\sinh(kx)+1\right)^2} \\ + \dfrac{k^2\left(-\left(e^{-\frac{1}{3}kx}+e^{\frac{2}{3}kx}\right)^3 + 8\cosh\left(\frac{1}{2}kx\right)^2\right)}{\cosh\left(\frac{1}{2}kx\right)^2\left(e^{-\frac{1}{3}kx}+e^{\frac{2}{3}kx}\right)^3} \end{array}\right]$$

as $p^n \to 0$

And

$$v(x,t) = \sum_{n=0}^{1} p^n v_n(x,t) \quad (43)$$

$$= \frac{1}{4}\frac{4+3e^{kx}}{\left(1+e^{kx}\right)^2} + \frac{1}{16}t\left[\begin{array}{l} \dfrac{-16-24e^{kx}+72e^{3kx}k^2+3e^{2kx}(-3+32k^2)}{\left(1+e^{kx}\right)^4} + \dfrac{4(4+3e^{kx})}{\left(e^{\frac{1}{2}kx}+e^{\frac{3}{2}kx}\right)^2\left(e^{-kx}+1\right)} \\ + \dfrac{\left(3\left(1+e^{kx}\right)^3\left(e^{-\frac{2}{3}kx}+e^{\frac{1}{3}kx}\right)^3\left(6\left(1+e^{kx}\right)^3+\left(e^{-\frac{2}{3}kx}+e^{\frac{1}{3}kx}\right)^3(4e^{kx}+3e^{2kx})\right)k^2\right)}{\cosh\left(\frac{1}{2}kx\right)^2\left(e^{-\frac{2}{3}kx}+e^{\frac{1}{3}kx}\right)^3\left(1+e^{kx}\right)^3} \end{array}\right]$$

as $p^n \to 0$

Example 2.

Consider the Boussinesq-Burger equations

$$u_t = u(1-u^2-v) + u_{xx}, \qquad 0 \le x \le 1, \qquad t > 0 \quad (44)$$

$$v_t = v(1-u-v) + v_{xx} \quad (45)$$

with initial conditions

$$u(x,0)=\frac{e^{kx}}{(1+e^{kx})} \tag{46}$$

$$v(x,0)=\frac{1+\frac{3}{4}e^{kx}}{\left(1+e^{kx}\right)^2} \tag{47}$$

Applying Laplace transform to both sides of equation (44) and (45) to obtain

$$L[u]=\frac{u(x,0)}{s}+\frac{1}{s}L\left[u(1-u^2-v)+u_{xx}\right] \tag{48}$$

$$L[v]=\frac{v(x,0)}{s}+\frac{1}{s}L\left[v(1-u-v)+v_{xx}\right] \tag{49}$$

The inverse Laplace transform of equation (48) and (49) implies that;

$$u=L^{-1}\left[\frac{u(x,0)}{s}\right]+L^{-1}\left[\frac{1}{s}L\left[u(1-u^2-v)+u_{xx}\right]\right] \tag{50}$$

$$v=L^{-1}\left[\frac{v(x,0)}{s}\right]+L^{-1}\left[\frac{1}{s}L\left[v(1-u-v)+v_{xx}\right]\right] \tag{51}$$

Substituting (9) and (10) with initial condition (46) and (47) into equation (50) and (51) and applying equation (15)-(20), we obtained

$$u_0=\frac{1}{\left(e^{-\frac{1}{2}kx}+1\right)^2} \tag{52}$$

$$v_0=\frac{1}{1+e^{\frac{1}{2}kx}} \tag{53}$$

$$u_1=\frac{1}{32}t\left[-\frac{32}{\left(e^{-\frac{1}{2}kx}+1\right)^4}+\frac{32}{\left(e^{-\frac{3}{4}kx}+e^{-\frac{1}{4}kx}\right)^2\left(1+e^{\frac{1}{2}kx}\right)}+\frac{\left(-16\cosh\left(\frac{1}{4}kx\right)^4+3\left(e^{-\frac{1}{3}kx}+e^{\frac{1}{6}kx}\right)^3\right)k^2}{\cosh\left(\frac{1}{4}kx\right)^4\left(e^{-\frac{1}{3}kx}+e^{\frac{1}{6}kx}\right)^3}\right] \tag{54}$$

β (55)

Therefore, the solution is given by

$$u(x,t)=\sum_{n=0}^{1}p^n u_n(x,t)$$

$$=\frac{1}{\left(e^{-\frac{1}{2}kx}+1\right)^2}+\frac{1}{32}t\left[-\frac{32}{\left(e^{-\frac{1}{2}kx}+1\right)^4}+\frac{32}{\left(e^{-\frac{3}{4}kx}+e^{-\frac{1}{4}kx}\right)^2\left(1+e^{\frac{1}{2}kx}\right)}+\frac{\left(-16\cosh\left(\frac{1}{4}kx\right)^4+3\left(e^{-\frac{1}{3}kx}+e^{\frac{1}{6}kx}\right)^3\right)k^2}{\cosh\left(\frac{1}{4}kx\right)^4\left(e^{-\frac{1}{3}kx}+e^{\frac{1}{6}kx}\right)^3}\right] \tag{56}$$

as $p^n \to 0$

And

$$v(x,t) = \sum_{n=0}^{1} p^n v_n(x,t)$$

$$= \frac{1}{1+e^{\frac{1}{2}kx}} + \frac{1}{16}t\left[-\frac{16}{\left(\left(\cosh\left(\frac{1}{2}kx\right) - \sinh\left(\frac{1}{2}kx\right) + 1\right)^2 \left(1+\cosh\left(\frac{1}{2}kx\right) + \sinh\left(\frac{1}{2}kx\right)\right)\right)} + \frac{k^2\left(-\left(e^{-\frac{1}{3}kx} + e^{\frac{1}{6}kx}\right)^3 + 8\cosh\left(\frac{1}{4}kx\right)^2\right)}{\cosh\left(\frac{1}{4}kx\right)^2 \left(e^{-\frac{1}{3}kx} + e^{\frac{1}{6}kx}\right)^3} \right] \quad (57)$$

as $p^n \to 0$

Example 3.

Consider the following nonlinear equations

$$u_t = \frac{1}{2}v_x - 2uu_x, \qquad 0 \le x \le 1, \qquad t > 0 \quad (58)$$

$$v_t = \frac{1}{2}u_{xxx} - 2(uv)_x \quad (59)$$

with initial conditions

$$u(x,0) = \frac{ck}{2} + \frac{ck}{2}\tanh\left(\frac{(-kx - \ln b)}{2}\right) \quad (60)$$

$$v(x,0) = \frac{-k^2}{8} + \sec h^2\left(\frac{(kx + \ln b)}{2}\right) \quad (61)$$

Applying Laplace transform to both sides of equation (58) and (59) to obtain

$$L[u] = \frac{u(x,0)}{s} + \frac{1}{s}L\left[\frac{1}{2}v_x - 2uu_x\right] \quad (62)$$

$$L[v] = \frac{v(x,0)}{s} + \frac{1}{s}L\left[\frac{1}{2}u_{xxx} - 2(uv)_x\right] \quad (63)$$

The inverse Laplace transform of equation (62) and (63) implies that;

$$u = L^{-1}\left[\frac{u(x,0)}{s}\right] + L^{-1}\left[\frac{1}{s}L\left[\frac{1}{2}v_x - 2uu_x\right]\right] \quad (64)$$

$$v = L^{-1}\left[\frac{v(x,0)}{s}\right] + L^{-1}\left[\frac{1}{s}L\left[\frac{1}{2}u_{xxx} - 2(uv)_x\right]\right] \quad (65)$$

Substituting (9) and (10) with initial condition (60) and (61) into equation (64) and (65) and applying equation (15)-(20), we obtained

$$u_0 = \frac{1}{2}ck\left(1 - \tanh\left(\frac{1}{2}kx + \frac{1}{2}\ln b\right)\right) \quad (66)$$

$$v_0 = -\frac{1}{8}k^2 + \sec h\left(\frac{1}{2}kx + \frac{1}{2}\ln b\right)^2 \quad (67)$$

$$u_1 = \frac{1}{4}t\left(-\tanh\left(\frac{1}{2}kx+\frac{1}{2}\ln b\right)k\left(\begin{array}{l} 2\sec h\left(\frac{1}{2}kx+\frac{1}{2}\ln b\right)^2 \\ +c^2k^3\left(1+\tanh\left(\frac{1}{2}kx+\frac{1}{2}\ln b\right)\right)^3 \\ -\tanh\left(\frac{1}{2}kx+\frac{1}{2}\ln b\right)^2 \end{array}\right)\right) \quad (68)$$

$$v_1 = \frac{1}{16}ck^3t\left(\tanh\left(\frac{1}{2}kx+\frac{1}{2}\ln b\right)^2 - 1\right)\left(\begin{array}{l} k\left(3\tanh\left(\frac{1}{2}kx+\frac{1}{2}\ln b\right)^2 - 1\right) \\ +8\sec h\left(\frac{1}{2}kx+\frac{1}{2}\ln b\right)^2 \tanh\left(\frac{1}{2}kx+\frac{1}{2}\ln b\right) \end{array}\right) \quad (69)$$

Therefore, the solution is given by

$$u(x,t) = \sum_{n=0}^{1} p^n u_n(x,t)$$

$$= \frac{1}{2}ck\left(1-\tanh\left(\frac{1}{2}kx+\frac{1}{2}\ln b\right)\right) + \frac{1}{4}t\left(\begin{array}{l} -\tanh\left(\frac{1}{2}kx+\frac{1}{2}\ln b\right) \\ k\left(\begin{array}{l} 2\sec h\left(\frac{1}{2}kx+\frac{1}{2}\ln b\right)^2 \\ +c^2k^3\left(1+\tanh\left(\frac{1}{2}kx+\frac{1}{2}\ln b\right)\right)^3 \\ -\tanh\left(\frac{1}{2}kx+\frac{1}{2}\ln b\right)^2 \end{array}\right) \end{array}\right) \quad (70)$$

as $p^n \to 0$

And

$$v(x,t) = \sum_{n=0}^{1} p^n v_n(x,t) \quad (71)$$

$$= -\frac{1}{8}k^2 + \sec h\left(\frac{1}{2}kx+\frac{1}{2}\ln b\right)^2 + \frac{1}{16}ck^3t\left(\tanh\left(\frac{1}{2}kx+\frac{1}{2}\ln b\right)^2 - 1\right)$$

$$\left(\begin{array}{l} k\left(3\tanh\left(\frac{1}{2}kx+\frac{1}{2}\ln b\right)^2 - 1\right) \\ +8\sec h\left(\frac{1}{2}kx+\frac{1}{2}\ln b\right)^2 \tanh\left(\frac{1}{2}kx+\frac{1}{2}\ln b\right) \end{array}\right)$$

as $p^n \to 0$

Results and Discussion

A new method using LHPM to numerically solve partial differential physical equation is presented in this research, three (3) examples where presented (a diffusion reaction equation and Boussinesq – Burger equation) and the result obtained are almost the same with the exact solution earlier obtained as shown in table 1-3 below

Table 1: Comparison between the exact solution and the present approximate solution for Example 1: $h_x = \frac{1}{10},\ \ h_t = \frac{1}{250}$

x	u_{exact}	v_{exact}	u_{approx}	v_{approx}	$\lvert u_{exact} - u_{approx} \rvert$	$\lvert v_{exact} - v_{approx} \rvert$
0	0.5009999	0.4365002	0.5002500	0.4377343	0.0003749	0.0012341
0.1	0.5259765	0.4116921	0.5255839	0.4129533	0.0003926	0.0012612
0.2	0.5508238	0.3873219	0.5504157	0.3886017	0.0004080	0.0012798
0.3	0.5754200	0.3635019	0.5749990	0.3647968	0.0004209	0.0012896
0.4	0.5996483	0.3403341	0.5992171	0.3416249	0.0004311	0.0012907
0.5	0.6233988	0.3179079	0.6229605	0.3191912	0.0004383	0.0012833
0.6	0.6465709	0.2962998	0.6461282	0.2975674	0.0004426	0.0012676
0.7	0.6690740	0.2755724	0.6686301	0.2768168	0.0004438	0.0012443
0.8	0.6908294	0.2557745	0.6903873	0.2569887	0.0004421	0.0012142
0.9	0.7117708	0.2369409	0.7113332	0.2381189	0.0004376	0.0011780
1.0	0.7318442	0.2190936	0.7314138	0.2202304	0.0004304	0.0011367

Table 2: Comparison between the exact solution and the present approximate solution for Example 2 $h_x = \frac{1}{10},\ \ h_t = \frac{1}{250}$

x	u_{exact}	v_{exact}	u_{approx}	v_{approx}	$\lvert u_{exact} - u_{approx} \rvert$	$\lvert v_{exact} - v_{approx} \rvert$
0	0.2505002	0.4995000	0.2503750	0.4995000	0.0001252	0
0.1	0.2631659	0.4870029	0.2630345	0.4869966	0.0001314	0.0000062
0.2	0.2761270	0.4745220	0.2759893	0.4745096	0.0001376	0.0000124
0.3	0.2893654	0.4620729	0.2892216	0.4620543	0.0001438	0.0000186
0.4	0.3028619	0.4496710	0.3027121	0.4496463	0.0001498	0.0000247
0.5	0.3165960	0.4373312	0.3164403	0.4373006	0.0001557	0.0000306
0.6	0.3305460	0.4250686	0.3303845	0.4250321	0.0001614	0.0000364
0.7	0.3446893	0.4128975	0.3445223	0.4128554	0.0001670	0.0000420
0.8	0.3590023	0.4008319	0.355883	0.4007843	0.0031193	0.0000475
0.9	0.3734611	0.3888853	0.3732837	0.3888326	0.0001774	0.0000527
1.0	0.3880408	0.3770707	0.3878586	0.3770131	0.0001822	0.0000576

Table 3: Comparison between the exact solution and the present approximate solution for Example 3 $h_x = \frac{1}{10},\ \ h_t = \frac{1}{250}$

x	u_{exact}	$D_t u + R_1(u,v) + N_1(u,v) = g_1$	u_{approx}	v_{approx}	$\lvert u_{exact} - u_{approx} \rvert$	$\lvert v_{exact} - v_{approx} \rvert$
0	-0.1668888	0.7644808	-0.1662222	0.7642263	0.0006667	0.0002545
0.1	-0.1781858	0.7924836	-0.1775912	0.7922841	0.0005946	0.0001994
0.2	-0.1898116	0.8170378	-0.1892995	0.8168948	0.0005121	0.0001430
0.3	-0.2017205	0.8377055	-0.2013003	0.8376193	0.0004202	0.0000862
0.4	-0.2138615	0.8541041	-0.2135409	0.8540741	0.0003205	0.0000299
0.5	-0.2261790	0.8659210	-0.2259644	0.8659461	0.0000214	0.0000250
0.6	-0.23861440	0.87292591	-0.23850990	0.87300422	0.00010440	0.00007831
0.7	-0.2511065	0.8749804	-0.2511143	0.8751098	0.0000077	0.0001294
0.8	-0.2635931	0.8720436	-0.2637129	0.8722215	0.0001197	0.0001779
0.9	-0.2760121	0.8641739	-0.2762415	0.8643976	0.0002294	0.0002237
1.0	-0.2883026	0.8515265	-0.2886723	0.8517932	0.0003346	0.0002666

Conclusion

In this research, LHPTM is applied in solving coupled system of linear and nonlinear partial differential equation. LHPTM is the combination of Laplace transformation and homotopy perturbation method. The method is applied directly without using linearization, discretization, perturbation or restrictive assumption in comparison with other existing methods. The approximate solutions were obtained by using the initial conditions only. The results obtained are compared with the exact solution and it revealed that LHPTM is a good method and has advantages over decomposition method. This is an effective method for solving linear and nonlinear differential equations.

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