FINITE DIFFERENCE SOLUTION FOR MAGNETOHYDRODYNAMICS THIN FILM FLOW OF A THIRD GRADE FLUID DOWN INCLINED PLANE WITH OHMIC HEATING.

BY

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Abstract.
This study investigates the numerical solution of Magnetohydrodynamic (MHD) thin film flow of a third grade fluid down inclined plane with heat transfer. This problem has been studied by Aiyesimi, Okedayo and Lawal [1] using the traditional and homotopy perturbation methods. In this present study, the governing non-linear ordinary differential equations are reduced to a system of non-linear algebra using implicit finite difference schemes and then solved using damped-Newton method. Also, the solutions obtain in [1] are only valid for the weak non-linearity. However the numerical solutions obtained in this study is valid for all values of the physical parameter and their effects on the velocity and temperature profile are presented graphically.

Keywords*: Third grade fluid, Magnetohydrodynamics flow, Thin film flow, Finite difference method, Ohmic heating.*

Introduction

Recently, [1] investigated viscous dissipation effect on the MHD thin film flow of a third grade fluid down inclined plane with ohmic heating. They solved the nonlinear equations governing the flow using traditional perturbation technique and homotopy perturbation method (HPM) whose solutions are valid only for weakly non-linearity. The disadvantage of traditional perturbation technique does not overcome by the HPM as shown in [2, 3]. Moreover, it is illustrated in [2, 3] that the HPM solutions are divergent for strong non-linearity when considering thin film flow of third grade fluid. Having noticed all these facts, it is required to apply a method which can solve strongly nonlinear equations. Finite difference method is very efficient in handling linear and nonlinear differential equations and it is widely used in all engineering discipline especially in mechanical and aerospace engineering. However, [4] reduced a system of nonlinear differential equations governing the flow and heat transfer of a third grade fluid past a porous vertical plate and then solved using implicit finite difference scheme.

These are few studies relating to the application of finite difference to strongly nonlinear fluid mechanics problems. Some of these can be seen in [5, 6] and references there in.

Governing Equation

The derivation of equations governing the flow and heat transfer of MHD thin film flow of third grade fluid is already given in [1]. Therefore, detailed derivation is omitted here. But their dimensionless form are directly taken from [1] which are given as

$$\frac{d^2u}{dy^2} + 6\beta\left(\frac{du}{dy}\right)^2 \frac{d^2u}{dy^2} - Mu + K = 0 \tag{1}$$

$$\frac{d^2T}{dy^2} + Br\left(\frac{du}{dy}\right)^2 + 2Br\beta\left(\frac{du}{dy}\right)^4 + BrMu^2 = 0 \qquad (2)$$

with the following boundary conditions

$$u = 0,\ T = 0 \text{ at } y = 0$$

$$\frac{du}{dy} = 0,\ T = 1 \text{ at } y = 0 \qquad (3)$$

where u and T are the velocity and temperature of the fluid respectively. β is the third grade parameter while K is the gravitational parameter, Br is the Brinkman number and M is the magnetic parameter. The values of these dimensionless parameter β, K, Br and M are constants.

Method of Solution

We attempt to compute a grid function consisting of values $u_0, u_1, \ldots u_m, u_{m+1}$ and $T_0, T_1, \ldots T_m, T_{m+1}$ where u_i and T_i are the approximation to the solutions $u(y_i)$ and $T(y_i)$ representing velocity and temperature of the fluid respectively. Here $y_i, T_i = ih$ and $h = 1/m+1$ is the mesh width, the distance between grid points. From the boundary conditions, we know that $u_0 = 0$, $T_0 = 0$ and $u_{m+1} = u_m$, $T_{m+1} = 1$. Therefore, we have m unknown values for $u_0, u_1, \ldots u_m, u_{m+1}$ and $T_0, T_1, \ldots T_m, T_{m+1}$ to compute. The centred difference approximation to the derivatives at nodes (ih), $i = 1, 2, \ldots\ldots, m+1$ are taken as

$$\frac{du}{dy} = \frac{u_{i+1} - u_{i-1}}{2h} \qquad 4)$$

$$\frac{dT}{dy} = \frac{T_{i+1} - T_{i-1}}{2h} \qquad (5)$$

$$\frac{d^2u}{dy^2} = \frac{u_{i+1} - 2u_i + u_{i-1}}{h^2} \qquad (6)$$

$$\frac{d^2T}{dy^2} = \frac{T_{i+1} - 2T_i + T_{i-1}}{h^2} \qquad (7)$$

Then equations (1) and (2) are discretised as

$$\frac{u_{i+1} - 2u_i + u_{i-1}}{h^2} + 6\beta\left(\frac{u_{i+1} - u_{i-1}}{2h}\right)^2 \frac{u_{i+1} - 2u_i + u_{i-1}}{h^2} - Mu_i + K = 0 \qquad (8)$$

$$\frac{T_{i+1} - 2T_i + T_{i-1}}{h^2} + Br\left(\frac{u_{i+1} - u_{i-1}}{2h}\right)^2 + 2Br\beta\left(\frac{u_{i+1} - u_{i-1}}{2h}\right)^4 + BrMu_i^2 = 0 \qquad (9)$$

The discretised form of boundary conditions for velocity and temperature obtained from equation (3) are

$$u_i = 0, \quad u_{m+1} = u_m \qquad i = 0, 1, 2, \ldots\ldots, m+1 \qquad (10)$$

$$T_i = 0, \quad T_{m+1} = 1 \qquad (11)$$

Since the first equation $(i=1)$ involves the values $u_0 = 0,\ T_0 = 0$ and the last equation $(i=m)$ involve the values $u_{m+1} = u_m$ and $T_{m+1} = 1$ respectively, the tridiagonal system of equation (4) with the boundary condition (6) is solved using Gaussian elimination method. The system of non-linear equation (5) with boundary conditions (7) is solved by Damped Newton method describes in [5] which is then solved using MAPLE because of the large output of results.

4. Results and Discussion

This study exhibits the fluid flow moving down with a velocity (u) in an inclined plane. The effects of magnetic parameter (M), elastic parameter (β) and Brinkman number (Br) on the velocity and temperature of the fluid are investigated respectively through simulation and results are produced as graphs. The numerical results for velocity profile obtained in this study when elastic parameter $\beta = 0.001$ and $K = 1$ are in good agreement with the results obtained in [1] are illustrated in Table 1.

Figure 1 shows the effect of M on velocity profile when $K = 1$ and $\beta = 0.001$. It is noticed that an increase in M decreases the velocity at any point of the fluid. This is because high magnetic filed reduce the boundary layer thickness thereby opposes the rate of fluid transport phenomenon.

Figure 2 and 3 shows the effect of viscoelastic parameter β on the velocity and temperature profile. It is observed that the velocity profile decreases as β (whether large or small value) increases when $K = 1,\ M = 5$ and $Br = 5$ while temperature profile remains steady. Therefore, results obtained in [1] are only valid for very small values of elastic parameter (i.e for $\beta < 0.1$) in the absence of magnetic field (i.e when $M = 0$). This shows that the perturbation results are valid only for small value of perturbation parameter as it is already known from the literature. But in the presence of magnetic field (i.e when $M > 0$), their results is correct as shown in this work whether the values of β is very small or large. The influence of Br on the temperature profile is shown in Figure 4. It depicts that temperature increases by an increase in Br when keeping magnetic and elastic parameter fixed.

5. Conclusion

In this work, the numerical solution for MHD flow of a third grade fluid down an inclined plane is studied using finite difference scheme. Damped Newton method is used to solve the system of nonlinear equations obtained by discretisation which is then solved using MAPLE. The numerical solutions obtained are valid for all values of viscoelastic parameters unlike perturbation method that are valid for small values of viscoelastic parameter. Also, the results are shown graphically and the influences of some parameters are discussed. It is observed that the magnetic parameter, viscoelastic parameter and Brinkman number can be used to control the boundary layer thickness and heat transfer respectively.

Table 1: Comparison between the present numerical solution with the solution obtained by Aiyesimi et al.[1] when $\beta = 0.001,\ K = 1$ and $M = 0, 5, 10, 15, 20$ for velocity profile.

$M=0$	Results by Aiyesimi et al. [1]		Present Numerical Results
y	$u(y)$	$v(y)$	$u(y)$
0	0	0	0
0.2	0.1743764	0.1773753	0.1797062
0.4	0.3075667	0.3075067	0.3196669
0.6	0.4091249	0.4091158	0.4195147
0.8	0.4670300	0.4671303	0.4795027
1	0.4793474	0.4793584	0.4795019
$M=5$			
0	-7.566E-11	2.65E-06	0
0.2	0.0700012	0.0698013	0.70019381
0.4	0.1136204	0.1132061	0.11362037
0.6	0.1396591	0.1390399	0.13965907
0.8	0.1534279	0.1526025	0.1534279
1	0.1577262	0.1567033	0.1577262
$M=10$			
0	2.31E-11	1.07E-07	0
0.2	0.0666270	0.0464011	0.0466269
0.4	0.0712103	0.0707764	0.0711884
0.6	0.0838367	0.0832102	0.0838366
0.8	0.0898013	0.0890134	0.0898013
1	0.0915487	0.0905231	0.0915486
$M=15$			
0	-7.622E-12	1.15E-08	0
0.2	0.0358902	0.0357100	0.0358902
0.4	0.0523752	0.0520000	0.0523755
0.6	0.0598101	0.0592018	0.0598479
0.8	0.0630209	0.0622010	0.0630209
1	0.0638949	0.0628731	0.0638948
$M=20$			
0	5.13E-12	1.95E-09	0
0.2	0.0295010	0.0293010	0.0295440
0.4	0.0416042	0.0415878	0.0416041
0.6	0.0465018	0.0458659	0.0464879
0.8	0.0484000	0.0475012	0.0483696
1	0.0489001	0.0478310	0.0488578

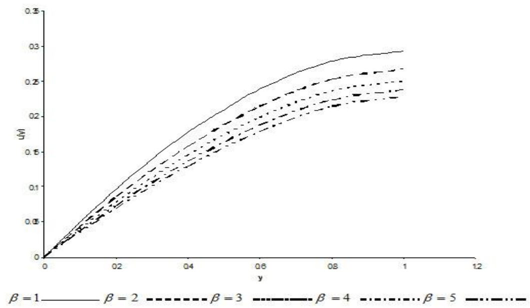

Figure 1: Velocity profile for different values of Magnetic parameter M when $K = 1$ and $\beta = 0.001$

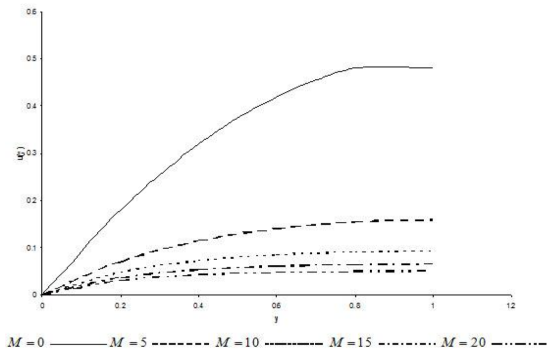

Figure 2: Velocity profile for different values of viscoelastic parameter β when $K = 1$ and $M = 1$

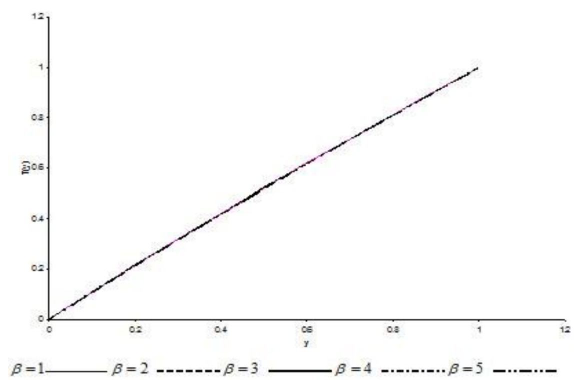

Figure 3: Temperature profile for different values of viscoelastic parameter β when $K = 1$, $M = 1$ and $Br = 1$

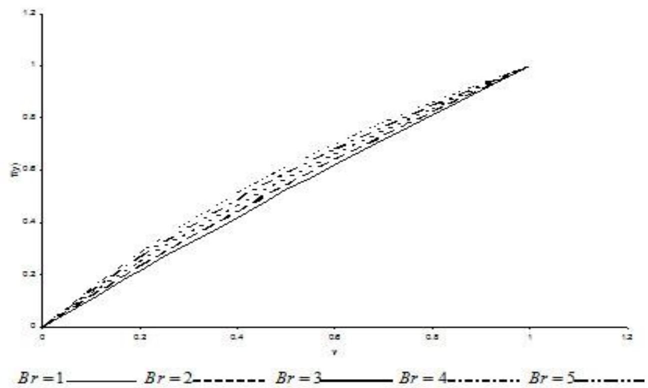

Figure 4: Temperature profile for different values of Brinkman number Br when $\beta = 1$, $K = 1$ and $M = 1$

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