RUN-UP FLOW OF AN ELECTRICALLY CONDUCTING FLUID IN THE PRESENCE OF TRANSVERSE MAGNETIC FIELD IN ANNULUS

by

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Abstract

In this paper, the run-up flow of an incompressible, viscous electrically conducting fluid in an annulus formed by two infinitely long concentric cylinders is considered. The flow is initially induced by a constant pressure gradient parallel to the direction of the fluid flow, when steady state is attained, the pressure gradient is suddenly withdrawn while the inner cylinder is impulsively started in the direction of the fluid flow and the outer cylinder continues to be at rest. For that, the governing equations are simplified by using reasonable dimensionless parameters. Laplace transform technique is then employed to obtain the solutions of the velocity in Laplace domain. Consequently, Riemann-sum approximation is then used to invert the obtained solutions into the corresponding time domain, and the skin friction on both cylinders are calculated. More so, expressions are obtained for the velocities for both cases of the applied magnetic field being fixed relative to either the fluid or the moving inner cylinder. As a result, the influence of the various parameters such as the Hartmann number, pressure gradient on the velocity and skin friction is discussed by the use of graphs. It is found that the fluid velocity increases with increase in time t and subsequently decreases. It is also discovered that the fluid velocity increases with the increase in pressure gradient G. In addition, a reversal of flow is observed when the orientation of the pressure gradient is reversed. The fluid velocity is seen to decrease with the increase in M when the magnetic lines of force are fixed relative to the fluid and increases when fixed relative to the moving inner cylinder. The results provided useful information to engineers to improve efficiency and performance of machines.

Keywords: *Run-up flow, MHD, Couette flow, Poissuielle flow, Annulus, Pressure gradient, Riemann-sum, Transverse magnetic field, and Impulsive motion*.

NOMENCLATURE

a Radius of the inner cylinder
b Radius of the outer cylinder
c Speed of light (m/s)
h Width of the plates
B_0 Constant uniform magnetic field
$\vec{B}_0$ Magnetic flux
$\vec{B}$ Magnetic field
$\vec{J}$ Current density
$\vec{E}$ Electric field
$\vec{F}$ Lorentz force
$\vec{f}$ Body force
I_0 Zero-order modified Bessel function of the first kind
K_0 Zero-order modified Bessel function of the second kind
I_1 First-order modified Bessel function of the first kind
K_1 First-order modified Bessel function of the second kind
K Dimensionless interaction coefficient

s Laplace parameter
M Hartmann number
t' Dimensional time
t Dimensionless time
W_0 Initial velocity
W Constant
W' Dimensionless velocity component of vector field $\vec{V}$
u',v', w' Components of velocity field $\vec{V}$
u, v, w Dimensionless velocity components
r' Dimensionless radial velocity
θ' Dimensionless angular velocity
z' Dimensionless axial velocity
p' Dimensional pressure
p Dimensionless pressure

GREEK
λ Annulus dimension ratio
ρ Mass density of the fluid (kg/m^3)
σ Electrical conductivity
τ Skin friction
υ Kinematic viscosity (m^2/s)
μ Dynamic viscosity (N/m^2s)
∇ Laplacian operator

Introduction
The first serious theoretical and experimental work on liquid metal MHD flows was carried out by Hartmann and Lazarus [5]. These flows received much attention from theoreticians because the equations are linear but the phenomena are neither trivially simple nor physically unattainable in the laboratory. Annular geometry is widely employed in the gas cooled nuclear reactors whereby cylindrical fissionable fuel elements are placed axially in vertical coolants channel parallel to the fuel element and also in drilling operations of oil and gas well. Makinde *et al.* [16] studied Magneto hydrodynamic viscous flow in a porous medium cylindrical annulus with an applied radial magnetic field. Jha and Apere [7] studied the unsteady MHD Couette flows in annuli: Riemann-sum Approximation Approach. It was found that; Hartmann number M has a decreasing effect on the fluid velocity when the magnetic field is fixed relative to the fluid. An increase in velocity with the increase in M is observed to occur when the magnetic field is fixed relative to the moving outer cylinder.

The pressure gradient force is responsible for triggering the initial movement of fluid. The fluid flow experiences phenomenon termed run-up flow which arises due to sudden withdrawal of the pressure gradient causing the fluid flow while one of its boundaries instantaneously move from rest. Under this phenomenon, steady flow in the unperturbed state gains unsteadiness later. This work studies the run-up flow of an electrically conducting fluid between annulus formed by two concentric cylinders in the presence of transverse magnetic field. The fluid motion is initially due to pressure gradient G acting in the same direction as the motion of the fluid. As the steady state is attained, the pressure gradient is suddenly withdrawn and the inner cylinder is set into motion instantaneously with a constant velocity in the direction of the applied pressure gradient. The problem is solved using Laplace transform technique. The solution from the Laplace domain is inverted to the time domain using *Riemann*-sum approximation method of Laplace inversion. The effects of pressure gradient and Hartmann number on the flow are investigated.

The run-up flow of viscous incompressible fluid between solid boundaries is an active area of study by researchers in recent years due to its potential applications in several branches of technology related to petroleum industry, chemical and bio-chemical industry, ceramic industry, paper technology, extraction of energy from geo-thermal regions, lubrication etc. Several investigations have been conducted in the field of run-up flow. However, to cite few works in this direction, Kazakia and Rivlin [10] started the study of run-up and spin-up flows. Rivlin [22, 23] later investigated the run-up and spin-up flows of viscoelastic fluids between rigid parallel plates and circular geometries. Run-up flow in a generalized porous medium is studied by Ramacharyulu and Raju [20]. Run-up flow of a couple stress fluids between parallel plates is discussed by Devaka and Iyengar [3]. They observed that there is a critical interval of the couple stress parameter, wherein as the parameter increases the velocity increases; outside this critical interval, for the range of values of the parameter taken, as it increases, the velocity decreases. A run-up flow of an incompressible micropolar fluid between parallel plates - A state space approach was discussed by Devaka and Iyengar [4]. It was found that the microrotation components are symmetric about $y = 0$ when the two plates move with the equal velocity. As the pressure gradient is increased, there is an increase in velocity. Qadri and Krishna [18] studied run-up flow of a Maxwell fluid through a parallel plate channel. It was found that the velocity increases with the increase, however in both Reynolds`s number and Maxwell`s fluid parameter and reduces with the increasing pressure gradient. MHD run-up flow of a Maxwell fluid through a porous medium in a parallel plate channel is investigated by Qadri and Krishna [19]. Reddy [21] studied numerical solution of run-up flow through a rectangular pipe. He found that at large Reynolds's number, the velocity distribution along the symmetric line $y = 0$ takes the form of a damping wave. At the initial stages, the velocity at any nodal point decreases with time in an uneven fashion. The run-up introduces local maxima and minima in the region. At small Reynolds`s number, the velocity distribution in the region $x = 0{:}2$, to $x = 0{:}8$ is almost constant for large time t.

Hussain and Ramacharyulu [6] considered an unsteady viscous incompressible flow in a porous medium between two impermeable parallel plates impulsively stopped from a relative motion. It is noticed that the effect of porosity on the velocity profiles is of flatten type with the velocity attaining maximum near the middle of the plates and due to friction, it decrease towards the plates. Hall effects on run-up flow of Rivlin-Ericksen fluid in a parallel channel bounded by porous bed on the lower half is investigated by Krishna and Qadri [12]. It is discovered that, the resultant velocity in the clean fluid region reduces with increasing Hartmann number M, inverse Darcy number D^1, or ratio of the viscosities, and enhances with visco-elastic parameter S irrespective of the thickness of the porous bed. Run-up Flow of Oldroyd-B Fluid through a Parallel plate channel is another research work by Krishna and Qadri [12]. It is found that: The magnitude of the velocity enhances with increasing Reynolds number, both material parameters as well as time. When pressure increases the velocity diminishes throughout the fluid region. Both the stresses rise with Reynolds number. Mass flux reduces with pressure and develops with Reynolds number, material parameters and time.

In all these analyses, none was carried out taking account the effect of electric conduction of the fluid when transverse magnetic field is acting normal to the direction of the fluid flow, a *Riemann*-sum approach. This paper, intends to examine the unsteady run-up flow of an electrically conducting fluid in the presence transverse magnetic field in annulus. The effect of pressure gradient, Hartmann number, and time are discussed with the aid of line graphs.

Formulation of the problem

Suppose the flow of a viscous, incompressible, electrically conducting fluid trapped in the annular gap between two infinite rigid non-conducting horizontal cylinders $r = 1\, and\, r = \lambda$ along the direction in the z axis in the presence of transverse magnetic field is considered. At $t^{'}$, the two cylinders are at rest and the constant pressure gradient G applied parallel to the flow set the fluid in motion. When the flow fully developed, at $t^{'} > 0$ the inner cylinder is set into motion with a uniform velocity Ut'^{n} in the direction of the applied pressure gradient and the outer cylinder is still at rest. Assuming the magnetic Reynolds's number is very small which corresponds to negative negligible induced magnetic field and the Hall effects of MHD are negligible. No applied polarization voltage exists ($\vec{E} = 0$), that is to say no energy is added or removed in the system, Sutton and Sherman [24]. The uniform magnetic field $\vec{B_r} \equiv (B_0, 0,0)$ since the magnetic field is along r-direction, $\vec{B}_\theta = \vec{B}_z = 0$ which the total magnetic field acting perpendicular to the direction of the flow. Using polar coordinate system (r', θ', z') with $V = (v', u', w')$ acting on each of the axes respectively. For unidirectional flow, $u = v = 0$, we take the velocity $w = (0,0, w(r,t))$ which satisfies the continuity equation.

MHD Initial state for run-up flow

In the initial state, consider the steady flow of a viscous, incompressible, electrically conducting fluid in annulus in the presence of transverse magnetic field under the influence of constant pressure gradient. Using the assumptions above for a cylindrical geometry, the equation of motion with electromagnetic force added for an incompressible electrically conducting fluid is:

$$\frac{\partial w\prime}{\partial t\prime} = -\frac{1}{\rho}\frac{\partial p\prime}{\partial z\prime} + \nu\left(\frac{\partial^2 w\prime}{\partial r'^2} + \frac{1}{r\prime}\frac{\partial w\prime}{\partial r\prime}\right) + \frac{F_z}{\rho} w' \quad (1)$$

Where F_z is the component of the magnetic force in the direction of z axis, the fluid.

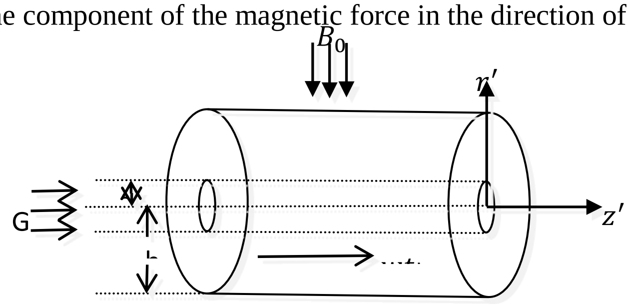

Figure 1(a) Schematic design of the

Now,

$$F_z = \sigma[w'i \times B_0 j] \times B_0 j \quad (2)$$

So that we have

$$\frac{F_z}{\rho} = \frac{\sigma B_0^2 w'}{\rho} \quad (3)$$

$$\frac{\partial w'}{\partial t'} = -\frac{1}{\rho}\frac{\partial p\prime}{\partial z\prime} + \upsilon\left(\frac{\partial^2 w'}{\partial r'^2} + \frac{1}{r\prime}\frac{\partial w}{\partial r'}\right) - \frac{B_0^2\sigma}{\rho}w \quad (4)$$

We define the dimensionless variables

$$z = \frac{z\prime}{a}, w = \frac{w'}{W'}, t = \frac{\nu t'}{a^2}, r = \frac{r'}{a} \quad M^2 = \frac{\sigma B_0^2 a^2}{\rho\nu}, \lambda = \frac{b}{a}, \quad p = \frac{a^2 p\prime}{\rho\nu^2} \quad (5)$$

Subject to the initial and boundary conditions;

$$t' \le 0 : w = w(r,0)\ for\ a \le r' \le b$$

$$t' > 0 ;\ w' = \begin{cases} w' = W, at\ r' = a \\ w' = 0,\ at\ r' = b \end{cases} \quad (6)$$

Since the motion is steady,

$\frac{\partial w}{\partial t} = 0$ and $-\frac{\partial p}{\partial z} = G$,

Equation (4) takes the dimensionless form; (7)

Equation (7) is solved subject to the boundary conditions;

$$t \le 0 : w = w(r,0)\ for\ a \le r \le \lambda$$

$$t > 0 ;\ w = \begin{cases} w = W,\ at\ r = 1 \\ w = 0,\ at\ r = \lambda \end{cases} \quad (8)$$

$$w(r) = C_1 I_0(Mr) + C_2 K_0(Mr) + \frac{G}{M^2} \quad (9)$$

$$C_1 = \frac{G}{M^2}\left[\frac{K_0(M) - K_0(M\lambda)}{K_0(M\lambda)I_0 - I_0(M\lambda)K_0(M)}\right]$$

$$C_2 = \frac{G}{M^2}\left[\frac{I(M\lambda) - I_0(M)}{K_0(M\lambda)I_0 - I_0(M\lambda)K_0(M)}\right]$$

I_0 and K_0 are the zero-order modified Bessel functions of the first and second kind respectively.

MHD Run-up flow

In this state, the two cylinders are hitherto stationary, if the inner cylinder is impulsively set into motion along the direction of the applied pressure gradient with a constant velocity W_0 while the applied pressure gradient in instantaneously withdrawn. As the resultant flow is time dependent,

we assume that

$w = w(r,t)$ and $-\frac{\partial p}{\partial z} = 0$

$$\frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} - M^2 W \quad (10)$$

Equation (10) is valid if the magnetic field is fixed relative to the moving fluid. If however, the magnetic field is fixed relative to the inner cylinder, then, equation (9) becomes;

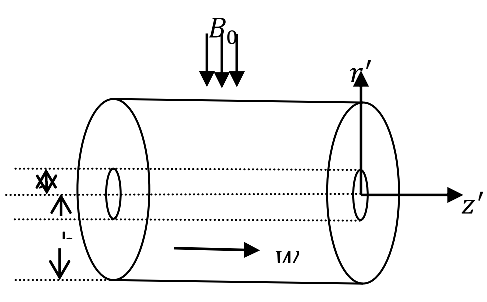

Figure2 (b) Schematic design of

$\frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} - M^2(w - KW_0)$
(11)
Where

$$K = \begin{cases} 0 \ \ when\ \ B_0\ is\ fixed\ relative\ to\ the\ fluid \\ 1 \ \ when\ \ B_0\ is\ fixed\ relative\ to\ the\ \ cylinder \end{cases}$$

Combining (10) and (11), we get

$$\frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} - M^2(w - KW_0) \qquad (12)$$

Equation (12) is solved, subject to the initial and boundary conditions;

$t \le 0 : w = w(r, 0)\ for\ a \le r \le \lambda$

$$t > 0\ ;\ w = \begin{cases} w = W_0, & at\ \ r = 1 \\ w = 0, & at\ r = \lambda \end{cases} \qquad (13)$$

Solution of the problem

Applying Laplace transform on equation (12), gives;

$$\frac{\partial^2 \overline{w}}{\partial r^2} + \frac{1}{r}\frac{\partial \overline{w}}{\partial r} - (M^2 + s)\overline{w} = -w(r, 0) - \frac{M^2 KW_0}{s}$$

OR

$$\frac{\partial^2 \overline{w}}{\partial r^2} + \frac{1}{r}\frac{\partial \overline{w}}{\partial r} - (M^2 + s)\overline{w} = -C_1 K_0(Mr) - \frac{G}{M^2} - \frac{M^2 KW_0}{s} \qquad (14)$$

Where

$$L[w(r,t)] = \overline{w}(r,s) = \int_0^\infty w(r,t)\exp(-st)\,dt, \qquad s > 0 \qquad (15)$$

The initial and boundary conditions (13) in Laplace domain are:

$t \le 0 : w = w(r, 0)$ for $a \le r \le \lambda$

$$t > 0 : \begin{cases} \overline{w}(r,s) = \frac{w_0}{s}, & at\ \ r = 1 \\ \overline{w}(r,s) = 0, & at\ r = \lambda \end{cases} \qquad (16)$$

Equation (14) subject to the initial and boundary conditions (16) have the following solution;

$$\overline{w}(r,s) = C_3 I_0(\delta r) + C_4 K_0(\delta r) + \frac{M^2 KW_0}{(M^2+s)s} + \left[\frac{C_1 I_0(Mr) + C_2 K_0(Mr)}{s}\right] + \frac{G}{(M^2+s)M^2} \qquad (17)$$

Where

$\mathrm{C}_1 = -\frac{\mathrm{G}}{\mathrm{M}^2}\left[\frac{K_0(M) - K_0(M\lambda)}{K_0(M\lambda)I_0(M) - I_0(M\lambda)K_0(M)}\right]$ $\mathrm{C}_2 = \frac{\mathrm{G}}{\mathrm{M}^2}\left[\frac{I_0(M\lambda) - I_0(M)}{K_0(M\lambda)I_0(M) - I_0(M\lambda)K_0(M)}\right]$

$C_3 = \frac{A_1[K_0(\lambda\delta) - K_0(\delta)]}{d_1} - \frac{W_0 K_0(\lambda\delta)}{sd_1}$

$C_4 = \frac{A_1[I_0(\delta) - I_0(\lambda\delta)]}{d_1} + \frac{W_0 I_0(\lambda\delta)}{sd_1}$

$\delta = \sqrt{M^2 + s}$

Riemann-sum approximation

Equation (17) is to be inverted so as to determine the velocity in the time domain. It is very difficult to invert this equation in closed form; applying a numerical procedure used in Jha and Apere [11] which is based on the Riemann-sum approximation. In this method, any function in the s domain can be inverted in the time domain as follows:

$$w(r,t) = \frac{e^{st}}{t}\left[\frac{1}{2}\overline{w}(r,t) + Re\sum_{n=1}^{N}\overline{w}\left(r, t + \frac{in\pi}{t}\right)(-1)^n\right] \qquad (18)$$

Where Re refers to the real part, $i = \sqrt{-1}$ is the imaginary number, N is the number of terms used in the Riemann-sum approximation and t is the real part of the Bromwich contour

that is used in inverting the Laplace transforms. The Riemann-sum for the Laplace inversion involves a single summation for the numerical process. Its accuracy depends on the value of t and the truncation error dictated by N. According to Jha and Apere [11], the value of t must be selected so that the Bromwich contour encloses all the branch points . For faster convergence, $\epsilon t = 4.7$ gives the most satisfactory results.
The skin friction coefficient τ which is the frictional force between the fluid and the boundaries is obtained by differentiating equation (17) with respect to r.

$$\tau = \frac{\partial w}{\partial r}\bigg|_{r=1,\lambda} = \frac{e^{\epsilon i}}{t}\left[\frac{1}{2} z(r,t) + Re \sum_{k=1}^{N} z\left(r, t + \frac{ik\pi}{t}\right)(-1)^k\right] \quad (19)$$

Where

$$z = \delta[C_3 I_1(\delta r) - C_4 K_1(\delta r)] + \frac{M}{s}[C_1 I_1(Mr) - C_2 K_1(Mr)] \quad (20)$$

The skin frictions on the inner and outer cylinders ($r = 1$) and ($r = \lambda$) respectively from equation (16);

$$\tau_1 = \frac{e^{st}}{t}\left[\frac{1}{2} z(1,t) + Re \sum_{k=1}^{N} z\left(1, t + \frac{ik\pi}{t}\right)(-1)^k\right] \quad (21)$$

I.e.

$$\tau_1 = \delta[C_3 I_1(\delta) - C_4 K_1(\delta)] + \frac{M}{s}[C_1 I_1(M) - C_2 K_1(M)] \quad (22)$$

And

$$\tau_\lambda = \frac{e^{st}}{t}\left[\frac{1}{2} z(\lambda,t) + Re \sum_{k=1}^{N} z\left(\lambda, t + \frac{ik\pi}{t}\right)(-1)^k\right] \quad (23)$$

I.e.

$$\tau_\lambda = \delta[C_3 I_1(\delta\lambda) - C_4 K_1(\delta\lambda)] + \frac{M}{s}[C_1 I_1(M\lambda) - C_2 K_1(M\lambda)] \quad (24)$$

Results and Discussion

To examine the effects of the various parameters on the fluid motion, MATLAB programme is adopted to generate graphs for velocity and skin frictions, and are presented in Figures. 3 - 8. Fig. 3 shows the velocity (w) profiles against the radius r for different values of t which depicts the effects of time t on the velocity. It is observed that the velocity increases with the increase in t gradient when the magnetic lines of force are fixed relative to the fluid ($K = 0$) or when fixed relative to the inner cylinder ($K = 1$). Figs.4 and 5 show the effect of pressure gradient G on the velocity. As G increases, the velocity increases for both ($K = 0$) and ($K = 1$). It is interesting to see that a reversal of flow occurs for $G = -2.5$ when $K = 0$ and $G = -3.75$ when ($K = 1$). Fig. 5 depicts the effect of Hartmann number M on the velocity. The velocity is observed to decrease with the increase in M for ($K = 0$) as shown in fig.5 (a) and increases with increase in M for ($K = 1$) as observed in fig. 5(b). The effect of skin friction on the surface of the two cylinders on time t for different M is presented in figures 7 and 8. It is observed that M has a decreasing effect on τ when ($K = 0$) and increasing when ($K = 1$) on both cylinders.

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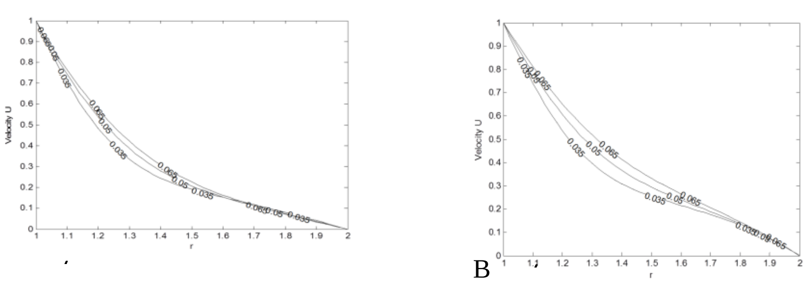
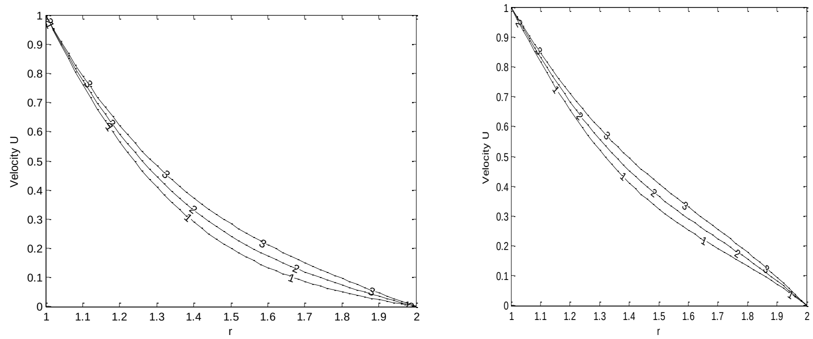


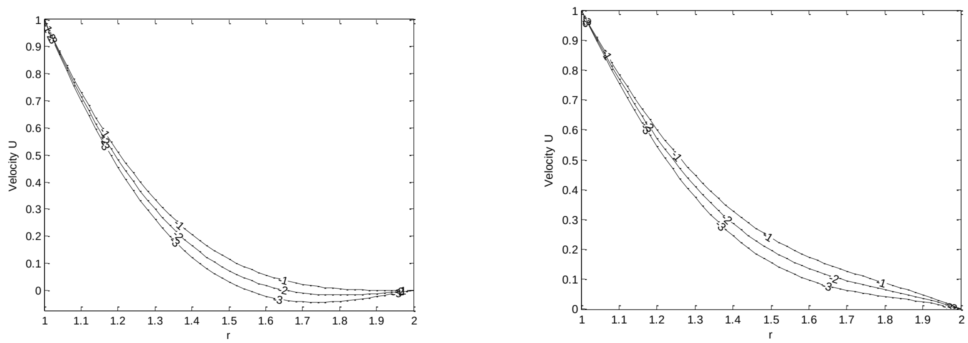
A B

Fig.3: Velocity profile for different values of t with K=0 and K=1 represented by (a) and (b) respectively with $W_0 = 1.0, G = 2.0, M = 2.0$



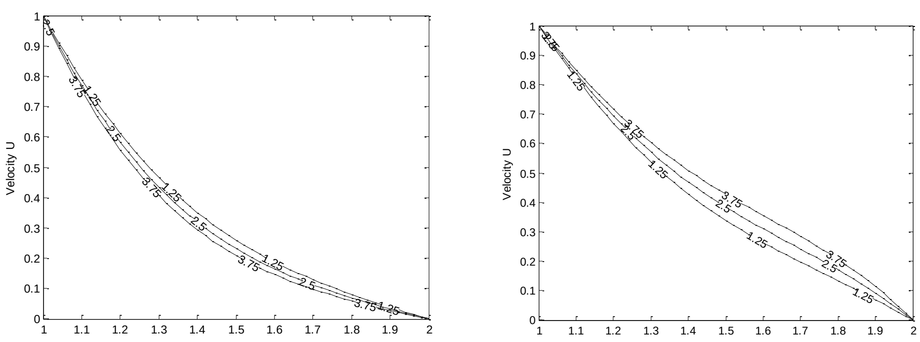
A B

ig.4: Velocity profile for different values of G with K=0 and K=1 represented by (a) and (b) respectively with $W_0 = 1.0, t = 0.08, M = 2.0$



A B

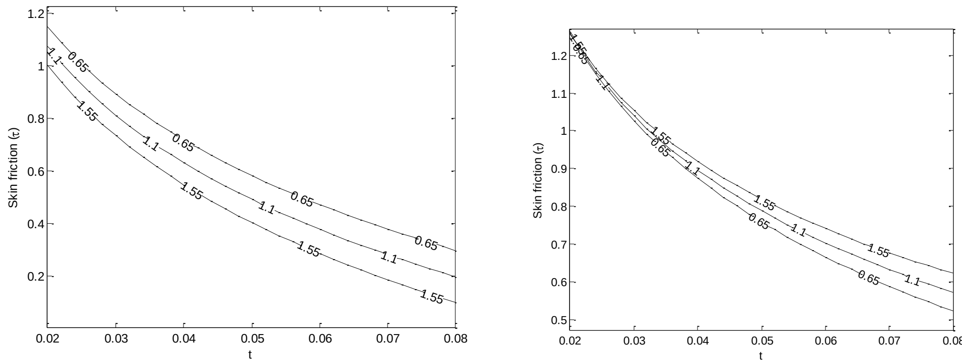
Fig.5: Velocity profile for different values of (-G) with K=0 and K=1 represented by (a) and (b) respectively with $W_0 = 1.0, t = 0.08, M = 2.0$



A

B

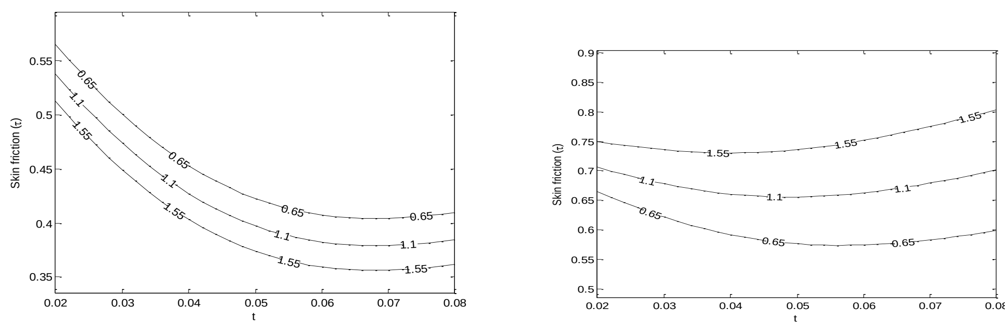
Fig.6: Velocity profile for different values of M with K=0 and K=1 represented by (a) and (b) respectively with $W_0 = 1.0, t = 0.08, G = 2.0$



A

B

Fig.7: Variation of the skin friction with time at the outer surface of the inner cylinder ($r = 1$) for different values of M with $K = 0$ and $K = 1$ represented by (a) and (b) respectively.



A

B

Fig.8: Variation of the skin friction with time at the inner surface of the outer cylinder ($r = 2$) for different values of M with $K = 0$ and $K = 1$ represented by (a) and (b) respectively.

Summary and Conclusion

This paper investigates the run-up flow of an electrically conducting fluid in the presence of trans-verse magnetic field taking account the effect of pressure gradient in annulus. A solution is presented for velocity and dimensionless skin frictions. The influence of each governing parameters on the fluid flow is discussed with aid of line graphs. The parameters in this work are pressure gradient (G) and Hartmann number (M). It is observed that the velocity increases with the increase in time and later fades away with time. As the pressure gradient G increases, the velocity increases. The velocity decreases with the increase in Hartmann number M when magnetic lines of force are fixed relative to the fluid ($K = 0$) and increases when fixed relative to the moving inner cylinder ($K = 1$). The skin friction τ is observed to be decreasing as a result in the increase in Hartmann number M when magnetic lines of force are fixed relative to either fluid ($K = 0$) or to the cylinder, and increase in τ is observed for $K = 1$ on both cylinders $r = 1$ and $r = 2$.

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