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AN INTRODUCTION OF THE CONCEPT OF N-LEVEL SOFT SET

by

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Abstract

The theory of soft sets and soft multisets are considered as useful tools for modeling uncertainty. In this paper, the concept of n-level soft set is introduced together with some of its properties. Both the first and the second decomposition theorems were established and proved.

Keywords*: Soft set, Multiset, Soft multiset, n-level Soft set*

1. Introduction

The theory of Soft set which was initiated with the aim of modeling uncertainty in real life situation has many applications in areas of decision making, medical diagnosis, data analysis, forecasting, game theory etc. as presented in [1, 2, 3, 4].

By violating a basic underlying set condition, the concept of multiset (mset, for short) which is an unordered collection of objects where multiples of objects are admitted was initiated with the aim of addressing repetition which is significant in real life situations. For a comprehensive account of the idea of multiset and its applications refer to [5, 6, 7, 8, 9].

Soft multiset which is a mapping from a set of parameters to the power set of a universal multiset was studied in different ways as can seen in [10, 11, 12, 13]. However, as multisets are generalization of sets [14], the idea of [13] serves as a generalization of soft sets.

The concept of n-level sets was introduced in [15] and studied by [16] together with some of their properties. In this paper the concept of n-level soft set is introduced and some related results were obtained.

2.1 Soft set

Definition 2.1.1 [17]

Let U be an initial universe set and E a set of parameters or attributes with respect to U. Let $P(U)$ denote the power set of U and $A \subseteq E$. A pair (F, A) is called a *soft set* over U, where F is a mapping given by $F: A \longrightarrow P(U)$.

In other words, a soft set (F, A) over U is a parameterized family of subsets of U. For $e \in A$, $F(A)$ may be considered as the set of e-elements or e-approximate elements of the soft set (F, A). Thus (F, A) is defined as

$$(F, A) = \{F(e) \in P(U): e \in E, F(e) = \emptyset \text{ if } e \notin A\}.$$

Definition 2.1.2 [4]

Let (F, A) and (G, B) be two soft sets over a common universe U, we say that

(a) (F,A) is a **soft subset** of (G,B), denoted $(F,A) \tilde{\subseteq} (G,B)$, if

- (i) $A \subseteq B$, and
- (ii) $\forall\ e \in A, F(e) \tilde{\subseteq} G(e)$.

(b) (F, A) is **soft equal** to (G, B), denoted $(F, A) = (G, B)$, if $(F,A) \tilde{\subseteq} (G,B)$ and $(G,B) \tilde{\subseteq} (F,A)$.

Definitions 2.1.3 [4, 18]

Let (F,A) and (G,B) be two soft sets over a common universe U.

(i) The **union** of (F,A) and (G,B), denoted $(F,A)\tilde{\cup}(G,B)$, is a soft set (H,C) where $C = A\cup B$ and $\forall\ e\in C$,

$$H(e)=\begin{cases} F(e), & e\in A-B \\ G(e), & e\in B-A \\ F(e)\cup G(e), & e\in A\cap B. \end{cases}$$

(ii) The **extended intersection** of (F,A) and (G,B) , denoted $(F,A)\tilde{\cap}(G,B)$, is a soft set (H,C) where $C = A\cup B$ and $\forall\ e\in C$,

$$H(e)=\begin{cases} F(e), \text{ if } e\in A-B \\ G(e), \text{ if } e\in B-A \\ F(e)\cap G(e), \text{ if } e\in A\cap B. \end{cases}$$

(iii) The **restricted intersection** of (F,A) and (G,B), denoted $(F,A)\cap_R(G,B)$, is a soft set (H,C) where $C = A\cap B$ and $\forall\ e\in C,\ H(e)=F(e)\cap G(e)$. If $A\cap B=\phi$ then $(F,A)\cap_R(G,B)=\tilde{\Phi}_\phi$.

(iv) The **restricted union** of (F,A) and (G,B), denoted $(F,A)\cup_R(G,B)$, is a soft set (H,C) where $C = A\cap B$ and $\forall\ e\in C$, $H(e)=F(e)\cup G(e)$. If $A\cap B=\phi$ then $(F,A)\cup_R(G,B)=\tilde{\Phi}_\phi$.

2.2 Multisets

Definition 2.2.1 [19]

An mset M drawn from the set X is represented by a function *Count* M or C_M defined as $C_M: X \longrightarrow \mathbb{N}$.

Let M be a multiset from X with x appearing n times in M. It is denoted by $x \in^n M$. $M = \{k_1/x_1, k_2/x_2, \dots, k_n/x_n\}$ where M is a multiset with x_1 appearing k_1 times, x_2 appearing k_2 times and so on.

Let M and N be two msets drawn from a set X. Then

$M \subseteq N$ iff $C_M(x) \le C_N(x)$ for all $x \in X$.

$M = N$ if $C_M(x) = C_N(x)$ for all $x \in X$.

$M \cup N = max\{C_M(x), C_N(x)\}$ for all $x \in X$.

$M \cap N = min\{C_M(x), C_N(x)\}$ for all $x \in X$.

$M - N = max\{C_M(x) - C_N(x), 0\}$ for all $x \in X$.

Definition 2.2.2 [20]

Let M be a multiset drawn from a set X. The support set of M denoted by M^* is a subset of X given by $M^* = \{x \in X: C_M(x) > 0\}$. Note that $M \subseteq N$ iff $M^* \subseteq N^*$.

The power multiset of a given mset M, denoted by $P(M)$ is the multiset of all submultisets of M, and the power set of a multiset M is the support set of $P(M)$, denoted by $P^*(M)$.

Example 2.2.1 Let $M = \{2/x, 2/y\}$, then $M^* = \{x, y\}$ and $P(M) = \{\emptyset, 2/\{1/x\}, 2/\{1/y\}, \{2/x\}, \{2/y\}, 4/\{1/x, 1/y\}, 2/\{2/x, 1/y\}, 2/\{1/x, 2/y\}, \{2/x, 2/y\}$. Moreover, $P^*(M) = \{\emptyset, \{1/x\}, \{1/y\}, \{2/x\}, \{2/y\}, \{1/x, 1/y\}, \{2/x, 1/y\}, \{1/x, 2/y\}, \{2/x, 2/y\}$.

Definition 2.2.4 [20]

Let $\{M_i : i \in I\}$ be a nonempty family of msets drawn from a set X. Then

(i) Their Intersection, denoted by $\bigcap_{i\in I} M_i$ is defined as

$$C_{\bigcap_{i\in I} M_i}(x) = \bigwedge_{i\in I} C_{M_i}(x), \forall x \in X,$$

where $\wedge$ is the minimum operation.

(ii) Their Union, denoted by $\bigcup_{i\in I} M_i$ is defined as

$$C_{\bigcup_{i\in I} M_i}(x) = \bigvee_{i\in I} C_{M_i}(x), \forall x \in X,$$

where $\vee$ is the maximum operation.

2.3 Soft Multiset

Definition 2.3.1 [13]

Let U be a universal multiset, E be a set of parameters and $A \subseteq E$. Then a pair (F, A) is called a soft multiset where F is a mapping given by $F : A \longrightarrow P^*(U)$. For all $e \in A$, the mset $F(e)$ is represented by a count function $C_{F(e)}: U^* \longrightarrow \mathbb{N}$.

Example 2.3.1 Let the universal mset $U = \{3/w, 2/x, 4/y, 1/z\}$, the parameter set $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$, $A = \{e_1, e_2, e_3\}$ and the mapping $F : A \longrightarrow P^*(U)$ be defined as $F(e_1) = \{2/w, 1/y, 1/z\}, F(e_2) = \{1/w, 2/x, 3/y\}$ and $F(e_3) = \{1/x, 2/y\}$. That is, (F, A) is a soft multiset such that for all $e \in A$, the multiset $F(e)$ is represented by a count function $C_{F(e)}: U^* \longrightarrow \mathbb{N}$ as

$$C_{F(e_1)}(w) = 2, \quad C_{F(e_1)}(x) = 0, \quad C_{F(e_1)}(y) = 1, \quad C_{F(e_1)}(z) = 1$$
$$C_{F(e_2)}(w) = 1, \quad C_{F(e_2)}(x) = 2, \quad C_{F(e_2)}(y) = 3, \quad C_{F(e_2)}(z) = 0$$
$$C_{F(e_3)}(w) = 0, \quad C_{F(e_3)}(x) = 1, \quad C_{F(e_3)}(y) = 2, \quad C_{F(e_3)}(z) = 0$$

Thus, $(F, A) = \{ (e_1, \{2/w, 1/y, 1/z\}), (e_2, \{1/w, 2/x, 3/y\}), (e_3, \{1/x, 2/y\})\}$.

Definition 2.3.3 [13]

Let (F, A) and (G, B) be two soft multisets over U. Then

(a) (F, A) is a soft submultiset of (G, B) written $(F, A) \sqsubseteq (G, B)$ if

i. $A \subseteq B$

ii. $C_{F(e)}(x) \leq C_{G(e)}(x), \forall x \in U^*, \forall e \in A$.

(b)$(F, A) = (G, B) \Leftrightarrow (F, A) \sqsubseteq (G, B)$ and $(G, B) \sqsubseteq (F, A)$.

Also, if $(F, A) \sqsubset (G, B)$ and $(F, A) \neq (G, B)$ then (F, A) is called a proper soft submset of (G, B) and (F, A) is a whole soft submset of (G, B) if $C_{F(e)}(x) = C_{G(e)}(x), \forall x \in F(e)$.

(c) **Union:**

$(F, A) \sqcup (G, B) = (H, C)$ where $C = A \cup B$ and $C_{H(e)}(x) = max\{C_{F(e)}(x), C_{G(e)}(x)\}, \forall e \in C, \forall x \in U^*$.

(d) **Intersection:**

$(F, A) \sqcap (G, B) = (H, C)$ where $C = A \cap B$ and $C_{H(e)}(x) = min\{C_{F(e)}(x), C_{G(e)}(x)\}, \forall e \in C, \forall x \in U^*$.

(e) **Difference:**

$(F, E) \backslash (G, E) = (H, E)$ where $C_{H(e)}(x) = max\{C_{F(e)}(x) - C_{G(e)}(x), 0\}, \forall x \in U^*$.

(e) **Null:**

A soft multiset (F, A) is called a Null soft multiset denoted by Φ, if $\forall e \in A$ $F(e) = \emptyset$.

(f) **Complement:**

The complement of a soft multiset (F, A), denoted by $(F, A)^c$, is defined by $(F, A)^c =$

(F^c, A) where $F^c\colon A \longrightarrow P^*(U)$ is a mapping given by $F^c(e) = U \backslash F(e), \forall e \in A$ where $C_{F^c(e)}(x) = C_U(x) - C_{F(e)}(x),\ \forall x \in U^*$.

3. n-Level Soft Set

Definition 3.1

Let (F, A) be a Soft multiset over a universal multiset U and a set of parameters E. Then, we define the *n-level soft set* of (F, A), denoted $(F, A)_n$ as

$$(F, A)_n = \{(e, \{x\}) \big| C_{F(e)}(x) \geq n, n \in \mathbb{N}, \forall e \in A, \forall x \in U^*\}.$$

Example 3.1

Let $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$, $A = \{e_1, e_2, e_3\}, B = \{e_1, e_2\}, U = \{7/x, 4/y, 3/z\}$,
$(F, A) = \{(e_1, \{2/x, 1/y, 3/z\}), (e_2, \{4/x, 3/y\}), (e_3, \{1/x\})\}$ and
$(G, B) = \{(e_1, \{2/x, 2/z\}), (e_2, \{4/x, 3/y\})\}$. Then,

$$(F, A)_1 = \{(e_1, \{x, y, z\}), (e_2, \{x, y\}), (e_3, \{x\})\}$$
$$(F, A)_2 = \{(e_1, \{x, z\}), (e_2, \{x, y\})\}$$
$$(F, A)_3 = \{(e_1, \{z\}), (e_2, \{x, y\})\}$$
$$(F, A)_4 = \{(e_2, \{x\})\}$$
$$(F, A)_n = \Phi, n \geq 5$$

and

$$(G, B)_1 = \{(e_1, \{x, z\}), (e_2, \{x, y\})\}$$
$$(G, B)_2 = \{(e_1, \{x, z\}), (e_2, \{x, y\})\}$$
$$(G, B)_3 = \{(e_2, \{x, y\})\}$$
$$(G, B)_4 = \{(e_2, \{x\})\}$$
$$(G, B)_n = \Phi, n \geq 5.$$

Definition 3.2

Let $(F, A)_n$ be the *n-level soft set* of (F, A), then

.

$$F_n(e) = \{x \in U^* \big| C_{F(e)}(x) \geq n, n \in \mathbb{N}, \forall e \in A\}.$$

Example 3.2

Observe that, from example 3.1, $F_1(e_1) = \{x, y, z\}, F_1(e_2) = \{x, y\}, F_1(e_3) = \{x\}$.

Theorem 3.1

Let (F, A) and (G, B) be Soft multisets over U and E, suppose $m, n \in \mathbb{N}$. Then,
(i) $((F, A) \sqcup (G, B))_n = (F, A)_n \sqcup (G, B)_n$,
(ii) $((F, A) \sqcap (G, B))_n = (F, A)_n \sqcap (G, B)_n$,
(iii) IF $(G, B) \sqsubseteq (F, A)$ then $(G, B)_n \sqsubseteq (F, A)_n$,
(iv) IF $m \leq n$ then $(F, A)_n \sqsubseteq (F, A)_m$,
(v) $(G, B) = (F, A)$ iff $(G, B)_n = (F, A)_n, \forall e \in A, \forall x \in U^*$.

Proof

(i) Let $x \in \big((F, A) \sqcup (G, B)\big)_n \Longrightarrow x \in (F, A) \sqcup (G, B),\ C_{F(e)}(x) \geq n,\ C_{G(e)}(x) \geq n$
$\Longrightarrow x \in (F, A), C_{F(e)}(x) \geq n$ or $x \in (G, B),\ C_{G(e)}(x) \geq n$
$\Longrightarrow x \in (F, A)_n$ or $x \in (G, B)_n$
$\Longrightarrow x \in ((F, A)_n \sqcup (G, B))_n$
i.e., $((F, A) \sqcup (G, B))_n \sqsubseteq (F, A)_n \sqcup (G, B)_n$ …(1)
Conversely, let $x \in (F, A)_n \sqcup (G, B)_n$
$\Longrightarrow x \in (F, A)_n$ or $x \in (G, B)_n$

$\Longrightarrow C_{F(e)}(x) \geq n, \forall e \in A$ or $C_{G(e)}(x) \geq n, \forall e \in B$
$\Longrightarrow x \in (F, A), C_{F(e)}(x) \geq n$ or $x \in (G, B),\ C_{G(e)}(x) \geq n$
$\Longrightarrow x \in (F, A)$ or $x \in (G, B),\ C_{F(e)}(x) \geq n, C_{G(e)}(x) \geq n$
$\Longrightarrow x \in (F, A) \sqcup (G, B),\ C_{F(e)}(x) \geq n,\ C_{G(e)}(x) \geq n$
$\Longrightarrow x \in \big((F, A) \sqcup (G, B)\big)_n$
i.e.,$(F, A)_n \sqcup (G, B)_n \sqsubseteq ((F, A) \sqcup (G, B))_n$ …(2)
From (1) and (2) the result follows.

Similarly for (ii).
(iii) Let $(G, B) \sqsubseteq (F, A)$ and suppose $x \in (G, B)_n$
$\Longrightarrow C_{G(e)}(x) \geq n, \forall e \in B$
Since $C_{G(e)}(x) \leq C_{F(e)}(x), \forall e \in B$ and $B \subseteq A$, we have $C_{F(e)}(x) \geq n, \forall e \in A$
$\therefore x \in (F, A)_n$
i.e., $(G, B)_n \sqsubseteq (F, A)_n$.

(iv) Let $m \leq n$ and suppose $x \in (F, A)_n$
$\Longrightarrow C_{F(e)}(x) \geq n, \forall e \in A$
$\Longrightarrow C_{F(e)}(x) \geq m, \forall e \in A$
$\Longrightarrow x \in (F, A)_m$
i.e., $(F, A)_n \sqsubseteq (F, A)_m$.

(v) Let $(G, B) = (F, A) \Longrightarrow A = B$ and $C_{F(e)}(x) = C_{G(e)}(x), \forall e \in A, \forall x \in U^*$
$\Longrightarrow \forall n \in \mathbb{N}$, if $C_{F(e)}(x) \geq n$ it imply $C_{G(e)}(x) \geq n, \forall e \in A$ and vice versa
Thus, $(G, B)_n = (F, A)_n$.
Conversely,
Let $(G, B)_n = (F, A)_n \Longrightarrow C_{F(e)}(x) \geq n, \forall e \in A$ and $C_{G(e)}(x) \geq n, \forall e \in B$
$\Longrightarrow C_{F(e)}(x) \geq n,$ and $C_{G(e)}(x) \geq n, \forall e \in A$
$\Longrightarrow C_{F(e)}(x) = C_{G(e)}(x), \forall e \in A$
$\Longrightarrow (G, B) = (F, A)$.

Definition 3.3

Let $S(U, E)$ be the class of all Soft multisets over U and E i.e. $S(U, A) = \{F: A \longrightarrow P^*(U), A \subseteq E\}$. Let $Q \subseteq U^*$, then, we define a soft multiset ${}_n(F, A) \in S(U, A)$ as

$${}_n(F, A) = \{(e, nQ) | C_{nQ}(x) = n, \forall e \in A, \forall n \in \mathbb{N}.$$

Example 3.3

Let $Q = \{x, y\}$, then

$${}_1(F, A) = \{(e, \{x, y\})\},\ {}_2(F, A) = \{(e, \{2/x, 2/y\})\},\ {}_3(F, A) = \{(e, \{3/x, /3y\})\}, \dots,\ {}_n(F, A) = \{(e, \{n/x, n/y\})\}, \forall e \in A, \forall n \in \mathbb{N}.$$

Theorem 3.2 (First Decomposition Theorem)

Let $(F, A)_n$ be a n-level soft set of a soft multiset (F, A), over U and E. Then,
$C_{(F,A)}(x) = C_{F(e)}(x), \forall e \in A = \sum_{n \in \mathbb{N}} \mathcal{X}(F_n(e))(x), \forall e \in A = \sum_{n \in \mathbb{N}} \mathcal{X}(F, A)_n\,(x)$ where $\mathcal{X}(F_n(e))$ is the characteristic function of $\big(F_n(e)\big), \forall e \in A$ and $\mathcal{X}(F, A)_n$ is the characteristic function of $(F, A)_n$.

Proof

Let $x \in F_r(e), \forall e \in A, r = r_1, r_2, \dots, r_m, m = cad(A)$ for $x \in U^*$. Observe that $x \notin F_{r+n}(e), n \in \mathbb{N}$. Then
$C_{(F,A)}(x) = C_{F(e)}(x) = r, \forall e \in A$. Now

$$\sum_{n\in\mathbb{N}} \mathcal{X}(F,A)_n(x) = \sum_{n\in\mathbb{N}} \mathcal{X}(F_n(e))(x), \forall e \in A = \sum_{n=1}^{r} \mathcal{X}\big(F_n(e)\big)(x) + \sum_{n\in\mathbb{N}} \mathcal{X}(F_{r+n}(e))(x), \forall e \in A$$
$$= [1 + 1 + \cdots r \text{ times}] + [0 + 0 + \cdots] = \text{r}, \forall e \in A.$$

Hence, $C_{(F,A)}(x) = \sum_{n\in\mathbb{N}} \mathcal{X}(F,A)_n(x)$.

Example 3.4

Consider $(F,A) = \{(e_1, \{2/x, 1/y, 3/z\}), (e_2, \{4/x, 3/y\}), (e_3, \{1/x\})\}$.

Now

$$C_{(F,A)}(x) = \{C_{F(e_1)}(x) + C_{F(e_2)}(x) + C_{F(e_3)}(x)\} = 2 + 4 + 1 = 7.$$

But

$$\mathcal{X}\big(F_1(e_1)\big)(x) = 1, \mathcal{X}\big(F_2(e_1)\big)(x) = 1, \mathcal{X}\big(F_n(e_1)\big)(x) = 0, n \geq 3,$$

$\mathcal{X}\big(F_1(e_2)\big)(x) = 1, \mathcal{X}\big(F_2(e_2)\big)(x) = 1, \mathcal{X}\big(F_3(e_2)\big)(x) = 1, \mathcal{X}\big(F_4(e_2)\big)(x) = 1, \mathcal{X}\big(F_n(e_2)\big)(x) = 0, n \geq 5, \mathcal{X}\big(F_1(e_3)\big)(x) = 1, \mathcal{X}\big(F_n(e_3)\big)(x) = 0, n \geq 2.$

and thus,

$$\sum_{n\in\mathbb{N}} \mathcal{X}(F,A)_n(x) = \sum_{n\in\mathbb{N}} \mathcal{X}(F_n(e))(x), \forall e \in A$$
$$= \sum_{n=1}^{2} \mathcal{X}\big(F_n(e_1)\big)(x) + \sum_{n=1}^{4} \mathcal{X}\big(F_n(e_2)\big)(x) + \mathcal{X}\big(F_1(e_3)\big)(x)$$
$$= \mathcal{X}\big(F_1(e_1)\big)(x) + \mathcal{X}\big(F_2(e_1)\big)(x) + \mathcal{X}\big(F_1(e_2)\big)(x) + \mathcal{X}\big(F_2(e_2)\big)(x)$$
$$+ \mathcal{X}\big(F_3(e_2)\big)(x) + \mathcal{X}\big(F_4(e_2)\big)(x) + \mathcal{X}\big(F_1(e_3)\big)(x) = 1 + 1 + 1 + 1 + 1 + 1 + 1$$
$$= 7$$

Theorem 3.3 (Second Decomposition Theorem)

Let $(F,A)_n$ be the n-level soft set of a soft multiset (F,A) over U and E. Then $(F,A) = \coprod_{n\in\mathbb{N}} {}_n(F,A)_n$ where $\sqcup$ is the soft multiset union.

Proof

Let $x \in U^*$ and $C_{(F,A)}(x) = t, \forall e \in A$. This imply that $x \in (F,A)_n$, for $n = 1,2,\dots,t$ and $x \notin (F,A)_n, \forall n \geq t+1, \ \forall e \in A$.

Now,

$$C_{\left(\coprod_{n\in\mathbb{N}} {}_n(F,A)_n\right)}(x) = \coprod\nolimits_{n\in\mathbb{N}} {}_n(F,A)_n\,(x)$$
$$= {}_1(F,A)_1 \sqcup {}_2(F,A)_2 \sqcup \dots \sqcup {}_t(F,A)_t \sqcup_{t+1}(F,A)_{t+1} \sqcup \dots$$
$$= \bigcup\{1,2,\dots,t,0,0,\dots\} = t, \forall e \in A.$$
$$= C_{(F,A)}(x), \forall e \in A, \forall x \in U^*$$
$$= (F,A).$$

Therefore, $(F,A) = \coprod_{n\in\mathbb{N}} {}_n(F,A)_n$.

Example 3.5

Let $(F,A) = \{(e_1, \{2/x, 1/y, 3/z\}), (e_2, \{4/x, 3/y\}), (e_3, \{1/x\})\}$, we have

$$(F,A)_1 = \{(e_1, \{x, y, z\}), (e_2, \{x, y\}), (e_3, \{x\})\}$$
$$(F,A)_2 = \{(e_1, \{x, z\}), (e_2, \{x, y\})\}$$
$$(F,A)_3 = \{(e_1, \{z\}), (e_2, \{x, y\})\}$$
$$(F,A)_4 = \{(e_2, \{x\})\}$$
$$(F,A)_n = \Phi, n \geq 5$$

and thus,

$${}_1(F,A)_1 = \{(e_1, \{1/x, 1/y, 1/z\}), (e_2, \{1/x, 1/y\}), (e_3, \{1/x\})\}$$
$${}_2(F,A)_2 = \{(e_1, \{2/x, 2/z\}), (e_2, \{2/x, 2/y\})\}$$

$$_{3}(F,A)_{3} = \{(e_1,\{3/z\}),(e_2,\{3/x,3/y\})\}$$
$$_{4}(F,A)_{4} = \{(e_2,\{4/x\})\}.$$

Now,

$$_{1}(F,A)_{1}\sqcup\, _{2}(F,A)_{2}\sqcup\, _{3}(F,A)_{3}\sqcup\, _{4}(F,A)_{4}\sqcup\, _{5}(F,A)_{5}\sqcup\, _{6}(F,A)_{6}\sqcup \ldots$$
$$= \{(e_1,\{1/x,1/y,1/z\}),(e_2,\{1/x,1/y\}),(e_3,\{1/x\})\}\sqcup\{(e_1,\{2/x,2/z\}),(e_2,\{2/x,2/y\})\}$$
$$\sqcup\{(e_1,\{3/z\}),(e_2,\{3/x,3/y\})\}\sqcup\{(e_2,\{4/x\})\}\sqcup\Phi\sqcup\Phi\sqcup \ldots$$
$$= \{(e_1,\{2/x,1/y,3/z\}),(e_2,\{4/x,3/y\}),(e_3,\{1/x\})\} = (F,A).$$

Conclusion

In this paper, the notion of n-level set is applied to Soft multisets. In addition, some theorems are established and proved.

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