# USING MATHEMATICAL MODELING OF UNDERSTANDING THE CAUSE AND THE WAY TO PREVENT OUTBREAK DISEASE (ZIKA VIRUS)

**[*1]D.I.Lanlege & [2]S.O Imoni**

*[*1,2] Department of Mathematical Science Federal University Lokoja kogi State, Nigeria.*

*loislanlege@yahoo.com david.lanlege@fulokoja.edu.ng*

*Sunday.imoni@fulokoja.edu.ng*

*Tell: +2348030528667, +2348060887418*

**Abstract**

*We developed a deterministic mathematical model describing the transmission dynamics of Zika virus. The model is a system of first order ordinary differential equation (ODE), where $S_H$ is susceptible human, $I_H$ is infected human, $R_H$ is recovered human ,human and mosquitoes interact to infect human, incorporating the use of condom, water hygiene and sanitation. The equilibrium states and the analytical solution use Homotopy Perturbation Method (HPM) and generating the reproductive number $R_o$, using Gauss Jordan elimination method. The demographic profile of French Polynesia was use in the model to show the effect of control measure at different rate (i.e lower, medium and higher) on French Polynesia population. A numerical simulation was carried out using maple software to show the effective reproductive number to determine whether the disease is under control or out of control.*

**Keywords:** *Zika Virus, Transmission Dynamics, Model Equations, Homotopy Perturbation Method (HPM), Ordinary Differential Equations (ODE), Population and Susceptible- Infected- Recovered (SIR)*

## 1.0 INTRODUCTION

Originally identified in Africa (Hayes, 2009) the first large reported outbreak of Zika virus (ZIKV) disease occurred in Yap in April to July 2007. Also, there was an outbreak in French Polynesia between October 2013 and April 2014 (Duffy, *et.al*, 2009,Cao-Lormeau, 2014) and cases in other Pacific countries (Musso 2015). In 2015, local transmission was also reported in South American countries, including Brazil (Campos, 2015, Colombia and Camacho 2016).

Zika virus is an emerging mosquito–borne virus that was also identified in Africa (Uganda) in 1947, it was subsequently discovered in humans in 1952 in Uganda and United Republic of Tanzania. The disease is caused by Zika virus which is spread to people primarily through the bite of an infected Aedes species mosquito, the most common symptoms of Zika virus are fever, rash, joint pain and conjunctivitis(red eye).

Transmission of Zika virus (ZIKV) is predominately vector-borne, but can also occur via sexual contact and blood transfusions (Musso, 2015). The virus is spread by the Aedes species of mosquito (Mallet, *et al.*, 2015), which is also the vector for dengue virus. (DENV), Zika virus (ZIKA) is therefore likely to be capable of sustained transmission in other tropical areas as well as causing symptoms such as fever and rashes, Zika virus infection has also been linked to increased incidence of neurological sequelae, including Guillain-Barr-e Syndrome (GBS) and microcephaly in infants born to mothers who were infected with Zika virus during pregnancy (Schuler-Faccini *et .al,*. 2016). On 1st February 2015, the World Health Organization declared a Public Health Emergency of International Concern in

response to the clusters of microcephaly and other neurological disorders reported in Brazil, possibly linked to the recent rise in Zika virus incidence. The same phenomena were observed in French Polynesia, with 42 GBS cases (Leparc-GoffartI,*et.al*, 2015) reported during the outbreak. In addition to the GBS cluster, there were 18 fetal or newborn cases with unusual and severe neurological features reported between March 2014 and May 2015 in French Polynesia, including cases with microcephaly and severe brain lesions, and 8 norm cephalic cases with severe anatomical or functional neurological abnormalities (Centre d'hygiene et de salubrit_epublique. 2014). Given the potential for Zika virus to spread globally, it is crucial to characterize the transmission dynamics of the infection. This includes estimates of key epidemiological parameters, such as the basic reproduction number, Ro (defined as the average number of secondary cases generated by a typical infectious individual in a fully susceptible population), and of how many individuals (including both symptomatic and asymptomatic) are typically infected during an outbreak. Such estimates could help assist with outbreak planning, assessment of potential countermeasures, and the design of studies to investigate putative associations between Zika virus infection and other conditions. Islands can be useful case studies for outbreak analysis. Small, centralized populations are less likely to sustain endemic transmission than a large, heterogeneous population (Keeling and Grenfell, 1997), which means outbreaks are typically self-limiting after introduction from external sources (Cao-Lormeau 2014). Further, if individuals are immunologically naive to a particular pathogen, it is not necessary to consider potential effect of pre-existing immunity on transmission dynamics (Ballesteros *et al.*, 2011).

Using a mathematical model of vector-borne infection, we examined the transmission dynamics of Zika virus on six archipelagos in French Polynesia during the 2013-2014 outbreaks. We inferred the basic reproduction number, and the overall size of the outbreak, and hence how many individuals would still be susceptible to infection in coming years (Adam *et.al*, 2016).

## 2.0 METHOD OF DATA COLLECTION

The information (Data) collected for the analysis of this paper work is purely secondary data that is, already made data. We used weekly reported numbers of suspected ZIKV infections from the main regions of French Polynesia between May 2015 and August 2016. The data are recorded on weekly basis so that at the end of each month the overall total will be calculated.

We used a susceptible-infected-recovered (SIR) model to simulate vector-borne transmission. Both human and mosquitoes were modeled using a susceptible--infectious-removed (SIR) framework. This model incorporated delays as a result of the intrinsic (human) and extrinsic (vector) latent periods. Since there is evidence that asymptomatic DENV-infected individuals are capable of transmitting DENV to mosquitoes, we assumed the same for ZIKV.

## 3.0 METHODS AND MATERIALS

### 3.1 Formulation of the Model

We develop a model to analysis the transmission of Zika virus through ordinary differential equation. The disease free and endemic equilibrium states are addressed and the value of effective basic reproductive number $R_o$ is expressed in terms of parameters, which determine whether the disease is under control or is out of control in the population. This

model divides the total population of human into three sub-classes namely: $S_H$, $I_H$ and $R_H$, while the population of the vector is divide into two classes namely: Adult Vector (Cv) and Pupae Vector (Av).

### 3.2 Basic Assumption

I. It is assumed that the new births of susceptible $S_{(t)}$ are susceptible.

II. It is assumed that the virus does not kill the vector i.e. their death can be natural or accidental.

III. The infected classes of the vector are divided into two: Adult Vector and Pupae Vector

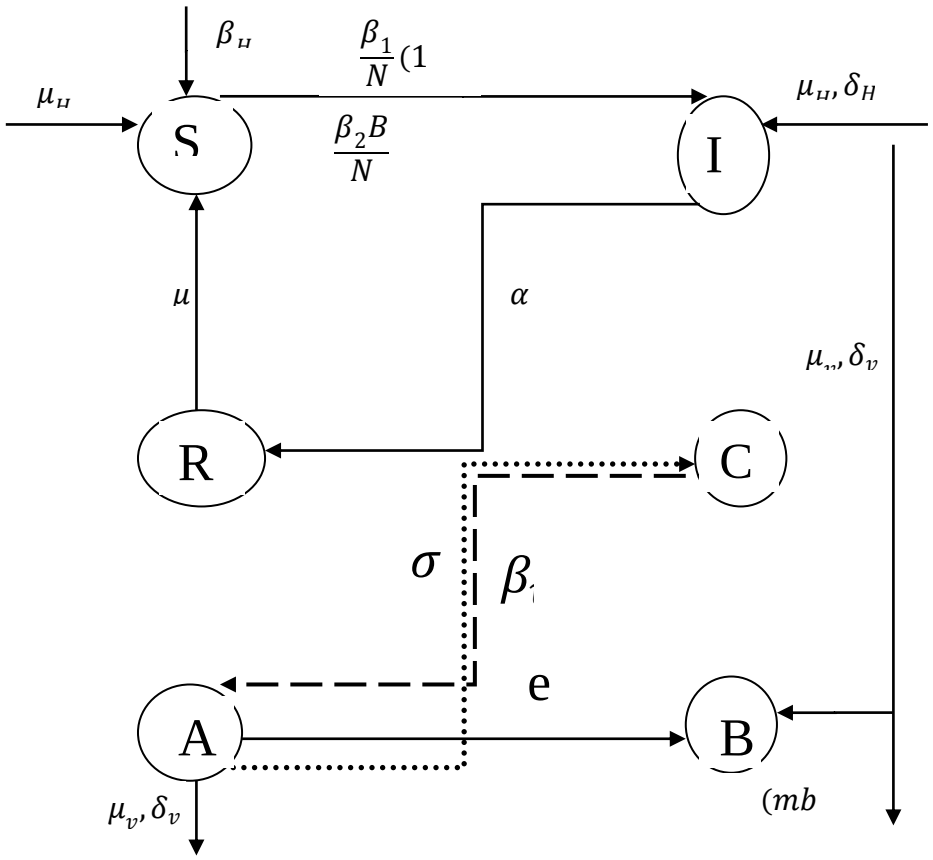


**Figure 1**: Shows the Schematic Diagram of the Mathematical Model for Zika Virus Transmission.

### 4.0 Model Equation

Applying the assumption, definition of variables and parameter and the relationship between the variables and parameters describe in the schematic diagram in the previous page, we developed a six ordinary differential equation for the transmission and control of Zika virus in a population. The differential equations are given below;

$$\frac{dS_H}{dt} = \beta_H N - \frac{\beta_1(1-\tau\varepsilon)I_H S_H}{N} + \varphi R_H - \mu_H S_H - \frac{\beta_2 B S_H}{N} \quad (1)$$

$\frac{dI_H}{dt} = \beta_1 \frac{(1-\tau\varepsilon)I_H S_H}{N} + \frac{\beta_2 B S_H}{N} - \mu_H I_H - \alpha I_H - \delta_H I_H$ (2)

$\frac{dR_H}{dt} = \alpha I_H - \mu_H R_H - \omega R_H$ (3)

$\frac{dC_V}{dt} = \beta_V A_V - \mu_V C_V - \sigma C_V - \delta_V C_V$ (4)

$\frac{dA_V}{dt} = \sigma C_V - \mu_V A_V - \delta_V A_V$ (5)

$\frac{dB}{dt} = eA_V - B(mb - nb)$ (6)

These equations are valid for $N_H > 0$ for all the parameters in the model are assumed positive and the total population size are;

$N_H = S_H + I_H + R_H$ (7)

$N_V = C_V + A_V$ (8)

The differential equation of the total population are ;

$\frac{dN_H}{dt} = (\beta_H - \mu_H)N_H - \delta_H I_H$ (9)

$\frac{dN_V}{dt} = \beta_V A_V - (\mu_V + \delta_V)N_V$ (10)

At the equilibrium states let $S_H = S_H^*$, $I_h = I_H^*$, $R_H = R_H^*$, $C_V = C_V^*$, $A_V = A_V^*$, B=$B^*$.

$\frac{dS_H}{dt} = \frac{dI_H}{dt} = \frac{dR_H}{dt} = \frac{dC_V}{dt} = \frac{dA_V}{dt} = \frac{dB}{dt} = 0$ (11)

So we rewrite the equation and substitute for the parameters and variables.

$\beta_H N - \frac{\beta_1(1-\tau\varepsilon)S_H^* I_H^*}{N} + \varphi R_H^* - \mu_H S_H^* - \frac{\beta_2 B^* S_H^*}{N} = 0$ (12)

$\frac{\beta_1(1-\tau\varepsilon)S_H^* I_H^*}{N} + \mu_H I_H^* - \propto I_H^* - \delta_H I_H^* + \frac{\beta_2 B^* S_H^*}{N} = 0$ (13)

$\propto I_H^* - \mu_H R_H^* - \varphi R_H^* = 0$ (14)

$\beta_V A_V^* - \mu_V C_V^* - \sigma C_V^* - \delta_V C_V^* = 0$ (15)

$\sigma C_V^* - \mu_V A_V^* - \delta_V A_V^* = 0$ (16)

$eA_V^* - B^*(mb - nb) = 0$ (17)

At the disease-free equilibrium

From equation (16)

$\sigma C_V^* - A_V^*(\mu_V + \delta_V) = 0$

$A_V^*(\mu_V + \delta_V) = \sigma C_V^*$

$A_V^* = \frac{\sigma C_V^*}{(\mu_V + \delta_V)}$ (18)

From equation (15) substitute for $A_V^*$

$\beta_V \left(\frac{\sigma C_V^*}{(\mu_V + \delta_V)}\right) - \mu_V C_V^* - \sigma C_V^* - \delta_V C_V^* = 0$

$C_V^* \left(\frac{\sigma}{(\mu_V + \delta_V)} - \mu_V - \sigma - \delta_V\right) = 0$

Since $\left(\frac{\sigma}{(\mu_V + \delta_V)} - \mu_V - \sigma - \delta_V\right) \neq 0$

$C_V^* = 0$

From equation (18) substitute for $C_V^*$

$A_V^* = \frac{\sigma(0)}{(\mu_V + \delta_V)}$

$A_V^* = 0$

Also from equation (17) substitute for $A_V^*$

$\mathrm{e}A_V^* - B^*(\mathrm{mb} - \mathrm{nb}) = 0$

$B^*(\mathrm{mb} - \mathrm{nb}) = \mathrm{e}A_V^*$

$$B^* = \frac{\mathrm{e}A_V^*}{(\mathrm{mb} - \mathrm{nb})}$$

$$B^* = \frac{\mathrm{e}(0)}{(\mathrm{mb} - \mathrm{nb})}$$

$B^* = 0$

From equation (13) we have

$$\frac{\beta_1(1-\tau\varepsilon)S_H^*I_H^*}{N} + \mu_{\mathrm{H}}I_H^* - \propto I_H^* + \frac{\beta_2 B^* S_H^*}{N} = 0$$

Substitute for $B^*$

$$\frac{\beta_1(1-\tau\varepsilon)S_H^*I_H^*}{N} + \mu_{\mathrm{H}}I_H^* - \propto I_H^* - \delta_H I_H^* + \frac{\beta_2(0)S_H^*}{N} = 0$$

$$I_H^*\left(\frac{\beta_1(1-\tau\varepsilon)S_H^*I_H^*}{N} - \mu_{\mathrm{h}} - \propto -\delta_H\right) = 0$$

$$\left(\frac{\beta_1(1-\tau\varepsilon)S_H^*I_H^*}{N} - \mu_{\mathrm{H}} - \propto -\delta_H\right) \neq 0$$

$I_H^* = 0$

From equation (14)

$\propto I_H^* - \mu_{\mathrm{H}}R_H^* - \varphi R_H^* = 0$

Substitute for $I_H^*$

$\propto (0) - \mu_{\mathrm{H}}R_H^* - \varphi R_H^* = 0$

$R_H^*(\mu_{\mathrm{H}} + \varphi) = \propto (0)$

$$R_H^* = \frac{\propto (0)}{(\mu_{\mathrm{h}} + \varphi)}$$

$R_H^* = 0$

Also from equation (11)

$$\beta_H N - \frac{\beta_1(1-\tau\varepsilon)S_H^*I_H^*}{N} + \varphi R_H^* - \mu_{\mathrm{H}}S_H^* - \frac{\beta_2 B^* S_H^*}{N} = 0$$

Substitutes for $I_h^*, R_h^*,$ and $B^*$

$$\beta_H N - \frac{\beta_1(1-\tau\varepsilon)S_H^*(0)}{N} + \varphi(0) - \mu_{\mathrm{H}}S_H^* - \frac{\beta_2(0)S_H^*}{N} = 0$$

$\beta_H N - \mu_{\mathrm{h}}S_H^* = 0$

$\mu_{\mathrm{H}}S_H^* = \beta_H N$

$$S_H^* = \frac{\beta_H N}{\mu_{\mathrm{H}}}$$

Hence, the disease-free equilibrium is

$$E_0 = (S_H^*, I_H^*, R_H^*, C_V^*, A_V^*, B^*) = \left(\frac{\beta_H N}{\mu_H}, 0, 0, 0, 0, 0\right)$$

For endemic equilibrium

$$S_H^* > 0, I_H^* > 0, R_H^* > 0, C_V^* > 0, A_V^* > 0, B^* > 0$$

For human population

Recall from equation (14)

$$\propto I_H^* - \mu_H R_H^* - \varphi R_H^* = 0$$

$$R_H^*(\mu_H + \varphi) = \propto I_h^*$$

$$R_H^* = \frac{\propto I_H^*}{(\mu_H + \varphi)}$$

Also from equation (12)

$$\beta_H N - \frac{\beta_1(1-\tau\varepsilon)S_H^* I_H^*}{N} + \varphi R_H^* - \mu_H S_H^* - \frac{\beta_2 B^* S_H^*}{N} = 0$$

$$-S_H^*\left(\frac{\beta_1(1-\tau\varepsilon)I_H^*}{N} + \mu_H + \frac{\beta_2 B^*}{N}\right) + \beta_H N + \varphi R_H^* = 0$$

$$S_H^*\left(\frac{\beta_1(1-\tau\varepsilon)I_H^*}{N} + \mu_H + \frac{\beta_2 B^*}{N}\right) = \beta_h N + \varphi R_H^*$$

$$S_H^* = \frac{\beta_H N + \varphi R_H^*}{\left(\frac{\beta_1(1-\tau\varepsilon)I_H^*}{N} + \mu_H + \frac{\beta_2 B^*}{N}\right)}$$

$$S_H^* = \frac{N(\beta_H N + \varphi R_H^*)}{\beta_1(1-\tau\varepsilon)I_H^* + N\mu_H + \beta_2 B^*}$$

Since

$$R_H^* = \frac{\propto I_H^*}{(\mu_H + \varphi)}$$

So the equation becomes

$$S_H^* = \frac{N\left(\beta_H N + \varphi\frac{\propto I_H^*}{(\mu_H + \varphi)}\right)}{\beta_1(1-\tau\varepsilon)I_H^* + N\mu_H + \beta_2 B^*}$$

$$S_H^* = \frac{N(\beta_H N(\mu_H+\varphi) + \varphi(\propto I_h^*))}{(\mu_H+\varphi)[\beta_1(1-\tau\varepsilon)I_H^* + N\mu_H + \beta_2 B^*]} \quad (19)$$

From equation (33)

$$\frac{\beta_1(1-\tau\varepsilon)S_H^* I_H^*}{N} + \mu_H I_H^* - \propto I_H^* + \frac{\beta_2 B^* S_H^*}{N} = 0$$

$$-I_H^*\left(-\frac{\beta_1(1-\tau\varepsilon)S_H^*}{N} + \mu_H + \propto + \delta_H\right) + \frac{\beta_2 B^* S_H^*}{N} = 0$$

$$I_H^*\left(-\frac{\beta_1(1-\tau\varepsilon)S_H^*}{N} + \mu_H + \propto + \delta_H\right) = \frac{\beta_2 B^* S_H^*}{N}$$

$$I_H^* = \frac{\beta_2 B^* S_H^*}{N(\mu_H + \propto + \delta_H) - \beta_1(1-\tau\varepsilon)S_H^*} \quad (20)$$

Similarly, for the vector population

From equation (15)

$\beta_V A_V^* - \mu_V C_V^* - \sigma C_V^* - \delta_V C_V^* = 0$

$-C_V^*(\mu_V + \sigma + \delta_V) + \beta_V A_V^* = 0$

$C_V^*(\mu_V + \sigma + \delta_V) = \beta_V A_V^*$

$$C_V^* = \frac{\beta_V A_V^*}{(\mu_V+\sigma+\delta_V)} \quad (21)$$

Also from equation (3.16) we have

$\sigma C_V^* - A_V^*(\mu_V + \delta_V) = 0$

$A_V^*(\mu_V + \delta_V) = \sigma C_V^*$

$$A_V^* = \frac{\sigma C_V^*}{(\mu_V+\delta_V)} \quad (22)$$

From equation (17)

$eA_V^* - B^*(mb - nb) = 0$

$B^*(mb - nb) = eA_V^*$

$$B^* = \frac{eA_V^*}{(mb-nb)} \quad (23)$$

Hence, the endemic equilibrium is given by

$$E_E = (S_H^*, I_H^*, R_H^*, C_V^*, A_V^*, B^*)$$

$$= \left\{ \frac{N\left(\beta_h N(\mu_H + \varphi) + \varphi(\propto I_H^*)\right)}{(\mu_H + \varphi)[\beta_1(1-\tau\varepsilon)I_H^* + N\mu_H + \beta_2 B^*]}, \frac{\beta_2 B^* S_H^*}{N(\mu_H + \propto + \delta_H) - \beta_1(1-\tau\varepsilon)S_H^*}, \frac{\propto I_H^*}{(\mu_H + \varphi)}, \frac{\beta_V A_V^*}{(\mu_V + \sigma + \delta_V)}, \frac{\sigma C_V^*}{(\mu_V + \delta_V)}, \frac{eA_V^*}{(mb - nb)} \right\}$$

## 5.0 Analytical Solution of the Model

### 5.1 Analytical solution of the model using Homotopy Perturbation Method (HPM)

Fundamentals of Homotopy Perturbation Method (HPM) were first proposed by[12].To illustrate the basic ideas of this Method, the following nonlinear differential equation was considered:

$$\boldsymbol{A(u) - f(r) = 0, r \in \Omega} \quad (24)$$

Subject to the boundary condition:

$$\boldsymbol{B\left(u, \frac{\partial u}{\partial n}\right) = o, r \in \Gamma} \quad (25)$$

Where A is a general differential operator, B a boundary operator, f(r) is a known analytical function and Γ is the boundary of the domainΩ. The operator A can be divided into two parts L and N, where L is the linear part, and N is the nonlinear component. Equation (16) may therefore be rewritten as:

$$L(u) + N(U) - f(r) = 0, r \in \Omega \quad (26)$$

The Homotopy perturbation structure is shown as follows

$$H(v,p) = (1-p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0 \quad (27)$$

Where:

$$v(r,p): \Omega \in [0,1] \rightarrow R \quad (28)$$

In equation (28) p ∈ [0,l]is an embedding parameter and $\boldsymbol{u_0}$ is the first approximation that satisfies the boundary condition. It can be assumed that the solution of equation (28) can be written as power series as follows:

$V = V_0 + PV_1 + P^2V_2 + \cdots$ (29)

And the best approximation for the solution is:

$u = \lim_{p\to 1} V = V_0 + PV_1 + P^2V_2 + \cdots$ (30)

The series (29) is convergent for the most cases. However, the convergence rate depends on the nonlinear operator A(v).

## 6.0 Solution of the Model Equation

$\frac{dS_H}{dt} - \beta_H N + \frac{\beta_1(1-\tau\varepsilon)I_H S_H}{N} - \varphi R_H + \mu_H S_H + \frac{\beta_2 B S_H}{N} = 0$ (31)

$\frac{dI_H}{dt} - \beta_1 \frac{(1-\tau\varepsilon)I_H S_H}{N} - \frac{\beta_2 B S_H}{N} + \mu_H I_H + \alpha I_H + \delta_H I_H = 0$ (32)

$\frac{dR_H}{dt} - \alpha I_H + \mu_H R_H + \varphi R_H = 0$ (33)

$\frac{dC_V}{dt} - \beta_V A_V + \mu_V C_V + \sigma C_V + \delta_V C_V = 0$ (34)

$\frac{dA_V}{dt} - \sigma C_V + \mu_V A_V + \delta_V A_V = 0$ (35)

$\frac{dB}{dt} - eA_V + B(mb - nb) = 0$ (36)

With the initial condition$S(0) = S_0$, $I(0) = I_0$, $R(0) = R_0$, $C_V(0) = C_0$, $A_V(0) = A_0$ and $B(0) = B_0$

Applying Homotopy Perturbation Method to equation (31)

$(1-P)\frac{dS_H}{dt} + P\left(\frac{dS_H}{dt} - \beta_H N + \frac{\beta_1(1-\tau\varepsilon)I_H S_H}{N} - \varphi R_H + \mu_H S_H + \frac{\beta_2 B S_H}{N}\right) = 0$ (37)

Let

$S_H = U_0 + PU_1 + P^2U_2 + \cdots$ (38)

$I_H = V_0 + PV_1 + P^2V_2 + \cdots$ (39)

$R_H = W_0 + PW_1 + P^2W_2 + \cdots$ (40)

$C_V = X_0 + PX_1 + P^2X_2 + \cdots$ (41)

$A_V = Y_0 + PY_1 + P^2Y_2 + \cdots$ (42)

$B = Z_0 + PZ_1 + P^2Z_2 + \cdots$ (43)

After Substitute for $S_H, I_H, R_H$and collecting the coefficient of power of P we have

$P^0 = U_0' = 0$ (44)

$P^1 = U_1' + \frac{\beta_1(1-\tau\varepsilon)V_0 U_0}{N} + \mu_h U_0 + \frac{\beta_2 Z_0 U_0}{N} - \varphi W_0 - \beta_h N = 0$ (45)

$P^2 = U_2' + \frac{\beta_1(1-\tau\varepsilon)V_1 U_1}{N} + \mu_h U_1 + \frac{\beta_2 Z_1 U_1}{N} - \varphi W_1 = 0$ (46)

Applying HPM to the equation 3.32

$$(1-P)\frac{dI_H}{dt} + P\left(\frac{dI_H}{dt} - \beta_1\frac{(1-\tau\varepsilon)I_H S_H}{N} - \frac{\beta_2 B S_H}{N} + \mu_H I_H + \alpha I_H + \delta_H I_H\right) = 0 \quad (47)$$

After the substitute for $S_H, I_H, R_H$ and collecting the coefficient of the power of P we have

$$P^0 = V_0' = 0 \quad (48)$$

$$P^1 = V_1' - \beta_1\frac{(1-\tau\varepsilon)V_0 U_0}{N} - \frac{\beta_2 Z_0 U_0}{N} + \mu_h V_0 + \alpha V_0 + \delta_h V_0 = 0 \quad (49)$$

$$P^1 = V_2' - \beta_1\frac{(1-\tau\varepsilon)V_1 U_1}{N} - \frac{\beta_2 Z_1 U_1}{N} + \mu_h V_1 + \alpha V_1 + \delta_h V_1 = 0 \quad (50)$$

Applying HPM to the equation (33)

$$(1-P)\frac{dR_H}{dt} + P\left(\frac{dR_H}{dt} - \alpha I_H + \mu_H R_H + \varphi R_H\right) = 0 \quad (3.51)$$

After the substitute for $\mathrm{I_H}, \mathrm{R_H}$ and collecting the coefficient of the power of P we have

$$P^0 = W_0' = 0 \quad (52)$$

$$P^1 = W_1' - \alpha V_0 + \mu_h W_0 + \varphi W_0 = 0 \quad (53)$$

$$P^2 = W_2' - \alpha V_1 + \mu_h W_1 + \varphi W_1 = 0 \quad (54)$$

Applying HPM to the equation 3.34

$$(1-P)\frac{\mathrm{dC_V}}{\mathrm{dt}} + P\left(\frac{\mathrm{dC_V}}{\mathrm{dt}} - \beta_V \mathrm{A_V} + \mu_V \mathrm{C_V} + \sigma \mathrm{C_V} + \delta_V \mathrm{C_V}\right) = 0 \quad (55)$$

After the substitute for $\mathrm{C_V}$ and collecting the coefficient of the power of P we have

$$P^0 = X_0' = 0 \quad (56)$$

$$P^1 = X_1' - \beta_V \mathrm{Y_0} + \mu_V \mathrm{X_0} + \sigma \mathrm{X_0} + \delta_V \mathrm{X_0} = 0 \quad (57)$$

$$P^2 = X_2' - \beta_V \mathrm{Y_1} + \mu_V \mathrm{X_1} + \sigma \mathrm{X_1} + \delta_V \mathrm{X_1} = 0 \quad (58)$$

Applying HPM to the equation 3.35

$$(1-P)\frac{\mathrm{dA_V}}{\mathrm{dt}} + P\left(\frac{\mathrm{dA_V}}{\mathrm{dt}} - \sigma \mathrm{C_V} + \mu_V \mathrm{A_V} + \delta_V \mathrm{A_V}\right) = 0 \quad (59)$$

After the substitute for $\mathrm{A_V}$ and collecting the coefficient of the power of P we have

$$P^0 = Y_0' = 0 \quad (60)$$

$$P^1 = Y_1' - \sigma \mathrm{X_0} + \mu_V \mathrm{Y_0} + \delta_V \mathrm{Y_0} = 0 \quad (61)$$

$$P^2 = Y_2' - \sigma \mathrm{X_1} + \mu_V \mathrm{Y_1} + \delta_V \mathrm{Y_1} = 0 \quad (62)$$

Applying HPM to the equation 36

$$(1-P)\frac{\mathrm{dB}}{\mathrm{dt}} + P\left(\frac{\mathrm{dB}}{\mathrm{dt}} - \mathrm{eA_V} + \mathrm{B(mb - nb)}\right) = 0 \quad (63)$$

After the substitute for B and collecting the coefficient of the power of P we have

$$P^0 = Z_0' = 0 \quad (64)$$

$$P^1 = Z_1' - \mathrm{eY_0} + Z_0(\mathrm{mb - nb}) = 0 \quad (65)$$

$$P^1 = Z_1' - \mathrm{eY_1} + Z_1(\mathrm{mb - nb}) = 0 \quad (66)$$

Applying the initial condition

$$\mathrm{U_0} = \mathrm{S_0} = S(0) \quad (67)$$

$$P^0 = U_0' = 0 \Rightarrow U_0' = 0 \quad (68)$$

Integrate it

$$\mathrm{U_0} = A$$

$$\mathrm{S_0} = A$$

Apply the same technique to other variables

$$\mathrm{I_0} = C = \mathrm{V_0}$$

$$\mathrm{R_0} = D = \mathrm{W_0}$$

$$\mathrm{C_0} = E = \mathrm{X_0}$$

$$\mathrm{A_0} = F = \mathrm{Y_0}$$

$$\mathrm{B_0} = G = \mathrm{Z_0}$$

Where A, C, D, E, F, G are the constant of the integral

$$P^1 = U_1' + \frac{\beta_1(1-\tau\varepsilon)\mathrm{V_0\,U_0}}{\mathrm{N}} + \mu_\mathrm{h}\mathrm{U_0} + \frac{\beta_2 Z_0 \mathrm{U_0}}{\mathrm{N}} - \varphi\mathrm{W_0} - \beta_h\mathrm{N} = 0$$

$$U_1' = -\frac{\beta_1(1-\tau\varepsilon)\mathrm{V_0\,U_0}}{\mathrm{N}} - \mu_\mathrm{h}\mathrm{U_0} - \frac{\beta_2 Z_0 \mathrm{U_0}}{\mathrm{N}} + \varphi\mathrm{W_0} + \beta_h\mathrm{N}$$

Integrating with respect to t we have

$$\mathrm{U_1} = \left(-\frac{\beta_1(1-\tau\varepsilon)\mathrm{V_0\,U_0}}{\mathrm{N}} - \mu_\mathrm{h}\mathrm{U_0} - \frac{\beta_2 Z_0 \mathrm{U_0}}{\mathrm{N}} + \varphi\mathrm{W_0} + \beta_h\mathrm{N}\right)t + J$$

Where J is constant of integration. Apply the initial condition $\mathrm{U_1}(0) = 0$

gives J = 0 and now we have

$$\mathrm{U_1}(t) = \left(-\frac{\beta_1(1-\tau\varepsilon)\mathrm{V_0\,U_0}}{\mathrm{N}} - \mu_\mathrm{h}\mathrm{U_0} - \frac{\beta_2 Z_0 \mathrm{U_0}}{\mathrm{N}} + \varphi\mathrm{W_0} + \beta_h\mathrm{N}\right)t \quad (69)$$

Therefore, the other variables

$$\mathrm{V_1}(t) = \left(\beta_1\frac{(1-\tau\varepsilon)V_0\,U_0}{N} + \frac{\beta_2 Z_0 U_0}{N} - \mu_h V_0 - \alpha V_0 - \delta_h V_0\right)t \quad (70)$$

$$\mathrm{W_1}(t) = (\,\alpha V_0 - \mu_h W_0 - \varphi W_0\,)t \quad (71)$$

$X_1(t) = (\beta_V Y_0 - \mu_V X_0 - \sigma X_0 - \delta_V X_0)t$ (72)

$Y_1(t) = (\sigma X_0 - \mu_V Y_0 - \delta_V Y_0)t$ (73)

$Z_1(t) = \big(eY_1 - Z_1(mb - nb)\big)t$ (74)

Substitute the

$S_h = U_0 + PU_1 + P^2U_2 + \cdots$

$$S_h = U_0 + P\left(-\frac{\beta_1(1-\tau\varepsilon)V_0 U_0}{N} - \mu_h U_0 - \frac{\beta_2 Z_0 U_0}{N} + \varphi W_0 + \beta_h N\right)t$$

Letting p = 1

$S_h(t) = \lim_{p\to 1} s \Longrightarrow$

$U_0 + U_1 + U_2 + \cdots$

Since $U_0 = S_0, V_0 = I_0, W_0 = R_0, X_0 = C_0, Y_0 = A_0, Z_0 = B_0$

Therefore

$S_H(t) = S_0 + \left(-\frac{\beta_1(1-\tau\varepsilon)I_0 S_0}{N} - \mu_h S_0 - \frac{\beta_2 B_0 S_0}{N} + \varphi R_0 + \beta_h N\right)t$ (75)

We follow the same process as above to the other variables

$I_H(t) = I_0 + \left(\beta_1 \frac{(1-\tau\varepsilon)I_0 S_0}{N} + \frac{\beta_2 B_0 S_0}{N} - \mu_h I_0 - \alpha I_0 - \delta_h I_0\right)t$ (76)

$R_H(t) = R_0 + (\alpha I_0 - \mu_h R_0 - \varphi R_0)t$ (77)

$C_V(t) = C_0 + (\beta_V A_0 - \mu_V C_0 - \sigma C_0 - \delta_V C_0)t$ (78)

$A_V(t) = A_0 + (\sigma C_0 - \mu_V A_0 - \delta_V A_0)t$ (79)

$B(t) = B_0 + \big(eA_0 - B_0(mb - nb)\big)t$ (80)

### 6.1 Effective basic reproduction number, $R_0$

One of the most important concerns about any infectious disease is its ability to invade a population. The basic reproduction number, $R_0$is a measure of the potential for disease spread in a population, and is inarguably "one of the foremost and most valuable ideas that mathematical thinking has brought to epidemic theory" (Heesrbeek and Dietz, 1996). It represents the average number of secondary cases generated by an infected individual if introduced into a susceptible population with no immunity to the disease in the absence of interventions to control the infection. If $R_0 < 1$, then on average, an infected individual produces less than one newly infected individual over the course of his infection period. In this case, the infection may die out in the long run. Conversely, if $R_0 > 1$ , each infected individual produces, on average more than one new infection, the infection will be able to spread in a population. A large value of Ro may indicate the possibility of a major epidemic. Using the next generation operator technique described by (Diekmann and Heesterbeek, 2000) subsequently analyzed by (Vanden and Watmough, 2005), we obtained the basic reproduction number of the model equations (1) - (6) with is the spectral radius ($\rho$)of the next generation Matrix, $K$ .

i.e.

$R_0 = \rho K$ where $K = FV^{-1}$

Now, to find the value of the basic Reproductive Number $R_0$, we must first find the matrix $FV^{-1}$,

where

$$F = \begin{bmatrix} \beta_1 \dfrac{(1-\tau\varepsilon) I_{\mathrm{H}} S_H}{N} & 0 & 0 & \dfrac{\beta_2 S_H}{N} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and $V^{-1} = \frac{adj(V)}{|V|}$. To find $V^{-1}$, we have to find V and V is defined as $V^- - V^+$ where

$$V^- = \begin{bmatrix} \mu_H + \propto + \delta_V & 0 & 0 & 0 \\ 0 & \mu_V + \sigma + \delta_V & 0 & 0 \\ 0 & 0 & \mu_V + \delta_V & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$V^+ = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_V & 0 \\ 0 & \sigma & 0 & 0 \\ 0 & 0 & e & mb - nb \end{bmatrix}$$

$$V = \begin{bmatrix} \mu_H + \propto + \delta_V & 0 & 0 & 0 \\ 0 & \mu_V + \sigma + \delta_V & -\beta_V & 0 \\ 0 & -\sigma & \mu_V + \delta_V & 0 \\ 0 & 0 & -e & nb - mb \end{bmatrix}$$

In order to get determinant of the matrix $V^{-1}$We use the gauss Jordan elimination method

$$V = \left\{ \begin{array}{cccc|cccc} \mu_H + \propto + \delta_V & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \mu_{\mathrm{V}} + \sigma + \delta_{\mathrm{V}} & -\beta_V & 0 & 0 & 1 & 0 & 0 \\ 0 & -\sigma & \mu_{\mathrm{V}} + \delta_{\mathrm{V}} & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -e & nb - mb & 0 & 0 & 0 & 1 \end{array} \right\}$$

To get $R_1 = \dfrac{R_1}{\mu_H + \propto + \delta_V}$

$$R_2 = \frac{R_2}{\mu_{\mathrm{V}} + \sigma + \delta_{\mathrm{V}}}$$

$$R_3 = \frac{R_3}{\mu_{\mathrm{V}} + \delta_{\mathrm{V}}}$$

$$R_4 = \frac{R_4}{nb - mb}$$

We have

$$\left\{\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{1}{\mu_H+\propto+\delta} & 0 & 0 & 0 \\ 0 & 1 & \frac{-\beta_V}{\mu_V+\sigma+\delta_V} & 0 & 0 & \frac{1}{\mu_V+\sigma+\delta_V} & 0 & 0 \\ 0 & \frac{-\sigma}{\mu_V+\delta_V} & 1 & 0 & 0 & 0 & \frac{1}{\mu_V+\delta_V} & 0 \\ 0 & 0 & \frac{-e}{nb-mb} & 1 & 0 & 0 & 0 & \frac{1}{nb-mb} \end{array}\right\}$$

To get new $R_2 = R_2 + \frac{\beta_V}{\mu_V+\sigma+\delta_V} \times R_3$

Let $k = \frac{(\mu_V+\delta_V)[\mu_V+\sigma+\delta_V]-\beta_V\sigma}{(\mu_V+\delta_V)[\mu_V+\sigma+\delta_V]}$ we have

$$\left\{\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{1}{\mu_H+\propto+\delta_V} & 0 & 0 & 0 \\ 0 & k & \frac{\beta_V}{\mu_V+\sigma+\delta_V} & 0 & 0 & \frac{1}{\mu_V+\sigma+\delta_V} & \frac{\beta_V}{(\mu_V+\delta_V)[\mu_V+\sigma+\delta_V]} & 0 \\ 0 & \frac{-\sigma}{\mu_V+\delta_V} & 1 & 0 & 0 & 0 & \frac{1}{\mu_V+\delta_V} & 0 \\ 0 & 0 & \frac{-e}{nb-mb} & 1 & 0 & 0 & 0 & \frac{1}{nb-mb} \end{array}\right\}$$

Let U be $U = \frac{[\mu_V+\sigma+\delta_V]}{[\mu_V+\sigma+\delta_V]-\beta_V\sigma}$

The new $R_2 = U \times R_2$

We have

$$\left\{\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{1}{\mu_H+\propto+\delta_V} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{(\mu_V+\sigma+\delta_V)-\beta_V\sigma} & \frac{\beta_V}{(\mu_V+\delta_V)[\mu_V+\sigma+\delta_V]-\beta_V\sigma} & 0 \\ 0 & \frac{-\sigma}{\mu_V+\delta_V} & 1 & 0 & 0 & 0 & \frac{1}{\mu_V+\delta_V} & 0 \\ 0 & 0 & \frac{-e}{nb-mb} & 1 & 0 & 0 & 0 & \frac{1}{nb-mb} \end{array}\right\}$$

To get new $R_3$we use $R_3 = \dfrac{\sigma}{\mu_V + \delta_V} \times R_2$

We have

$$\left\{\begin{array}{cccc|cccc} & & & & \frac{1}{\mu_H + \propto + \delta_V} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & & & & \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{(\mu_V + \sigma + \delta_V) - \beta_V \sigma} & \frac{\beta_V}{(\mu_V + \delta_V)[\mu_V + \sigma + \delta_V] - \beta_V \sigma} & 0 \\ 0 & 0 & 1 & 0 & & & & \\ 0 & 0 & \frac{-e}{nb - mb} & 1 & 0 & \frac{\sigma}{(\mu_V + \delta_V)[\mu_V + \sigma + \delta_V] - \beta_V \sigma} & \frac{(\mu_V + \sigma + \delta_V)}{(\mu_V + \delta_V)[\mu_V + \sigma + \delta_V] - \beta_V \sigma} & 0 \\ & & & & 0 & 0 & 0 & \frac{1}{nb - mb} \end{array}\right\}$$

To get new $R_4$, we use $R_4 = R_4 + \dfrac{e}{nb - mb} \times R_3$ and let

$$M = \frac{e\sigma}{(nb - mb)(\mu_V + \delta_V)[\mu_V + \sigma + \delta_V] - \beta_V \sigma}$$

also $P = \dfrac{e(\mu_V + \sigma + \delta_V)}{(nb - mb)(\mu_V + \delta_V)[\mu_V + \sigma + \delta_V] - \beta_V \sigma}$

$$D = (\mu_V + \delta_V)[\mu_V + \sigma + \delta_V] - \beta_V \sigma$$

$$= \left\{\begin{array}{cccc|cccc} & & & & \frac{1}{\mu_H + \propto + \delta_V} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & & & & \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{(\mu_V + \sigma + \delta_V) - \beta_V \sigma} & \frac{\beta_V}{D} & 0 \\ 0 & 0 & 1 & 0 & & & & \\ 0 & 0 & 0 & 1 & 0 & \frac{\sigma}{D} & \frac{(\mu_V + \sigma + \delta_V)}{D} & 0 \\ & & & & 0 & M & P & \frac{1}{(nb - mb)} \end{array}\right\}$$

$$\left\{\begin{array}{cccc|cccc} & & & & \frac{1}{\mu_H+\propto+\delta_V} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & \frac{1}{(\mu_V+\sigma+\delta_V)-\beta_V\sigma} & \frac{\beta_V}{D} & 0 \\ 0 & 1 & 0 & 0 & 0 & \frac{\sigma}{D} & \frac{(\mu_V+\sigma+\delta_V)}{D} & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{M}{(nb-mb)} & \frac{P}{(nb-mb)} & \frac{1}{(nb-mb)} \\ 0 & 0 & 0 & 1 & & & & \end{array}\right\}$$

Therefore for $V^{-1} =$

$$\left[\begin{array}{cccc} \frac{1}{\mu_H+\propto+\delta_V} & 0 & 0 & 0 \\ 0 & \frac{1}{(\mu_V+\sigma+\delta_V)-\beta_V\sigma} & \frac{\beta_V}{(\mu_V+\delta_V)[\mu_V+\sigma+\delta_V]-\beta_V\sigma} & 0 \\ 0 & \frac{\sigma}{(\mu_V+\delta_V)[\mu_V+\sigma+\delta_V]-\beta_V\sigma} & \frac{(\mu_V+\sigma+\delta_V)}{(\mu_V+\delta_V)[\mu_V+\sigma+\delta_V]-\beta_V\sigma} & 0 \\ 0 & \frac{e\sigma}{(nb-mb)(\mu_V+\delta_V)[\mu_V+\sigma+\delta_V]-\beta_V\sigma} & \frac{e(\mu_V+\sigma+\delta_V)}{(nb-mb)(\mu_V+\delta_V)[\mu_V+\sigma+\delta_V]-\beta_V\sigma} & \frac{1}{(nb-mb)} \end{array}\right]$$

So $FV^{-1}$ we have

$$= \left\{\begin{array}{cccc|cccc} \beta_1\frac{(1-\tau\varepsilon)I_H S_H}{N} & 0 & 0 & \frac{\beta_2 S_H}{N} & \frac{1}{\mu_H+\propto+\delta_V} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{(\mu_V+\sigma+\delta_V)-\beta_V\sigma} & \frac{\beta_V}{D} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sigma}{D} & \frac{(\mu_V+\sigma+\delta_V)}{D} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{M}{(nb-mb)} & \frac{P}{(nb-mb)} & \frac{1}{(nb-mb)} \end{array}\right\}$$

$$\begin{bmatrix} \frac{\beta_1(1-\tau\varepsilon)S_H^*}{N(\mu_H+\propto+\delta_V)} & \frac{\beta_2 S_H^* e\sigma}{N(nb-mb)D} & \frac{\beta_2 S_H^* e\sigma[\mu_V+\sigma+\delta_V]}{N(nb-mb)D} & \frac{\beta_2 S_H^*}{N(nb-mb)} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then, for the Basic Reproductive Number $R_{0,}$ we have

$$R_{0,} = \frac{\beta_1(1-\tau\varepsilon)S_H^*}{N(\mu_H+\propto+\delta_V)}$$

Since

$$S_H^* = \frac{\beta_H N}{\mu_H}$$

So we have

$$R_{0,} = \frac{\beta_1(1-\tau\varepsilon)\beta_H}{\mu_H(\mu_H+\propto+\delta_V)} \qquad (81)$$

## 7.0 RESULT

**Population data for main regions of French Polynesia**

The total population value of region of French Polynesia is taking to be 162,470,000 and the life expected at birth is given as 52.05 for the year 2015 (UNICEF, 2015). The birth rate is given as 39.23 births for 1000 peoples and the natural death rate as $\frac{1}{52} = 0.0192$ for the year 2015 (WHO, 2015)

$$\beta_h = \frac{39.32}{1000} = 0.03923 \qquad \mu_h = 0.0192 \qquad (82)$$

For, $\beta_1$, the probability of transmission of infection for an infectious human to be susceptible human given that a contact between the two occurs is $\rho = 0.43$ (CDC, 2000) and assuming the average number of contacts is equal to 2=c

Then,

$$\beta_1 = \rho \times c = 2 \times 0.43 = 0.86 \qquad (83)$$

For $\beta_2$, the probability of transmission of infection from Zika to a susceptible human given that they come in contact with each other is $\rho = 0.05$ (CDC, 2000) and assuming the average number of contacts is c=1

Then, $\beta_2 = 0.05$ (84)

Table 1 The parameter value for main region of French Polynesia

| Parameters | Value | Detail |
|---|---|---|
| $\mu_H$ | 0.0192 | UNICEF, 2015 |
| $\beta_H$ | 0.0392 | UNICEF, 2015 |
| $\delta_H$ | 0.000011 | WHO |
| $\alpha$ | 0.03836 | CDC |
| $\varepsilon$ | 0.8 | Estimate/Assumed |
| $\beta_1$ | 0.86 | CDC, 2000 |
| $\beta_2$ | 0.05 | CDC, 2000 |
| Mb | 0.0175 | CDC |
| Nb | 0.0233 | CDC |
| $\beta_V$ | 0.0167 | WHO |
| E | 0.015 | Garba and Gumel (2010) |
| $\sigma$ | 0.06667 | Whitney |
| $\mu_V$ | 0.0082 | Spyghana |
| $\tau$ | 0-1 | Abdulraham (2014) |

Effective Basic Reproduction Number $R_0$

From equation

When the control measure (compliance to the use of condom, insecticide, water hygiene and sanitation) is 0.25

$$R_0 = \frac{0.86(0.8)(0.03923)}{(0.0192)[0.0192+0.03836+0.25]}$$

$$R_0 = 4.57$$

When the control measure (compliance to the condom usage, insecticide, water hygiene and sanitation) is 0.50

$$R_0 = \frac{(0.86)(0.60)(0.03923)}{(0.0192)[0.0192+0.03836+0.50]}$$

$$R_0 = \frac{0.0202272}{0.010705}$$

$$R_0 = 1.89$$

When the control measure (compliance to condom usage, insecticide, water hygiene and sanitation) is 0.75

$$R_0 = \frac{(0.86)(0.40)(0.03923)}{(0.0192)[0.0192+0.03836+0.75]}$$

$$R_0 = \frac{0.0134846}{0.015505}$$

$R_0 = 0.87$

When the control measure (compliance to condom usage, insecticide, water hygiene and sanitation) is 1.00

$$R_0 = \frac{(0.86)(0.20)(0.03923)}{(0.0192)[0.0192 + 0.03836 + 1]}$$

$$R_0 = \frac{0.00674756}{0.020305152}$$

$$R_0 = 0.332 \quad (82)$$

Table 2 Description of effective basic reproductive number and the rate of compliance to the use of vaccination

| Control measure (compliance to the use of vaccination)$\tau$ $and$ $\delta_v$ | Effective basic reproductive number $R_0$ |
|---|---|
| 0.1 | 16.04 |
| 0.2 | 5.73 |
| 0.3 | 3.73 |
| 0.4 | 2.61 |
| 0.5 | 1.89 |
| 0.6 | 1.39 |
| 0.7 | 1.02 |
| 0.8 | 0.74 |
| 0.9 | 0.51 |

**8 Discussion of Results**

The effective basic reproductive number tells us how important each parameter is to disease transmission. Such information is crucial not only to experimental design, but also to data assimilation and reduction of complex nonlinear model. When we have low control measure we see that the reproductive number is greater than one and again when we use 75% of control measure and above we see that the reproductive number is lesser than one and when this occur it means the disease is under control. These indicate that we can use effective basic reproductive number to determine whether disease is of control or out of control.

**9 Conclusion**

In this paper, a Mathematical Model with standard incidence is developed and analyzed to study the transmission and control of Zika virus. Mathematically we modeled Zika virus as a 6 –dimensional system of non-linear ordinary differential equation. We first show that there exist a domain *D* where our model is Mathematically and Epidemiologically well posed The Model incorporates two control parameters, condom usage and personal hygiene efficacy ($\varepsilon\tau$) and compliance ($\delta_v$) and which is the rate at which both the dormant and active vector are killed due to the use of insecticide. The Disease Free Equilibrium points of the model were obtained, and analyzed for stability. We obtained an important threshold parameter Effective Reproductive Number$R_0$, it is known that when $R_0 < 1$the disease dies out, and when $R_0 > 1$ the disease persists in the population.However, there are many human activities that will militate against the achievement of a Zika virus free society. The factors are:

I. Refusing of uses of condom doing intercourse
II. Lack of health education and low awareness in most rural areas.

III. Keeping of bushes and stagnant water around human homes.

**10 References**


Baba A. (2006). Research Journal of Medicine and Medical Sciences,1(2): 57-62.

Cao-Lormeau V.M, Roche C., Teissier A., Robin E., Berry A.L.and Mallet H.P. (2014). Zika virus, French polynesia, South pacific, Emerging infectious diseases. 20 (6), 1085

Campos G.S., Bandeira A.C. and Sardi S.I.(2015). Zika virus outbreak, Bahia, Brazil. 219 Emerging infectious diseases. 21(10):1885

Corsica F. (2015) Zika virus transmission from French Polynesia to Brazil. Emerging Infectious Diseases. 222.

Camacho .E, Paternina-Gomez .M, Blanco P.J, Osorio J.E, and Aliota M.T (2016) Detection of Autochthonous Zika Virus Transmission in Sincelejo, Colombia. Emerging Infectious Diseases; 22(5)..

[Central Intelligence Agency (2015).World fact book for the year 2014.*Retrieved on 20 February 2016 from http:/www.cia/library/publication/the-world fact book/geos/ni.htm*.

Diekmann. O., and Heesterbeek, J. A. P. (2000). Mathematical epidemiology of infectious diseases: *Model building, analysis and integration. New York: John Wiley*.

Duffy .M.R, Chen T.H., Hancock W.T, Powers A.M., Kool J.L. and Lanciotti R.S, (2009): Zika virus outbreak on Yap Island, federated states of Micronesia. New England Journal of Medicine; 360(24):2536-2543..

Heesterbeek, J. A. P. and Dietz, K. (1996). The concept of $\mathbf{R_0}$ in epidemic theory. Stat. Neerl. 50, 89- 110.

Herbert W. Hethcote (2005) Species coexistence and periodicity in host-host pathogen models, Math .J. Biol. 51.

Helena R. C., Araújo, Danilo O. C., Rafaella S. I. Costa-da-Silva A. L., and

Margareth L. C. (2015), *6, 576-594; doi:10.3390/insects 6020576 ISSN 2075-4450.* Health, 100 Galvin Hall, University of Notre Dame, Notre Dame, in 46556 USA..

Hayes E.B.(2009) Zika virus outside Africa. Emerging infectious diseases ;Vol, 15(9):1347.

Ji-Haun H.E (2000) . A coupling method of a homotopy technique and a perturbation *techniquefor non-linear problem, 37-43.*

Musso. D., Cao-Lormeau V.M. and Gubler D.J. (2015) Zika virus: following the path of dengue Andchikungunya? The Lancet.;386(9990):243{244. 214.

*M*usso D., Roche C., Robin E., Nhan T., Teissier A. and Cao-Lormeau V.M. (2015) Potential Sexual Transmission of ZikaVirus. Emerging infectious diseases. 21(2):359.

Mallet HP., Vial A.L., Musso D.and . Bilan D.E (2015) epidemic a virus ZIKA Polynesie 257 Francaise 2013-2014. Bulletin d'Information Sanitaires, Epidemiologiqueset 258 Statistiques ;Available from: http: 259 //www.hygiene-publique.gov.pf/IMG/pdf/no13 mai 2015_-_zika.pdf.

Naresh R., Surabli P. and Misra A.K. (2008). Analysis of Vaccination Model for Carrier Infectious Disease with Environmental Effects, Nonlinear Analysis: *Modelling and Control, 13, No. 3, 331-350.*

Roth A., Mercier A., Lepers C., Hoy D., Duituturaga S. and Benyon E. (2014) Concurrent outbreaks of dengue, chikungunya and Zika virus infections an 216unprecedented epidemic wave of mosquito-borne viruses in the Pacific 2012-2014. 217 Euro Surveill;

Sharon. B, William F. F, Justin .C. and Folashade .A. (2016). It is made available under a CC-BY-ND 4.0 International license doi: http://dx.doi.org/10.1101/041897.

Alex P.T, Amir S. S, Corrine W. R., Moritz U.G., Kraemer and Andrew J. T. (2016). Department of Biological Sciences and Eck Institute for Global.Diseases

UNICEF, (2011), United Nation International Children's Emergency Fund *annual report*

Vanden, P. D. and Watmough, J. (2002).Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. *Mathematicals Biosciences, 180, 29 – 48.*

World Health Organisation(2016).Western Pacific Region, Zika virus; Available 278 from:http: 279 //www.wpro.who.int/mediacentre/factsheets/fs_05182015_zika/en/.