THE THERMO-DIFFUSION EFFECTS ON HEAT AND MASS TRANSFER FLOW IN A POROUS ANNULUS DUE TO MAGNETIC FIELD

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Abstract
The focus of this paper is on the investigation of simultaneous effects of thermal-diffusion and magnetic field in an annulus with a porous medium. The dimensionless forms of the equations were solved analytically using Perturbation method. The solutions of velocity, temperature and concentration equations are derived. Skin-friction, Nusselt number and Sherwood number at the outer surface of inner cylinder are obtained. Selected sets of graphical results illustrating the effects of various controlling parameters involved in the problem on flow formation are discussed.

Keywords: *Thermo-diffusion, magnetic field, porous medium.*

Nomenclature B_0 External magnetic field

σ Electrical conductivity
k Thermal conductivity
C_p Specific heat at constant pressure
g Acceleration due to gravity
n Dimensional scalar constant
K Dimensional permeability of the porous medium
T_0 Initial temperaturee
T_w Temperature of the outer surface
K^* Absorption coefficient for thermal radiation
h_1 The velocity slip parameter
h_2 Temperature jump parameter
h_3 Concentration jump parameter
Pr Prandtl number
M magnetic parameter
Gr Grashof number
R stark number
Nu Nusselt number

Greek symbol
θ Fluid Temperature
τ Skin friction
ρ Density
υ Kinematic Viscosity
λ Suction/Injection parameter
β Coefficient of volume expansion
ϕ concentration

1.0 Introducton

The flow through a porous medium is of great interest in oil refineries, chemical sciences, life sciences and medical sciences. In agriculture sector, the proper distribution of fertilizers and pesticides is insured using the annulus porous medium. The flows through porous annulus type configurations are encountered in many industries in one or other ways for cooling purpose or for heat connection processes. The pioneer work of Chuhan and Rastogi (2010) reported on radiation effect on natural convection MHD flows in a rotating vertical porous channel partially filled with a porous medium. Jha *et al*. (2015) reported unsteady MHD free convection Couette flow between vertical porous plates with thermal radiation. However, Jha *et al*. (2015) studied MHD natural convection flow in a vertical parallel plate microchannel.

Gundagami *et al*. (2013) reported unsteady magneto hydrodynamic free convective flow past a vertical porous plate. Ahmad *et al*. (2016) studied the chemical reaction effect on natural convection flow between fixed vertical plates with suction and injection. Farhad *et al*. (2013) studied the influence of thermal radiation on unsteady free convection MHD flow of Brinkman type fluid in a porous medium with Newtonian heating. Shuny *et al.* (2016) reported on flow downward penetration of vertical parallel plates natural convection with as asymmetrically heated wall. Idowu *et al.* (2013) carried out investigation on Heat and mass transfer of magneto hydrodynamic (MHD) and dissipative fluid flow pass a moving vertical Porous plate with variable suction. Nagarajue *et al*. (2013) investigated the steady flow of an electrically conducting incompressible micro-polar fluid in a narrow gap between two concentric rotating vertical cylinders with porous lining on inside of outer annulus under an imposed axial magnetic field. Kuo and Leong (2013) investigated analysis of a conducting fluid in a thin annulus with rotating insulated walls under radial magnetic effect. Yeh *et al.* (2014) derived analytical solution for MHD flow of a magnetic fluid within a thick porous annulus. Shihhao *et al.* (2014) studied analytical solution for MHD flow of a magnetic fluid within a thick porous annulus. Abbas *et al.* (2013) dealt with laminar flow and heat transfer of an electrically conducting viscous fluid over a stretching cylinder in the presence of thermal radiations through a porous medium. Aldoss (2014) studied the MHD mixed convection flow about a vertical cylinder embedded in a non-Darlian Porous medium with variable heat transfer boundary. Suneetha and Bhasker (2014) analyzed the interaction of free convection with thermal radiation of a viscous in compressible unsteady MHD flow past a moving vertical cylinder with heat and mass transfer in a porous medium. Sharma *et al.* (2014) studied MHD flow and heat transfer through a circular cylinder partially filled with non–Darcy porous media. Yadav and Shama (2014) investigated the effect of porous medium on MHD fluid along a stretching cylinder. Jhaa nd Odengle (2016) studied a semi analytical solution for start-up flow in an annulus partially filled with porous material. In the present work, we shall investigate a Diffusion-thermo effect on Hydromagnetic heat and mass transfer flow through a circular cylinder.

2.0 MATHEMATICAL ANALYSIS

This problem considers the natural convection heat and mass transfer flow in an infinite vertical annulus formed by two infinite concentric vertical cylinders in presence of thermal radiation and transverse magnetic field. Figure 1, shows the physical configuration of the problem. A uniform magnetic field of strength B_0 is imposed normal to the porous cylinder. The z - axis is along the upward direction of the annulus and x - axis (radial co-ordinate) normal to it. The radiation term q_r fluid under consideration is viscous and incompressible which occupies infinite vertical annulus space of distance $b¢$ and $a¢$ dimensional inner radius of the cylinder.

Applying Rosseland approximation, the radiative heat transfer takes the form following Siegel and Howell (1972), respectively as follows:

$$q_r = -\frac{4\sigma^*}{3K^*}\frac{\partial T'^4}{\partial z'};\ \overline{q_r} = -\frac{4\sigma^*}{3K^*}\frac{\partial T^4}{\partial z'}. \qquad (1)$$

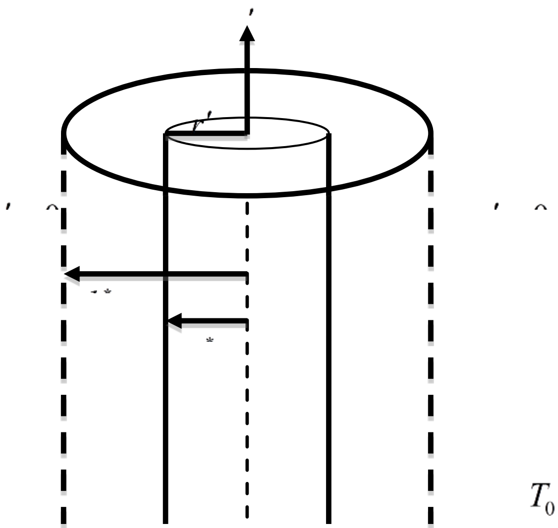

Figure 1: Physical configuration of the problem

Following Raptis (1998), the temperature function in (1) can be expressed as a linear function of temperature.

Expanding T'^4 and $\overline{T^4}$ in a Taylor series about T_d and neglecting higher –order terms, we obtain

$$T'^4 = 4T_d^3T' - 3T_d^4 \quad \text{and} \quad \overline{T^4} = 4T_d^3\overline{T} - 3T_d^4 \quad . \qquad (2)$$

Next introducing the following non-dimensional quantities,

$$r = \frac{z'}{d},\ \alpha = \frac{h}{d},\ \alpha = \frac{h}{d},\ \lambda = \frac{w_0 d}{\upsilon},\ U = \frac{\bar{u}}{w_0},\ \bar{V} = \frac{\bar{v}}{w_0},\ u = \frac{u'}{w_0},\ v = \frac{v'}{w_0},\ \theta = \frac{\bar{T} - T_d}{T_0 - T_d}, \qquad (3)$$

with the help of equations (1) - (3), the dimensionless governing equations for the presence problem are given by:

$$l\,\frac{1}{r}\frac{du}{dr} = \frac{1}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) + Grq + Nf - \frac{1}{r^2}\left(\frac{1}{K} + M^2\right)u \qquad (4)$$

$$\lambda\frac{1}{r}\frac{d\theta}{dr} = \left(\frac{3R+4}{3R\Pr}\right)\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta}{dr}\right) + Df\left(\frac{d^2\phi}{dr^2} + \frac{1}{r}\frac{d\phi}{dr}\right) \qquad (5)$$

$$\lambda\frac{1}{r}\frac{d\phi}{dr} = \frac{1}{r}\frac{d}{dr}\left(r\frac{d\phi}{dr}\right) \qquad (6)$$

where $Pr=\frac{\mu C_p}{k}$, $R=\frac{kK^*}{4\sigma^* T_d^3}$, $M^2=\frac{\sigma}{\mu}B_0^2 d^2$, $Gr=\frac{g\beta(T_0-T_d)d^2}{\upsilon w_0}$

the boundary conditions are

$$u=1+h_1\frac{du}{dr},\ \theta=1+h_2\frac{d\theta}{dr},\ \phi=1+h_3\frac{d\phi}{dr},\ at\ r=1 \quad (7)$$

$$u=0,\ \theta=0,\ \phi=0,\ at\ r=\alpha$$

To obtain solution to equations (4) to (6), Euler method was applied by assuming r to be e^t, concentration, temperature velocity, and reads:

$$\phi=A_6+B_6r^a \quad (7)$$

Substitute equation (7) into equation (5) and using equation (9) subject to boundary condition (7) we obtain temperature profile as follows:

$$\theta=A_7+B_7r^{c_2}+L_1r^a \quad (8)$$

Substitute equations (7) and (8) into equation (4) and using equation (8) the required solution of velocity subject to boundary condition (7) reads as:

$$u=A_8r^{L_4}-B_8r^{L_5}+L_6r^2+L_7r^{\lambda+2}+L_8r^{c_2+2} \quad (9)$$

From (9), the steady-state skin friction on the outer surface of the cylinder at $r=1$is:

$$t_1=\left.\frac{du}{dr}\right|_{r=1}=L_4A_8+B_8L_5+2L_6+L_7(l+2)+L_8(c_2+2) \quad (10)$$

Also from (8), the steady-state Nusselt numberon the outer surface of the cylinder at $r=1$is:

$$Nu_1=\left.\frac{d\theta}{dr}\right|_{r=1}=c_2B_7+\lambda L_1 \quad (11)$$

Finally, from (7), the Sherwood number on the outer surface of the cylinder at $r=1$is:

$$Sh_1=\left.\frac{d\phi}{dr}\right|_{r=1}=\frac{\lambda}{1-\alpha^{\lambda}-h_3\lambda} \quad (12)$$

3.0 RESULTS AND DISCUSSION

The expressions of the physical situation presented in (4) to (6) are solved employing the Euler method subject to the boundary conditions (7). The energy equation is coupled to the mass transfer by composition gradients parameter and mass transfer equation is coupled to the velocity by the sustention parameter N as illustrated in equations (4) and (6) respectively.

The controlling parameters that governed this flow are Grashof number $Gr(\pm 5)$which corresponding to both cooling and external heating of the channel plate and is employed for practical applications in nuclear technology and also in geophysical and naval energy system applications (1998)**,** Prandtl number Pr chosen as 0.71 and 7.0 that physically represent two fluids air and water respectively. However, suction/ injection (λ), which is simultaneously opposite to annular at the same rate, Stark number (R), magnetic parameter (M), the velocity slip parameter(h_1), temperature jump parameter $(h_2)$and concentration jump parameter (h_3) are choosing arbitrary.

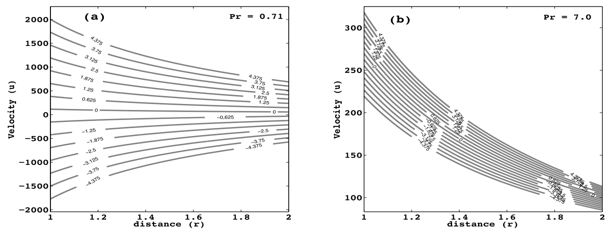

Figure 2: Effect of Gr on velocity when $Df = 0.1, h_1 = 0.34,\ h_2 = 0.34,\ h_3 = 0.34, K = 0.9$
$M = 1, N = 0.5, Sc = 1, / = 0.5$ *and* $a = 2$

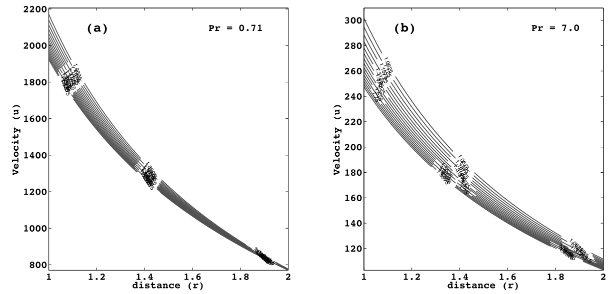

Figure 3: Effect of K on velocity when $Df = 0.1, Gr = 5, h_1 = 0.34,\ h_2 = 0.34,\ h_3 = 0.34,$
$M = 1, N = 0.5, Sc = 1, / = 0.5$ *and* $a = 2$

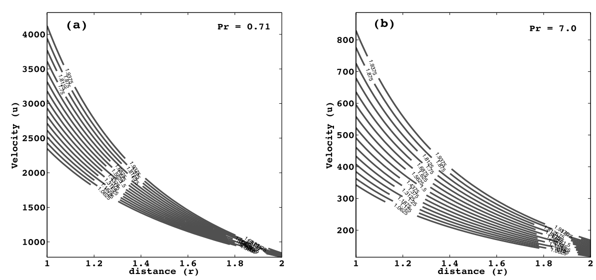

Figure 4: Effect of M on velocity when $Df = 0.1, Gr = 5, h_1 = 0.34,\ h_2 = 0.34,\ h_3 = 0.34, K = 0.9$
$N = 0.5, Sc = 1, / = 0.5$ *and* $a = 2$

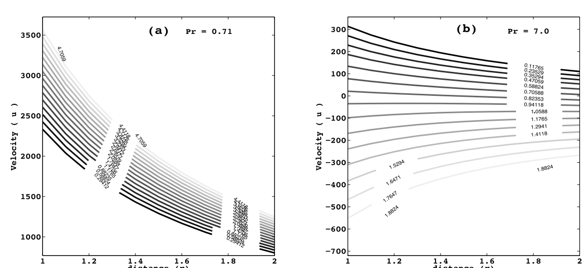

Figure 5: Effect of Df on velocity when $Gr = 5, h_1 = 0.34,\ h_2 = 0.34,\ h_3 = 0.34, K = 0.9$
$M = 1,\ N = 0.5, Sc = 1, / = 0.5$ *and* $a = 2$

Figure (2) shows the effect of Grashof number on velocity when other parameters are fixed. From figure (2a) and (2b), when $Gr > 0$ the velocity of the air ($Pr = 0.71$) is significantly higher than that of water ($Pr = 7.0$). This corresponds to the external cooling of the cylinder, which results in thickening the boundary layer and consequently assist the velocity. However, for $Gr < 0$ the velocity of the air takes reverse flow to the negative direction as depicted in figure (2a).

Figure (3) illustrates the effect of porous medium parameter (K) on velocity when other parameters are fixed. As porous medium (K) increases, the velocity of the fluid also increases for $Gr > 0$. Furthermore, the velocity of the air is higher than that of water because air move faster than water, since density of water is higher than that of the air see figures (3a) and (3b).

Figure (4) depicts the effect of Magnetic parameter (M) on velocity. When Magnetic parameter (M) increases, the velocity of the fluid decreases, this physically reveals that the hydromagnetic drag embodies in the term $-M^2u$ of the velocity equation (4) decreases the velocity in the cooling channel of the free convection current as illustrated in figure (4a) and (4b). However, the velocity of air is higher than the velocity of water.

Figure (5) presents the effect of thermal- diffusion parameter (Df) on velocity when other parameters are treated constant**.** From figure (5a) the values of velocity increases as Df increases while reverse trend is reveled in figure (5b), with increase in thermal- diffusion parameter (Df).

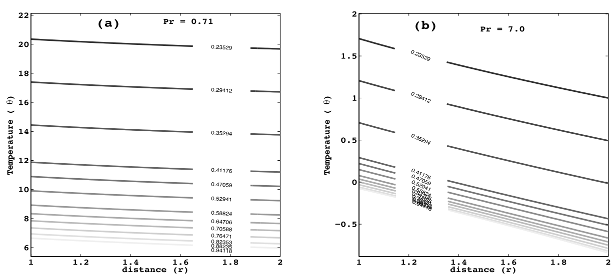

Figure 6: Effect of R on temperature when $h_2 = 0.34,\ h_3 = 0.36\ and\ \lambda = -0.5$

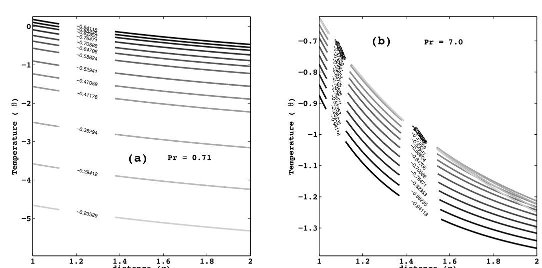

Figure 7: Effect of suction/injection parameter $(\lambda < 0)$ on temperature when $h_2 = 0.34,\ h_3 = 0.36\ and\ R = 1$

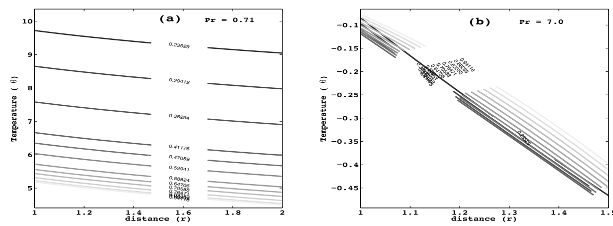

Figure 8: Effect of suction/injection parameter $(\lambda > 0)$ on temperature when h_2 =0.34, $h_3 = 0.36$ and R=1

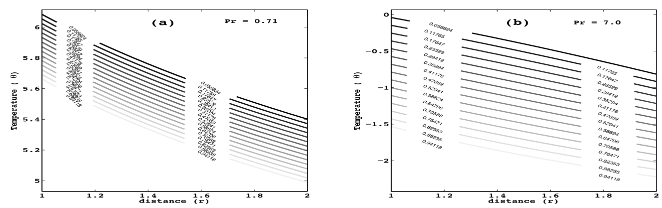

Figure 9: Effect of Df on temperature when R=1, λ =0.5, h_1 =0.34, h_2 =0.34, h_3 =0.36,Sc=1

Figure (6) illustrate the effect of stark number (R) on temperature. An increase in Stark number R decreases the temperature as observed from figure (6a) and (6b). In addition, the temperature is higher on air (Pr=0.71) than water (Pr=7.0).

Figure(7) and (8) represent the effect of suction/injection on temperature.When suction/injection(λ)parameter increases, the temperature reduces, this is due to the decrease in thermal radiation flux.

Figure (9) show the effect of thermal- diffusion parameter (Df) on temperature when other parameters are treated constant. It is observed that quantitatively as Df increases, the value of temperature reduces as shown in figures (9a) and (b).

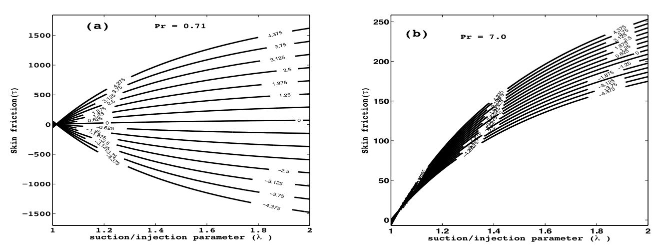

Figure 10: Skin friction for different values of Gr at $y = 1$

Figures (10a) and (10b) present skin friction against Grashof number at the outer surface of the cylinder $(r = 1)$. It is found that for Grashof number $Gr > 0$ the skin friction increases. This corresponds to the external cooling of the outer surface of the cylinder, which results in thickening the boundary layer and consequently assist the Skin friction. It also observed that for $Gr < 0$ the skin friction values takes negative values.

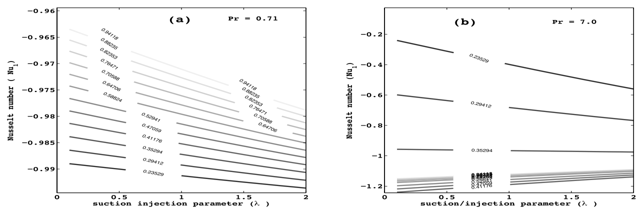

Figure 11: Nusselt number for different values of R at $y=1$

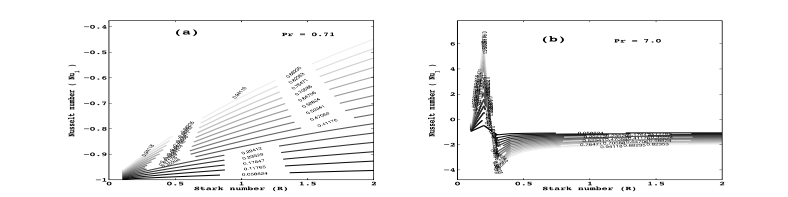

Figure 12: Nusselt number for different values of Df at $y=1$

Figure (11) and (12) also present the landmark influence of suction/injection parameter $(/)$ on Nusselt number (rate of heat transfer) at the outer surface of the cylinder $(r=1)$. Form figures (11a, b) and (12a, b), it is reported that as suction/injection increases Nusselt number decrease for the fixed values of R and Pr respectively. Furthermore, it is remarkable to point out that the impacts of suction/injection on Nusselt number are prominent either in air or water, from the figure (11a) and (11b) for $(/ < 0)$ in comparison with figure (12a) and (12b).

4.0 CONCLUSION

The radiation effect on natural convection inside annulus with porous medium is investigated analytically using Perturbation method. The expressions for velocity, temperature, concentration, skin friction, Nusselt number, and Sherwood number are obtained. The outcomes from the result show that:

i. The results show that the velocity increases with increase in Grashof number $(Gr>0)$, thermal- diffusion parameter (Df) and porous medium parameter (K) while decreases with Grashof number $(Gr<0)$ and Magnetic parameter (M).

ii. The increase of Stark number (R), thermal- diffusion parameter (Df) and \ suction/injection parameter (λ) decreases the temperature.

iii. The increase of Grashof number (Gr) increases Skin friction.

iv. An increase in suction/ injection (λ) reduces the Nusselt number.

v. Sherwood number increases with increase in suction/ injection (λ).

Appendix

$$B_6 = \frac{1}{1-\alpha^{\lambda} - h_3\lambda}; \quad A_6 = \frac{-\alpha^{\lambda}}{1-\alpha^{\lambda} - h_3\lambda}; \quad C_1 = \frac{3R+4}{3R\,\mathrm{Pr}}; \quad C_2 = \frac{\lambda}{C_1};$$

5.0 REFERRENCES

Chauhan, D.S.,and Rastogi, P., (2010) Radiation effect on natural convection MHD flow in a rotating vertical porous channel partially filled with a porous medium. *Journal of Applied Mathematical Science* .**4**(13): 643-655.

Chauhan, D. S. and Kumar, V., (2011) On radiation effects on mixed convection flow and viscous heating in a vertical channel partially filled with a porous medium. *Tamkang Journal of Science and Engineering.***14**(2): 97-106.

Jha, B. K., Isah, B. Y. and Uwanta, I. J., (2015) Unsteady MHD free convection Couette flow between vertical porous plates with thermal radiation. *Journal of King Saudi University –Science*. **27**: 338-348.

Jha, B. K., Aina, B., and Ajiya, A. T., (2015) MHD natural convection flow in a vertical parallel plate microchannel.*Ain Shams Engineering Journal***6**: 289-295.

Raju, M.C.,(2011) Unsteady MHD free convection oscillatory couette flow through a porous medium with periodic wall temperature. *I-manager Journal on Future Engineering and Technology*. **6**(4): 7-12.

Gundagami, M., sheri, S., Paul, A. and Reddy, M.C.K., (2013) Unsteady magneto hydrodynamic free convective flow past a vertical porous plate. *International journal of Applied Science and Engineering.***11**(3): 267-275.

Ahmad, S. K., Isah, B. Y. and Altine M. M. (2016) Chemical reaction effect on natural convection flow between fixed vertical plates with suction and injection. *Journal of the Nigerian Association of Mathematical Physics*. **36**(2): 131-140.

Farhad, A. Ilyas, K. Sami U.I. and Sharidan S. (2013) Influence of thermal radiation on unsteady free convection MHD flow of Brinkman type fluid in a porous medium with Newtonian heating. *Mathematical Problem in Engineering.***13**: 1-13.

Shuny,W. U. and Chao W.S. and Li C. G. (2016) Flow downward penetration of vertical parallel plates natural convection with as asymmetrically heated wall. *International Communications in Heat and Mass Transfer.***74**: 55- 62.

Idowu, A. S. Dada, M. S. and Jimoh, A. (2013) Heat and mass transfer of magneto hydrodynamic (MHD) and dissipative fluid flow pass a moving vertical Porous plate with variable suction. *Mathematical Theory and modeling*. **3**(3): 80-104.

Nagarajue,G., Murthy, J. V. F. and Sai, K. S.,(2013) The steady flow of an electrically conducting incompressible micro-polar fluid in a narrow gap between two concentric rotating vertical cylinders with porous lining on inside of outer annulus under an imposed axial magnetic field. *ActaTechnicacorvininesis –Blletin of Engineering.***3**(3): 115-132.

Kuo, J. C. and Leong, J. C., (2013) Analysis of a conducting fluid in a thin annulus with rotating insulated walls under radial magnetic effect. *Applied Mathematical Modeling.***37**(5): 3021-3035.

Yeh, S., Chen, T. J. and Leong, J.,(2014) Analytical solution for MHD flow of a magnetic fluid within a thick porous annulus. *Journal of Applied Mathematics.***12**: 1-10.

Shihhao, Y., Tsai, J. C. and Leong, J. C., (2014) Analytical solution for MHD flow of a magnetic fluid within a thick porous annulus. *Journal of Applied Mathematics.* **14:** 732-744.

Abbas, Z., Majeed, A. and Javed, T., (2013) Laminar flow and heat transfer of an electrically conducting viscous fluid over a stretching cylinder in the presence of thermal radiations through a porous medium. *Alexandria Engineering Journal.***54**(4): 1029-1036.

Aldoss, T. K., (2014) MHD mixed convection flow about a vertical cylinder embedded in a non-Darlian Porous medium with variable heat transfer boundary. *International Communication in Heat and Mass Transfer*. **23**(4): 517-530.

Suneetha, S. and Bhasker, R. N., (2014) Interaction of free convection with thermal radiation of a viscous in compressible unsteady MHD flow past a moving vertical cylinder with heat and mass transfer in a porous medium. *Theoretical Applied Mechanics.***15**: 345-359

Sharma, M. K. Singh, K. and Kumar A., (2014) MHD flow and Heat Transfer through a circular cylinder partially filled with non – Darcy porous media. *International Journal of Innovation Technology and Exploring Engineering.***4** (7): 30-42.

Yadav, R. S., Shama, P. R., (2014) Effects of porous medium on MHD fluid flow along a stretching cylinder. *Annals of Pure and Applied Mathematics.***6**(1): 104-113.

Jha, B. K., and Odengle, J. O., (2016) A semi-analytical solution for start-up flow in an annulus partially filled with porous material. *Transport in Porous Media*. **113**(1) : 1-19.