ORDERING POLICY FOR AMELIORATING INVENTORY WITH LINEAR DEMAND RATE AND UNCONSTRAINED RETAILER'S CAPITAL

by

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Abstract

Items that incur a gradual increase in quality, quantity or both while in inventory are referred to as ameliorating items. Fruits, wine, high breed fishes in breeding yard (fish culture facility), fast growing animals like broiler, goose, rabbit etc. in farming yard provide good examples. When these items are stoked in the inventory or in the production centre, they undergo amelioration at some stages of their storage. This paper proposes a model that determines the optimal replenishment cycle time, such that the total variable cost is minimized.The amelioration rate and holding cost are constants, the demand rate throughout the cycle is linear time dependent, and shortages are not allowed. Numerical examples are provided to illustrate the application of the model developed.

Keywords:*Inventory, Amelioration, Ordering Policy, Linear demand rate, Unconstrained retailer's capital*

1.0 Introduction

In the classical inventory models, one of the assumptions was that the items preserved their physical characteristics while they were kept in the inventory or in the production centers. This assumption is not always true because some items are subject to risks of breakage, damage, spoilage, evaporation, obsolescence, etc. The decay that prevents items from being used for their original purpose is termed as deterioration.Although degradation (or loss) of value or utility or quantity of some physical goods is a commonexperience, Moon *et al. (*2003)observed that, there are some items whose value or utility increase over time by amelioration activation,e.g. wine. It is a common experience in wine manufacturing circle that utility or value of some kind of wine increases by time. Other examples can be seen with fruits (like orange, pineapple, mango etc.), high breed fishes in breeding yard (fish culture facility)or fast growing animals like broiler, goose, rabbit etc. in farming yard. These items at the initial stage of their storage or production environment undergo amelioration.

Two basic key factors are necessary in the development of models for ameliorating items these are: demand and amelioration rate. Demand acts as driving force of the entire inventory system and the amelioration rate stands for the characteristics of the ameliorating items. Li *et al*. (2010) classified demand into two types: the one that can be determined over certain period of time (Deterministic demand), for example, constant demand, time-dependent demand, inventory-level-dependent demand, price-dependent demand and among them Ramp-type demand are all deterministic demand, and Stochastic demand (the one that is characterized by a known distribution and the one that is characterized by arbitrary distribution). Hill (1995) was the first researcher to use Ramp type demand in his inventory model followed by Mandal and Pal (1998). Ameliorating rate and deteriorating rate are other key factors to be considered in developing model of ameliorating or deteriorating items. Earlier researchers like Ghare and Schrader (1963), Shah and Jaiswal (1977), Aggarwal (1978), Padmanabhana and Vrat (1995) and many others considered constant deteriorating rates in their models. However, recent researchers considered several scenarios of relationship between time and deteriorating rates. Some of these scenarios include: deteriorating rate as linear function of time, two-parameter Weibull distribution, three-parameter Weibull distribution and or deteriorating rate as other function of time.It is common experience in the market palace that the demand for inventory items increases with time in the growth phase, and decreases in the decline phase. So researchers commonly use a time-varying demand pattern to reflect sales in different phases of product circle. It all started with Silver and Meal in the early 1970’s who came up with replenishment lot size model with deterministic time varying demand rate. Later, Donaldson (1977) came up with inventory replenishment policy for a linear trend in demand-an analytical solution. Recently, Ahmad and Musa (2016) developed an EOQ model with time dependent exponential declining demand.

The existing literature on inventory seems to ignore or give little attention to ameliorative inventories. Hwang (1997)was the first researcher to develop EOQ models for ameliorating items with the assumption that the ameliorating time follows the Weibull distribution. Again Hwang(1999) came up with other models for both ameliorating and deteriorating items separately considering LIFO and FIFO issuing policies. A partial selling inventory model for ameliorating items under profit maximization was developed by Mondal et al. (2003). Singh *et al.*(2011)used genetic algorithm to propose an optimal replenishment policy for ameliorating items under inflation and time value of money. Shortages were allowed and back-ordering was considered to be a decreasing function of waiting time. Panda *et al*. (2013) provides a note on inventory model for ameliorating items with time dependent second order demand rate. Harvest and sale decision problem of fresh agricultural products considering both amelioration of field items and deterioration of stored items was proposed by Chen (2011). This profit model of the farmer was developed via two situations: when the fresh agricultural products are harvested at maturing point and when they are harvested at critical ripeness point. Recent research on ameliorating items was carried out by Gwanda and Sani (2011). The model determined an optimum order quantity in which the demand rate, the amelioration rate and holding cost are all constants.

In the present article, an attempt has been made to propose an inventory model for ameliorating items in which the ameliorating rate and holding cost are constant. The demand rate is linear function of time throughout the cycle and retailer’s capital is unconstrained, that is, the payment is affected on the receipt of the items in the inventory.

2.0 Assumptions and Notations

The model is developed based on the following assumptions and Notations:

i. Both the amelioration rate and holding cost are constants.

ii. The replenishment rate is instantaneous, lead time is zero.

iii. The inventory system involves only one single item and one stocking point.

iv. Shortages are not allowed.

v. Amelioration occurs when the items are effectively in stock.

vi. The demand rate λ is time dependent and linear i.e $\lambda(t) = \alpha + \beta t$.

vii.T is the length of the cycle and it is the time when the inventory level reaches zero.

$viii$ $I(t)$ is the inventory level at any time t.

ix Q is the optimal ordering quantity per cycle.

x. A_o is the fixed ordering cost per order.

xi. I_o is the initial inventory at $t = 0$.
xii. A_m is the ameliorated amount.
$xiii$. h is the inventory holding cost per unit per unit of time.
xiv c is the cost of each ameliorated item.
xv. $TVC(T)$ is the total (average) inventory cost per unit time.

3. 0 Mathematical formulation and solution

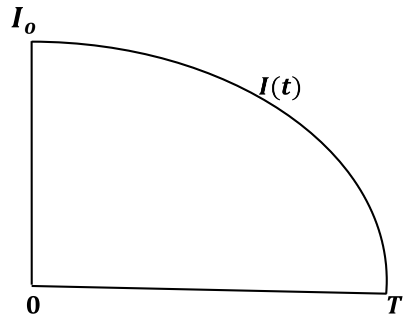

Fig 1: The inventory depletion in a constant amelioration system with no shortages

The depletion of the inventory during the interval $[0, T]$is a function of the ameliorating rate, demand rate and the remaining inventory level at the inventory system.Thus, the differential equation that describes the state of the inventory level $I(t)$during the time interval $(0 \leq t \leq T)$is given by: $\frac{dIt)}{dt} - AI(t) = -\lambda(t) 0 \leq t \leq T$Eq. (1)
Equation (1) is first order linear differential equation given by:

$$\frac{dI(t)}{dt} - AI(t) = -(\alpha + \beta t)$$

The integrating factor, $\rho = e^{-\int A dt} = e^{-At}$.
The solution is given by:
$e^{-At} I(t) = -\int e^{-At} (\alpha + \beta t) dt$.
$\Longrightarrow I(t) = \frac{\alpha+\beta t}{A} + \frac{\beta}{A^2} + ke^{At}$, k is a constant, Eq. (2)
At $t = 0, I(0) = I_0$,

$$\therefore \qquad I_0 = \frac{\alpha}{A} + \frac{\beta}{A^2} + k,$$

$\Longrightarrow$ $k = I_0 - \left(\frac{\alpha}{A} + \frac{\beta}{A^2}\right)$. Eq. (3)
Substituting (3) in (2),

$$I(t) = \frac{\alpha + \beta t}{A} + \frac{\beta}{A^2} + \left[I_0 - \left(\frac{\alpha}{A} + \frac{\beta}{A^2}\right)\right] e^{At}$$

$$= \frac{\alpha+\beta t}{A} + \frac{\beta}{A^2} + I_0 e^{At} - \left(\frac{\alpha}{A} + \frac{\beta}{A^2}\right) e^{At} \qquad \text{Eq. (4)}$$

At $t = T$, $I(t) = 0$,
Equation (4) becomes,

$$I_0 e^{AT} = \left(\frac{\alpha}{A} + \frac{\beta}{A^2}\right) e^{AT} - \frac{\alpha + \beta T}{A} - \frac{\beta}{A^2}.$$

$\Longrightarrow I_0 = \frac{\alpha}{A} + \frac{\beta}{A^2} - \frac{\alpha+\beta T}{A} e^{-AT} - \frac{\beta}{A^2} e^{-AT}.$ Eq. (5)

Substituting (5) in (4),

$I(t) = \frac{\alpha+\beta t}{A} + \frac{\beta}{A^2} + \left(\frac{\alpha}{A} + \frac{\beta}{A^2} - \frac{\alpha+\beta T}{A} e^{-AT} - \frac{\beta}{A^2} e^{-AT}\right) e^{At} - \left(\frac{\alpha}{A} + \frac{\beta}{A^2}\right) e^{At}.$

$= \left(\frac{\alpha}{A} + \frac{\beta}{A^2}\right) - \left(\frac{\alpha+\beta T}{A} + \frac{\beta}{A^2}\right) e^{A(t-T)}.$ Eq. (6)

The ameliorated amount, A_m is given as Total demand in the cycle− The beginning inventory level.

$$\Longrightarrow A_m = \int_0^T \lambda(t)dt - \left[\left(\frac{\alpha}{A} + \frac{\beta}{A^2}\right) - \left(\frac{\alpha+\beta T}{A} + \frac{\beta}{A^2}\right) e^{-AT}\right].$$

$= \frac{\alpha+\beta T}{A} e^{-AT} - \frac{\beta}{A^2}(1 - e^{-AT}) + \alpha\left(T - \frac{1}{A}\right) - \frac{1}{2}\beta T^2.$ Eq. (7)

The total inventory carried in the cycle T,H_T is given as:

$$H_T = \int_0^T I(t)dt = \int_0^T \left[\left(\frac{\alpha}{A} + \frac{\beta}{A^2}\right) - \left(\frac{\alpha+\beta T}{A} + \frac{\beta}{A^2}\right) e^{A(t-T)}\right]$$

$= \left(\frac{\alpha+\beta T}{A^2} + \frac{\beta}{A^3}\right)(e^{-AT} - 1) + \left(\frac{\alpha}{A} + \frac{\beta}{A^2}\right) T.$Eq.(8)

The total average cost per unit time $TVC(T)$ is given by:

$TVC(T) = \frac{1}{T}[ordering\ cost, A_0 + holding\ cost, H_c - cost\ of\ ameliorated\ items].$

$= \frac{1}{T}[A_0 + hH_T - cA_m].$

$$= \frac{A_0}{T} + \frac{h}{T}\left[\left(\frac{\alpha+\beta T}{A^2} + \frac{\beta}{A^3}\right)(e^{-AT} - 1) + \left(\frac{\alpha}{A} + \frac{\beta}{A^2}\right) T\right] - \frac{c}{T}\left[\frac{\alpha+\beta T}{A} e^{-AT} - \frac{\beta}{A^2}(1 - e^{-AT}) + \alpha\left(T - \frac{1}{A}\right) - \frac{1}{2}\beta T^2\right]$$

Eq. (9)

The necessary condition for $TVC(T)$ to be minimized is given by: $\frac{dTVC(T)}{dT} = 0.$

$$\Longrightarrow -\frac{A_0}{T^2} + \frac{1}{A^2T^2}\left[\big((cA-h)AT + (c-h)\big)(\alpha+\beta T)e^{-AT} + \left(\frac{\beta h}{A} - c\beta\right)(1 - e^{-AT}) + \alpha(h - cA) + c\right]$$

$-\frac{c\beta}{2} = 0$ Eq.(10)

Multiplying equation (10) by T^2 we get:

$$-A_0 + \frac{1}{A^2}\left[\big((cA-h)AT + (c-h)\big)(\alpha+\beta T)e^{-AT} + \left(\frac{\beta h}{A} - c\beta\right)(1 - e^{-AT}) + \alpha(h - cA) + c\right]$$

$-\frac{c\beta}{2}T^2 = 0$ Eq. (11)

The Economic Order Quantity, EOQ, is given by:

The total demand in a cycle period – ameliorated mount.

$\Longrightarrow$ EOQ $= \int_0^{\square} \square(\square)\square\,\square - \left[\frac{\square+\square\,\square}{\square}\square^{-\square\,\square} - \frac{\square}{\square^2}(1 - \square^{-\square\,\square}) + \square\left(\square - \frac{1}{\square}\right) - \frac{1}{2}\square\square^2\right]$

$$= \square\,\square + \frac{\square}{2}\square^2 - \left[\frac{\square+\square\,\square}{\square}\square^{-\square\,\square} - \frac{\square}{\square^2}(1 - \square^{-\square\,\square}) + \square\left(\square - \frac{1}{\square}\right) - \frac{1}{2}\square\square^2\right]$$

$= \frac{\square}{\square^2}(1 - \square^{-\square\,\square}) + \frac{\square}{\square}[(\square + \square\,\square) + 1] + \square\square^2$Eq. (12)

4.0 Numerical examples

For the purpose of numerical examples, eight parameter values in proper units are considered (as input) and the output of the model using Maple (2015) Mathematical Software gives the corresponding Optimal cycle length (□), the minimum total inventory cost(□ □ □) and Economic Order Quantity (EOQ) in the table below:

Table 1: EOQ, Total variable cost and optimal cycle length for ameliorating items with linear demand rate.

S/N	□□	□	h	C	□	□	□	□ □ □	□ □ □
1	1000	0.25	0.03	10	3	700	0.0423 (9 days)	21678	291
2	1000	0.35	0.03	10	3	700	0.0176 (6 days)	50396	175
3	1000	0.45	0.03	10	3	700	0.0037 (1 day)	229785	89
4	4000	0.33	0.75	30	10	500	0.0100 (4 days)	279537	367
5	4000	0.33	0.45	30	10	500	0.0198 (7 days)	285231	497
6	4000	0.33	0.25	30	10	500	0.0104 (4 days)	268802	507
7	4500	0.15	0.4	15	5	400	0.0016 (1 day)	245321	222
8	4500	0.15	0.2	15	5	400	0.0014 (1 day)	280365	225

5.0 Discussion of the results

From the above numerical examples we observe that amelioration rate and the holding cost affect the EOQ. It is clear that the higher the amelioration rate, the lower the EOQ. Thus, the stockiest is advised to purchase less in items with higher amelioration rate but purchase more in items with lower rate of amelioration. This is obvious for the stockiest to avoid over stocking which leads to the increase in total variable costs.Also we observe that the EOQ reduces with the rise in holding cost. Of course as the holding cost rises, it will be more economical to reduce the order quantity in order to maximize profit.

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