MATHEMATICAL ANALYSIS OF A MODEL TO INVESTIGATE THE DYNAMICS OF POVERTY AND CORRUPTION

BY

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Abstract

The concept of corruption has been actively studied by mathematicians for decades with the aim of obtaining optimal strategy of corruption control. This study investigates the relationship between corruption and poverty. This paper mainly concentrates on corruption and poverty free states and not on the endemic case. The poverty free equilibrium and corruption free equilibrium of the sub models were obtained and analyzed separately for local and global stability. The full model for poverty and corruption were also analyzed for local and global stability. Numerical simulations were carried out which showed that corruption can be reduced to the barest minimum within a shorter period as compared to poverty even with high control strategy.

Keywords*:* *Corruption, Equilibrium, Global Stability, Local Stability and Poverty*

Introduction

The phenomenon of corruption has adverse impact on the economy and society. In everyday use, corruption is a term which conveys an element of moral disapproval. The World Bank has defined corruption as "the abuse of public office for private gain". Additionally, Transparency International (TI) defined corruption as "the misuse of entrusted power for private benefit" Corruption is a pressing global problem and no country in the world is totally free of its menacing grip. It is a worm within the body of the society and a cancer to economic, social, and political development. Lack of transparency in rules, laws, and processes makes a breeding ground for corruption (Abdulrahman, 2014). Caulkins (2008) defined corruption as "an abuse of position or inducement of an abuse of position for an undeserved benefit, advantage or relief". Thus, forms of corruption include: bribery, extortion, fraud, embezzlement, nepotism, cronyism, appropriation of public institutions assets and properties for private use, and influence peddling. They stated some fundamental factors that engender corrupt practices in less developed nations to include (i) the great inequality in the distribution of wealth (ii) political office as the primary means of gaining access to wealth (iii) conflict between changing moral codes (iv) the weakness of social and governmental enforcement mechanism (v) absence of a strong sense of national community. However, obsession with materialism, compulsion for a short-cut to affluence, glorification and approbation (of ill-gotten wealth) by the general public are among the reasons for the persistence of corruption (Eduda, 2017)

Although the possible positive aspects of corruption is often debated as confirmed in the work of Maoro (1995) where it was shown that corruption impacts negatively on a variety of economic and social aspects of life. Ultimately, corruption slows economic growth, as it becomes more difficult to attract investment. Poverty is much more than just having enough money to meet basic needs including food, clothing and shelter. The world health organization has described poverty in this way; ‘poverty is hunger, poverty is lack of shelter, poverty is being sick and not being able to see the doctor, poverty is not having access to school and not knowing how to read, poverty is not having a job, it is fear for the future, living one day at a time

Corruption has contributed immensely to the poverty and misery of a large segment of the population. Most of the studies which have investigated the link between corruption and poverty have shown that corruption impacts negatively on a variety of economic and social aspects of life. Corruption is a cause of poverty and a barrier to successful poverty eradication. It could destroy the efforts of developing countries in order to alleviate poverty. Corruption’s relation to poverty are numerous and common. In the public sector, corruption delays and diverts economic growth and deepens poverty. Alternatively, poverty invites corruption as it weakens economic, political and social institutions. Corruption is one of the major determinants of poverty. Combating corruption is therefore a crucial part in the poverty reduction process. High levels of corruption aggravate the living conditions of the poor by distorting the entire decision making process connected with public sector programs. Corruption deepens poverty by hampering productive programs such as education and health care at the expense of larger capital intensive projects which can provide better opportunities to extract illegal incomes. Alternatively, social and income inequalities in poor countries make greater imbalances in the distribution of power and encourage corruption (Ndikumana, 2006).

Poverty and Corruption have been actively studied by mathematicians for nearly forty years: the first mathematical model dedicated to corrupt structures appeared in 1975 (Rose-Ackerman, 1975).The techniques of population dynamics and compartmental models have been extensively used to study possible correlation between poverty and corruption (Negin, 2010). This paper provides an investigation into possible strategies aimed at reducing and controlling poverty and corruption using mathematical modeling approach.

Model Formulation

We consider a closed population that is homogeneous, so that space variations can be neglected and the only independent variable is time, t. Society is divided into five subclasses with the following assumptions;

(a) Susceptible individuals $S(t)$ are those who have never been involved in any corrupt practices that will have harmful effects on a country’s economic growth.
(b) Corrupt individuals $C(t)$ are individuals who are often involved in corrupt practices and are capable of influencing a susceptible individual to become corrupt.
(c) Poverty/Poor Individuals $P(t)$ are those who lack a usual or socially acceptable amount of money or material possessions and also the means to satisfy their basic needs
(d) Prosecuted/Jailed individuals $J(t)$ are those who have been convicted of corrupt practices and imprisoned for a specific period of time during which he cannot be involved in any corrupt acts and cannot influence others
(e) Honest Individuals are those who can never be corrupt no matter the condition they find themselves.

We present a model for poverty and corruption in a population written in terms of Ordinary Differential Equations (ODEs) as given below.

$$\frac{dS}{dt} = \Lambda - \frac{\alpha_1 CS}{N} - \frac{\alpha_2 PS}{N} - (\mu + \sigma)S$$
$$\frac{dC}{dt} = \frac{\alpha_1 CS}{N} + \frac{\gamma_2 PC}{N} + \psi(1-\phi)J - (\mu + \beta_c + \omega + \gamma_1)C$$
$$\frac{dP}{dt} = \frac{\alpha_2 PS}{N} + \gamma_1 C - \frac{\gamma_2 PC}{N} - (\mu + \beta_p)P \qquad (1)$$
$$\frac{dJ}{dt} = \omega C - \left(\psi(1-\phi) + \psi\phi + \mu\right) J$$
$$\frac{dH}{dt} = \beta_c C + \beta_p P + \psi\phi J + \sigma S - \mu H$$
$$\alpha_1 = \pi_1(1-\tau_1) \ ; \quad \alpha_2 = \pi_2(1-\tau_2)$$

Table 1: Description of Parameters

S/N	Variables	Parameters
1	Λ	Recruitment rate into the susceptible population
2	μ	Death removal rate
3	π_1	Corruption transmission probability per contact
4	π_2	Poverty transmission probability per contact
5	τ_1	Effort rate against corruption
6	$\alpha_1 = \pi_1(1-\tau_1)$	Effective corruption contact rate
7	τ_2	Effort rate against poverty
8	$\alpha_2 = \pi_2(1-\tau_2)$	Effective poverty contact rate
9	γ_1	Rate at which corrupt individuals become poor
10	γ_2	Rate at which poverty individuals become corrupt
11	ω	Proportion of corrupt individuals prosecuted and jailed
12	$1/\psi$	Average period prosecuted individuals spend in prison
13	$\psi(1-\phi)$	Transition rate from jailed to corrupt
14	$\psi\phi$	Transition rate from jailed to honest
15	β_c	Proportion of corrupt individuals that progress to H compartment
16	β_p	Proportion of poverty individuals that progress to H compartment
17	σ	Proportion of susceptible individuals that progress to H compartment

Model Analysis

The model above is qualitatively studied and in particular, poverty and corruption- free equilibrium states and are defined and analyzed as follows.

Corruption Only Model

The corruption only model is obtained by setting p=0, so that

$$
\begin{aligned}
\frac{dS}{dt} &= \Lambda - \frac{\alpha_1 CS}{N} - (\mu+\sigma)S \\
\frac{dC}{dt} &= \frac{\alpha_1 CS}{N} + \psi(1-\phi)J - (\mu+\beta_c+\omega)C \\
\frac{dJ}{dt} &= \omega C - \left(\psi(1-\phi)+\psi\phi+\mu\right)J \\
\frac{dH}{dt} &= \beta_c C + \psi\phi J + \sigma S - \mu H \\
\alpha_1 &= \pi_1(1-\tau_1)\ ; \quad \alpha_2 = \pi_2(1-\tau_2)
\end{aligned}
\qquad (2)
$$

Boundedness
Lemma1

Consider the region $D_1 = \left\{ \left(S,C,J,H\right) \in \mathbb{R}_+^4 : N \le \frac{\Lambda}{\mu} \right\}$. It can be shown that the set D_1 is positively invariant and an attractor of all positive solution of the system (2)

Proof
The rate of change of the total population for the corruption only model gives

$$\frac{dN}{dt} = \Lambda - \mu N$$

Using the integrator factor method, we have

$N(t) = N(0)e^{-\mu t} + \frac{\Lambda}{\mu}\left(1-e^{-\mu t}\right)$. In particular, if $N(0) < \frac{\Lambda}{\mu}$, then $N(t) \le \frac{\Lambda}{\mu}$ for all $t > 0$.

Hence D_1 is positively invariant and an attractor so that no solution path leaves through any boundary of D_1 and it is sufficient to consider the dynamics of system (2) in D_1.
In this region, the system (2) is considered as being mathematically and epidemiologically well posed.

Local Stability of Corruption Free Equilibrium
The state in which the population is free of corruption is called the corruption free equilibrium and setting the derivatives to zero in system (2) we obtain the corruption free equilibrium given by

$$C_0 = \left(S^0, C^0, J^0, H^0\right) = \left(\frac{\Lambda}{\mu+\sigma}, 0, 0, \frac{\sigma\Lambda}{\mu(\mu+\sigma)}\right) \qquad (3)$$

We use the next generation operator method described by Van Den Driessche and Watmough (2002) to obtain the basic reproduction number as follows;
In (2), let

$$k_2 = (\mu+\beta_c+\omega), \quad k_3 = (\psi+\mu) \qquad (4)$$

then

$$F = \begin{pmatrix} \frac{\alpha_1 S}{N} & 0 \\ 0 & 0 \end{pmatrix} \qquad (5)$$

$$V = \begin{pmatrix} k_2 & -\psi(1-\phi) \\ -\omega & k_3 \end{pmatrix} \qquad (6)$$

It follows that the control reproduction number for corruption only model is

$$R_{oc} = \frac{\alpha_1 S k_3}{N(k_2 k_3 - \omega\psi(1-\phi))} \qquad (7)$$

R_{oc} is the minimum number of corruption generated by a single corrupt individual in the susceptible population in his entire corrupt period.

Lemma2

The corruption-free equilibrium C_0 *for system (2) is locally asymptotically stable if* R_{oc} *<1*

Proof

Linearization at C_0 gives the Jacobian matrix below

$$J(C_0) = \begin{pmatrix} -k_1 & \frac{-\alpha_1 S}{N} & 0 & 0 \\ 0 & -k_2 & \psi(1-\phi) & 0 \\ 0 & \omega & -k_3 & 0 \\ \sigma & \beta_c & \psi\phi & -\mu \end{pmatrix}$$

Simplifying the Jacobian from the above gives

$$(-\mathrm{k}_1-\lambda)(-\mu-\lambda)\left[(-\mathrm{k}_2-\lambda)(-\mathrm{k}_3-\lambda)-\psi(1-\phi)\omega\right]=0 \qquad (9)$$

So that,

$$\lambda_1 = -\mathrm{k}_1 < 0\,; \quad \lambda_2 = -\mu < 0 \qquad (10)$$

$$(-\mathrm{k}_2-\lambda)(-\mathrm{k}_3-\lambda)-\psi(1-\phi)\omega = 0 \qquad (11)$$

Simplifying the above gives

$$\lambda^2 + \lambda(\mathrm{k}_2+\mathrm{k}_3) + \frac{\alpha_1 S\,\mathrm{k}_3}{NR_{oc}} = 0 \qquad (12)$$

The corresponding characteristics equation is

$$\lambda^2 + a_1\lambda + a_2 = 0 \qquad (13)$$

Using the Routh-Hurwitz criterion the above polynomial has negative roots since

$a_1 > 0$; $a_2 > 0$ if $R_{oc} < 1$

Hence, the poverty-free equilibrium P_0 is locally asymptotically stable if R_{oc} <1 and unstable if R_{oc} >1. Epidemiologically, this implies that corruption will be eliminated from the population whenever R_{oc} <1 if the initial size of the sub-population are in the basin of attraction of the DFE i.e. small influx of corrupt individuals into the community will not generate a large corruption outbreak and corruption dies out in time.

Global Stability of Corruption-Free Equilibrium

The Lyapunov function approach is a suitable technique used in studying global stability of DFE whenever $R_{0c} < 1$ and we proceed as follows;

$if\ R_{0c} < 1$

Lemma 3: *The corruption-free equilibrium C_0 of system (2) is globally asymptotically stable*

Proof:

Consider the Lyapunov function

$$V = k_3 C + \psi(1-\phi)J \tag{14}$$

where $k_3 = \psi + \mu$

$$\dot{V} = k_3 \dot{C} + \psi(1-\phi)\dot{J} \tag{15}$$

$$\dot{V} = k_3\left(\frac{\alpha_1 CS}{N} + \psi(1-\phi)J - k_2 C\right) + \psi(1-\phi)\left(\omega C - k_3 J\right) \tag{16}$$

$$= C\left(\frac{k_3\alpha_1 S}{N} - k_2k_3 + \omega\psi(1-\phi)\right) + \mathrm{J}\left(k_3\psi(1-\phi) - k_3\psi(1-\phi)\right) \tag{17}$$

$$= C\left(k_2k_3 - \omega\psi(1-\phi)\right)\left(\frac{\alpha_1 S k_3}{N\left(k_2k_3 - \omega\psi(1-\phi)\right)} - 1\right) \tag{18}$$

with

$k_2k_3 - \omega\psi(1-\phi) > 0$

On $D_1,\ S \le N \le \dfrac{\Lambda}{\mu},$ hence $\dfrac{S}{N} \le 1.$ So we have

$$\dot{V} \le C\left(k_2k_3 - \omega\psi(1-\phi)\right)\left(R_{oc} - 1\right) \tag{19}$$

Clearly, $\dot{V} \le 0$ if $R_{oc} \le 1.$ Equality is achieved at $\mathrm{C} = \mathrm{J} = 0.$ For $R_{oc} \le 1,$ we have $\dot{V} \le 0$

Since V is a Lyapunov function in D_1, thus by the La salle's invariance principle (La salle's, 1976) the largest invariant set is the DFE which is globally assymptotically

Poverty Only Model

Setting $C = 0$ in (1) gives the poverty only model given below

$$\frac{dS}{dt} = \Lambda - \frac{\alpha_2 PS}{N} - (\mu + \sigma)S$$

$$\frac{dP}{dt} = \frac{\alpha_2 PS}{N} - (\mu + \beta_p)P \tag{20}$$

$$\frac{dH}{dt} = \beta_p P + \sigma S - \mu H$$

Boundedness
Lemma 4

The region $D_2 = \left\{ \left(S, P, H \right) \in \mathbb{R}_+^3 : N \le \frac{\Lambda}{\mu} \right\}$ is positively invariant and attracts all positive solution of the system (20)

Proof
Adding all the equations in (20) gives the rate of change of the total population for the poverty only model. That is

$$\frac{dN}{dt} = \Lambda - \mu N$$

Solving the equation above using the integrator factor method gives

$N(t) = \frac{\Lambda}{\mu} + \left(N(0) - \frac{\Lambda}{\mu} \right) e^{-\mu t}$. From the above differential equation, clearly $\frac{dN}{dt} < 0$

solution enters D_2 in finite time or $N(t)$ approaches $\frac{\Lambda}{\mu}$ as $t \to \infty$. Hence the region D_2 attracts all

if $N(t) > \frac{\Lambda}{\mu}$ and if $N(0) < \frac{\Lambda}{\mu}$, then $N(0) \le \frac{\Lambda}{\mu}$ for all $t > 0$. If $N(0) > \frac{\Lambda}{\mu}$ then either the

solutions in $\mathbb{R}_+^3$.

Local Stability of Poverty Free Equilibrium
The poverty free equilibrium is the state in which the population is free of poverty and setting the derivatives to zero in system (3) we obtain the poverty free equilibrium given by

$$P_0 = \left(S^0, P^0, H^0 \right) = \left(\frac{\Lambda}{\mu + \sigma}, 0, \frac{\sigma \Lambda}{\mu(\mu + \sigma)} \right) \qquad (21)$$

The next generation operator method described by Van den Driessche and Watmough (2002) will be used to derive the basic reproduction number as follows;

In (20), let $k_1 = \mu + \sigma,\ k_2 = \mu + \beta_p$ then

$$F = \frac{\alpha_2 S}{N} \qquad (22)$$

$$V = k_2 \qquad (23)$$

It follows that the control reproduction number for poverty only model is

$$R_{op} = \frac{\alpha_2 S}{N(\mu + \beta_p)} \qquad (24)$$

R_{op} is the expected number of poverty created by a single poor individual in his entire poverty period

Lemma5

The poverty-free equilibrium P_0 *is locally asymptotically stable if* R_{op} *<1*

Proof

Linearization at P_0 gives the Jacobian matrix below

$$J(P_0)=\begin{pmatrix} -k_1 & \frac{-\alpha_2 S}{N} & 0 \\ 0 & \frac{-\alpha_2 S}{N}-k_2 & 0 \\ \sigma & \beta_p & -\mu \end{pmatrix}$$

Simplifying gives the eigen values

$$\left(-\mu-\lambda\right)\left(-k_1-\lambda\right)\left(\frac{-\alpha_2 S}{N}-k_2-\lambda\right)=0 \qquad (26)$$

Which gives

$$\lambda_1=-k_2<0; \quad \lambda_1=-\mu<0; \quad \lambda_3=\frac{\alpha_2 S}{N}-k_2<0 \ if \qquad (27)$$

$$\frac{\alpha_2 S}{Nk_2}<1 \ \Rightarrow R_{op}<1$$

Hence, the poverty only equilibrium is locally asymptotically stable if $R_{op}<1$. i.e. small influx of poor individuals into the community will not generate a large poverty outbreak and poverty dies out in time.

Global Stability of Poverty-Free Equilibrium

We can construct a suitable Lyapunov function to establish a sufficient condition for global asymptotic stability of DFE when $R_{op}\leq 1$ and we proceed thus

Lemma 6: *The poverty - free equilibrium* P_0 *of system* (3) *is globally asymptotically stable if* $R_{0p}<1$

Proof:

Consider the Lyapunov function

$$V=P \qquad (28)$$

Clearly, $V>0$ since $P>0$

Differentiating gives

$$\dot{V}=\dot{P} \qquad (29)$$

$$\dot{V} = P\left(\frac{\alpha_2 S}{N} - (\mu + \beta_p)\right) \quad (30)$$

$$\dot{V} = P(\mu + \beta_p)\left(\frac{\alpha_2 S}{N(\mu + \beta_p)} - 1\right) \quad (31)$$

Clearly, on $D_2, S \leq N \leq \frac{\Lambda}{\mu}$, so that $S \leq N$ so we have

$$\dot{V} \leq P(\mu + \beta_p)\left(R_{op} - 1\right) \quad (32)$$

and $\dot{V} \leq 0$ if $R_{0p} < 1$. Equality is achieved at p=0.Since V is a Lyapunov function in D_2 from Lasalle's invariance principle (Lasalle' 1976), the largest invariant set is the singleton DFE which is globally asymptotically stable in D_2

Poverty and Corruption Model

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda - \frac{\alpha_1 CS}{N} - \frac{\alpha_2 PS}{N} - k_1 S \\
\frac{dC}{dt} &= \frac{\alpha_1 CS}{N} + \frac{\gamma_2 PC}{N} + \psi(1-\phi)J - k_2 C \\
\frac{dP}{dt} &= \frac{\alpha_2 PS}{N} + \gamma_1 C - \frac{\gamma_2 PC}{N} - k_3 P \\
\frac{dJ}{dt} &= \omega C - k_4 J \\
\frac{dH}{dt} &= \beta_c C + \beta_p P + \psi\phi J + \sigma S - \mu H
\end{aligned} \quad (33)$$

where

$$\begin{aligned}
&\alpha_1 = \pi_1(1-\tau_1)\ ; \quad \alpha_2 = \pi_2(1-\tau_2) \\
&k_1 = (\mu + \sigma) \\
&k_2 = (\mu + \beta_c + \omega + \gamma_1) \\
&k_3 = (\mu + \beta_p) \\
&k_4 = \left(\psi(1-\phi) + \psi\phi + \mu\right)
\end{aligned} \quad (34)$$

The qualitative properties of the corruption and poverty model can be studied by establishing the positive invariance of the closed set

$$D_3 = \left\{\left(S, C, P, J, H\right) \in \mathbb{R}_+^5 : N \leq \frac{\Lambda}{\mu}\right\}$$

and it can be shown that in particular if $N(0) < \frac{\Lambda}{\mu}$ then $N(t) \le \frac{\Lambda}{\mu}$ for all $t > 0$ and if

$N(0) > \frac{\Lambda}{\mu}$

then either the solution enters D_3 in finite time or $N(t)$ approaches $\frac{\Lambda}{\mu}$ as $t \to \infty$. Hence the region D_3 attracts all solutions in $\mathbb{R}^5_+$.

Local Stability of Poverty and Corruption Free Equilibrium

The poverty and corruption free equilibrium is a state where the society is free of poverty and corruption and setting the derivatives to zero gives

$$CP_0 = \left(S^0, C^0, P^0, J^0, H^0\right) = \left(\frac{\Lambda}{\mu+\sigma}, 0, 0, 0, \frac{\sigma\Lambda}{\mu(\mu+\sigma)}\right) \quad (35)$$

The next generation operator method described by Van den Driessche and Watmough (2002) is used here to obtain the basic reproduction number as follows

$$F = \begin{pmatrix} \frac{\alpha_1 S}{N} & 0 & 0 \\ 0 & \frac{\alpha_2 S}{N} & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (36)$$

$$V = \begin{pmatrix} k_2 & 0 & -\psi(1-\phi) \\ -\gamma_1 & k_3 & 0 \\ -\omega & 0 & k_4 \end{pmatrix} \quad (37)$$

It follows that the poverty and corruption control reproduction number is

$$R_{pc} = \frac{\alpha_2 S}{Nk_3} + \frac{\alpha_1 S k_4}{N(k_2 k_4 - \omega\psi(1-\phi))} = R_p + R_c \quad (38)$$

$$R_p = \frac{\alpha_2 S}{Nk_3} = \frac{\alpha_2 S}{N(\mu+\beta_p)} \quad (39)$$

$$R_c = \frac{\alpha_1 S k_4}{N(k_2 k_4 - \omega\psi(1-\phi))} \quad (40)$$

R_c is the minimum number of corruption created by a single corrupt individual in the poverty population

R_p is the minimum number of poor individuals generated by a single poor individual in the corruption population

Lemma7

The poverty and corruption-free equilibrium PC_0 *is locally asymptotically stable if* $R_p < 1$ *and* $R_c < 1$

Proof

Since the variable H does not appear in the first four equations of system (33), then the relation
$H = N - (S + C + P + J)$ allows us to study the first four equations of (A). The Jacobian stability technique is used to prove the local asymptotic stability of poverty and corruption free equilibrium states. Linearization at PC_0 gives the Jacobian matrix below

$$J(PC_0) = \begin{pmatrix} -k_1 & \frac{-\alpha_1 S}{N} & \frac{-\alpha_2 S}{N} & 0 \\ 0 & \frac{\alpha_1 S}{N} - k_2 & 0 & \psi(1-\phi) \\ 0 & \gamma_1 & \frac{\alpha_2 S}{N} - k_3 & 0 \\ 0 & \omega & 0 & -k_4 \end{pmatrix} \quad (41)$$

At PC_0 we can deduce that (33) becomes

$$\Lambda = \frac{\alpha_1 CS}{N} + \frac{\alpha_2 PS}{N} + k_1 S$$
$$\frac{\alpha_1 S}{N} - k_2 = -\frac{1}{C}\left(\frac{\gamma_2 PC}{N} + \psi(1-\phi)J\right) \quad (42)$$
$$\frac{\alpha_2 PS}{N} + \gamma_1 C = \frac{\gamma_2 PC}{N} + k_3 P$$
$$\omega C = k_4 J$$

Applying elementary row operation, equation (34) gives

$$J(PC_0) = \begin{pmatrix} -k_1 & \frac{-\alpha_1 S}{N} & \frac{-\alpha_2 S}{N} & 0 \\ 0 & \frac{\alpha_1 S}{N} - k_2 & 0 & \psi(1-\phi) \\ 0 & 0 & M_1 & \gamma_1 \psi(1-\phi) \\ 0 & 0 & 0 & M_3 \end{pmatrix} \quad (43)$$

$$M_1 = -\left(\frac{\alpha_1 S}{N} - k_2\right)\left(\frac{\alpha_2 S}{N} - k_3\right) \quad (44)$$

$$M_3 = \omega\psi(1-\phi) + k_4\left(\frac{\alpha_1 S}{N} - k_2\right) \quad (45)$$

The eigen values are

$$\lambda_1 = -k_1 < 0 \quad (46)$$

$\lambda_2 = \frac{\alpha_1 S}{N} - k_2 < 0$ from equation (35) since

$$\frac{\alpha_1 S}{N} - k_2 = -\frac{1}{C}\left(\frac{\gamma_2 PC}{N} + \psi(1-\phi)J\right) \quad (47)$$

$$\lambda_3 = M_1 = -\left(\frac{\alpha_1 S}{N} - k_2\right)\left(\frac{\alpha_2 S}{N} - k_3\right) < 0 \ \ if \ \frac{\alpha_2 S}{Nk_3} < 1 \quad (48)$$

$$\lambda_4 = M_3 = \omega\psi(1-\phi) + k_4\left(\frac{\alpha_1 S}{N} - k_2\right) < 0 \text{ if } \frac{\alpha_1 S k_4}{N(k_2 k_4 - \omega\psi(1-\phi))} < 1 \quad (49)$$

Hence, $\lambda_4 < 0 \ if \ R_c < 1$

Therefore, the poverty and corruption free equilibrium is locally asymptotically stable if $R_p < 1 \ and \ R_c < 1$. Epidemiologically, the above signifies that poverty and corruption can be brought under control irrespective of the initial size of the sub-population in the society if R_c, R_p can each be significantly reduced below unity

Global Stability of Poverty and Corruption Free Equilibrium

A linear Lyapunov function will be constructed to study the global asymptotic properties of the poverty and corruption free equilibrium if $R_c < 1$, $R_p < 1$ as shown below

Lemma 8: *The Poverty and Corruption - free equilibrium* PC_0 *of system* (3) *is globally asymptotically stable if* $R_p < 1$ *and* $R_c < 1$

Proof:

Consider the Lyapunov function

$$V = k_4 C + \psi(1-\phi)J + k_4 P \quad (50)$$

Clearly, $V > 0$, since $C > 0,\ J > 0, P > 0$ Differentiating V with respect to time gives

$$\dot{V} = k_4\dot{C} + \psi(1-\phi)\dot{J} + k_4\dot{P} \quad (51)$$

$$\dot{V} = k_4\left(\frac{\alpha_1 CS}{N} - \gamma_1 C + \frac{\gamma_2 PC}{N} + \psi(1-\phi)J - k_2 C\right) + \psi(1-\phi)\left(\omega C - k_4 J\right)$$
$$+ k_4\left(\frac{\alpha_2 PS}{N} + \gamma_1 C - \frac{\gamma_2 PC}{N} - k_3 P\right) \quad (52)$$

$$= C\left(\frac{\alpha_1 k_4 S}{N} - (k_2 k_4 - \omega\psi(1-\phi))\right) + P\left(\frac{\alpha_2 k_4 S}{N} - k_3 k_4\right) \quad (53)$$

$$= C\left(k_2 k_4 - \omega\psi(1-\phi)\right)\left(\frac{S\alpha_1 k_4}{N\left(k_2 k_4 - \omega\psi(1-\phi)\right)} - 1\right) + Pk_3 k_4\left(\frac{S\alpha_2}{Nk_3} - 1\right) \quad (54)$$

On D_3, $S \le N \le \frac{\Lambda}{\mu}$, so that $\frac{S}{N} \le 1$. Since all model parameters are non-negative and $k_2k_4 - \omega\psi(1-\phi) > 0$ we can deduce that

$$\dot{V} \le C\left(k_2k_4 - \omega\psi(1-\phi)\right)\left(R_c - 1\right) + Pk_3k_4\left(R_p - 1\right) \qquad (55)$$

With equality at $R_c = 1,\ R_p = 1,\ P = 0,\ C = 0.$ For $R_p \le 1,\ R_c \le 1,$ we have $\dot{V} \le 0.$

And since V is a Lyapunov function in D_3 it follows from Lasalle's invariance principle that every solution to the model is contained in the largest invariant set $\left\{(S,C,P,J,H) \in \mathbb{R}^5_+ : \dot{V} = 0\right\}$ which is a singleton DFE. This means $\left(C(t), P(t), J(t)\right) \to (0,0,0)$ as $t \to \infty$. Substituting $C = P = J = 0$ into (33) gives $S(t) \to \frac{\Lambda}{\mu+\sigma},\ H(t) \to \frac{\sigma\Lambda}{\mu(\mu+\sigma)}$ as $t \to \infty$ so that $(S,C,P,J,H) = \left(\frac{\Lambda}{\mu+\sigma}, 0, 0, 0, \frac{\sigma\Lambda}{\mu(\mu+\sigma)}\right)$ as $t \to \infty$ for $R_p \le 1,\ R_c \le 1,$ and so the DFE is globally asymptotically stable in D_3.

NUMERICAL SIMULATIONS

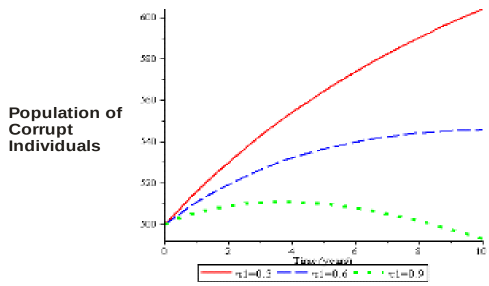

Figure 1: Graph of Corruption against time for corruption only model

Figure 1 shows that the higher the effort against corruption, the lower the population of corrupt. This decrease in the population of corrupt implies movement of corrupt individuals into the class of the honest

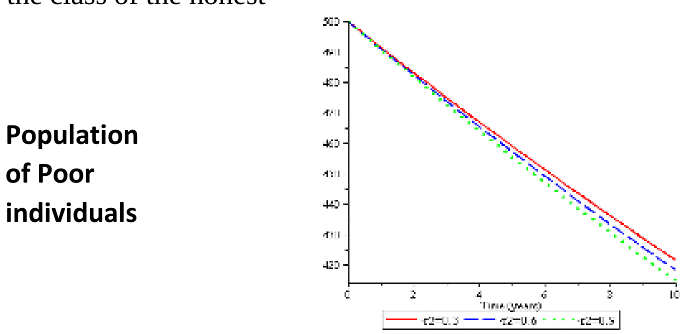

Figure 2: Graph of Poverty against time for poverty only model

Figure 2 shows that if much effort is adopted towards curbing poverty, then the population of poor in the society will gradually reduce. This reduction in poverty level is as a result of progression into the class of the honest

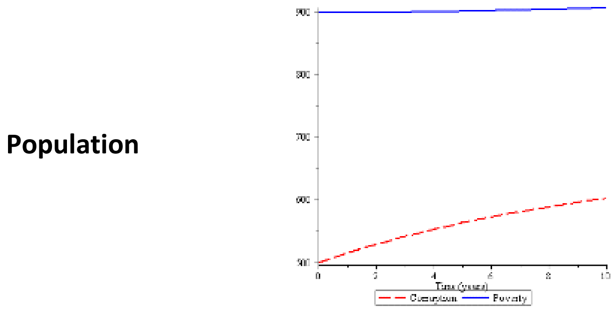

Figure 3: Comparison between Corruption and Poverty with low control strategy

Figure 3 shows that the lower the anti-poverty and corruption strategies adopted, corruption still increases at a faster rate than poverty.

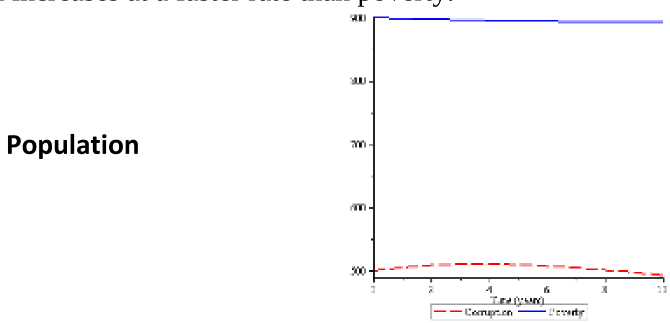

Figure 4: Comparison between Corruption and Poverty with high control strategy

Figure 4 shows that with increased measures targeted at reducing poverty and corruption, corruption dies out at a faster rate than poverty.

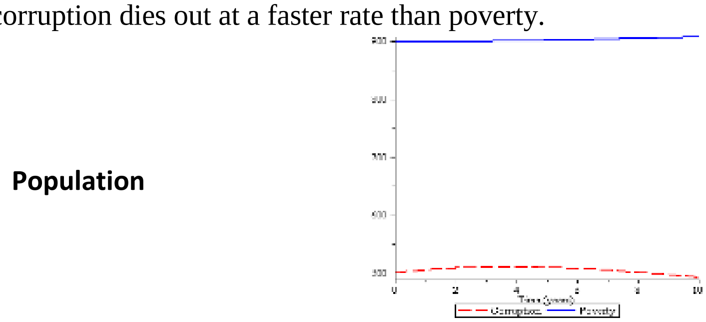

Figure 5: Comparison between Corruption and Poverty with high control for corruption and low control for poverty

Figure 5 shows that with high effort against corruption and low effort against poverty, corruption level eventually reduces to the barest minimum while poverty level keeps increasing slowly. This implies that an attempt to reduce corruption does not have significant effect in reducing poverty.

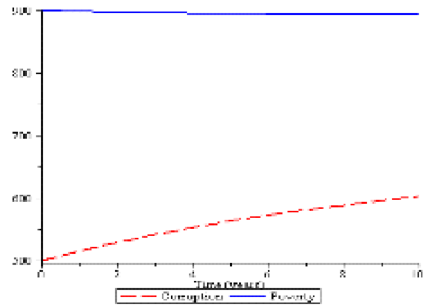

Figure 6: Comparison between Corruption and Poverty with low control for corruption and high control for poverty

Figure 6 shows that with low effort against corruption and high effort against poverty, corruption level keeps increasing in the society while poverty level decreases at an insignificant rate. This implies that an attempt to reduce poverty does not have significant effect in reducing corruption.

Conclusion Human societies are generally assumed to be corruption and poverty prone and many politicians and ruling elites of many countries across the world and the whole of the political spectrum are currently involved in processes of corruption. As a result a model for poverty and corruption in a population written in terms of Ordinary Differential Equations is formulated. The poverty free equilibrium, poverty only reproduction number and stability were found and analysed for the poverty only model while the corruption free equilibrium of the corruption sub model was obtained and analysed separately for local and global stability. The combined model for poverty and corruption were also analysed for local and global stability.

Corruption is costly, but it deprives citizens of more than money. From the simulation it is much easier to control corruption in the society than poverty. It can be deduced that high and rising corruption increases income inequality and poverty. Findings suggest that corruption and poverty go together, with causality running in both directions. In other words, the attempts to reduce poverty must be complemented by serious efforts to reduce corruption. Hence, it is necessary to address the integrated strategy to reduce poverty and fight corruption. Corruption adversely affects income distribution and poverty and various anti-corruption strategies including reforms and regulations that improve market access, distribution of information to the poor about opportunities for employment, asset ownership, empowerment, political freedom and stability of local and international prices can serve as control measures for poverty and corruption. Furthermore, to curb the menace of poverty and corruption, the central authority, the government and or central agencies, should play an important role to deterrence and to control the evolution of corruption.

References

Abdulrahman, S. (2014) Stability Analysis of the Transmission Dynamics and Control of Corruption. *The Pacific Journal of Science and Technology.* 15(1), 99-113

Caulkins J.P., Feichtinger G.,Grass D.,Hartl R.F.,Kort P.M.,Novak A.J., Seidl A.,Wirl F. A.(2011). Dynamic Analysis of Schelling's Binary Corruption Model: A Competitive Equilibrium Approach, Operations Research and Control Systems.

Eguda, F.Y.; Oguntolu, F. & Ashezua, T.T.(2017). Understanding the Dynamics of Corruption Using Mathematical Modeling Approach, *International Journal of Innovative Science*. 4(8)

LaSalle, J.P. (1976). "The Stability of Dynamical Systems". Regional Conference Series in Applied Mathematics, *SIAM*: Philadelphia, PA.

Maoro, P.(1995) "Corruption and Growth". *Quarterly Journal of Economics.* 110(3), 681-712

Ndikumana, L. (2006). Corruption and pro-poor growth outcomes: evidence and lessons for Africancountries. *Working Paper* Series No. 120, Political Economy Research Instituted.

Negin, V.; Rashid, Z.A. & Nikopour, H. (2010). The Casual Relationship between Corruption and Poverty: A Panel Data Analysis, Munich

Rose-Ackerman, S. (1975)"The economics of corruption". *Journal of Public Economics*, 4(2), 187–203

Van Den Driessche P, and Watmough J.,(2002) Reproductive numbers and the sub-threshold endemic equilibria for compartmental models of disease transmission. *J. Math. Biol*, 180, 29-48