PROBABILISTIC MODELING OF FAILURE DISTRIBUTION AND RELIABILITY ANALYSIS OF SMALL WELDING MACHINE

by

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Abstract

The need for performance evaluation and planned maintenance of industrial machines is an imperative for reliability study. This ensures that users have needed information on the likelihood of failure, when to repair and how long the repair would last. In this work, the failure distribution of a locally fabricated welding machine was shown to follow gamma distribution with shape parameter, $\hat{\alpha} = 1.3$ *and scale parameter,* $\hat{\beta} = 1386$ *using the chi squared goodness-of-fit test with the aid of easyfit (5.6) software. Failure density function, distribution function and failure rate were obtained. Also, the following reliability indices were obtained using mathematical expressions; mean time to repair (MTTR=1802 hours), mean time between failure (MTBF = 3393 hours) and the availability factor, A = 95.5% while the maintainability factor is M=4.5%. The probability functions of the failure distribution gave apt description of the failure behavior of the machine while the reliability indices obtained show that the small welding machine is in good working condition. These were complementarily used to formulate a maintenance policy for effective maintenance of the machine.*

Keywords: *Reliability, Small welding Machine, Failure rate, Probability functions, Gamma Distribution*

Introduction

The assessment of the operating condition of industrial machines is necessary to guarantee optimum production time, adequate performance and planned maintenance. Reliability is a yardstick of the capability of an equipment to operate without failure when put into service. It is considered as one of the most important characteristics for industrial products and systems. Reliability is the probability that a component, equipment, or a system will perform its intended function adequately for a specific period of time under a given set of conditions' Lewis (1987). Thus, the basic elements of reliability are; probability, adequate performance, duration of adequate performance and operating condition.

In assessing system reliability, it is necessary to categorize the different types of system failure. These are; complete and catastrophic failure (that is sudden and total failure from which recovery is impossible), gradual rate of failure, and rapid but non-constant rate of failure. Also, there are three basic ways in which the pattern of failures can change with time: the hazard rate may be decreasing, increasing or constant. A constant hazard rate characterizes failures which are caused by the application of loads in excess of the design strength at a constant average rate. Examples include overstress failures due to accidental or transient circuit overload and maintenance induced failures of mechanical equipment which occur randomly at a generally constant rate. In practice, different probabilistic and reliability approaches have been applied in reliability study based on the physics of the failure mechanism and failure mode of the equipment with the sole aim of assessing the operating condition of the equipment to aid maintenance plan. Karlen (2012) performs probabilistic modeling of fatigue failures with material scatter as the main thrust and made use of the Weakest Link (WL) integral (Weibull, (1939)) to model material scatter resulting from experimental result. This yielded the fatigue failure probability for the specimen. Consequently, the WL integral also evaluated at the specimen surface area and as a volumetric phenomenon was used to design a structure with respect to fatigue failure probability instead of the usual peak stress. In his work, other mode of failures were not of interest and detail analysis of reliability indices which examines the state of health of the system was not considered. 'Burn-in' of electronic parts is a good example of the way in which knowledge of a decreasing hazard rate is used to generate an improvement in reliability. Pourgolmohamad, Moghaddam, Soleimani and Chakherlou (2018) studied the reliability of the components of Micro-Electro-Mechanical systems (MEMS) devices using probabilistic physics of failure. Failure Modes and Effects Analysis (FMEA) was used to identified sticking caused by dielectric charging as the dominant failure mode. The probabilistic variables were modeled with the aid of Markov Chain Monte Carlo (MCMC) simulation. The Bayesian method was used to reduce the observed variability in the variables in a range of posterior probability distributions. However, the reliability indices of the system that could aid planned maintenance were not evaluated. Also, Zhu, Huang, Li, Liu and Yang (2013) combined a nonlinear damage accumulation model, a probabilistic S-N curve, and a one-to-one probability density functions transformation technique to model damage accumulation and analyze the time-dependent fatigue reliability of railway axle steels. The damage accumulation is characterized as a distribution in a general degradation path, which mean and variability change with time. The result of evolution and estimates of the probabilistic distribution of fatigue damage over time were used to develop a framework for fatigue reliability assessment and service life prediction. Again, Mathisen, Ronold and Sigurdsson (2004) considered the failure modes in jacket reliability analysis with ultimate limit state for jackets in relatively shallow water. The various areas that concerns the analysis included; failure modes and some requirements to load and resistance analysis, directionality in loading and resistance, random periods of individual extreme waves and foundations - axial and lateral capacity modeling for multiple piles and model uncertainty for pile capacity. In another development, probabilistic structural mechanics consisting flaw-distributed model (which follows the lognormal distribution) and defect size distribution was applied to evaluate potential effectiveness of In-service Inspection (ISI) program to reduce the failure probabilities of the components associated with fatigue crack growth from pre-existing fabrication flaws as the failure mechanism, Khaleel and Simonen (2009). Critical inputs (weld process and wall thickness) to fracture mechanics calculations of the parameters that characterizes the number and sizes of fabrication flaws in piping welds were considered. The results of their work was used to support the development and implementation of risk-informed ISI of piping and vessels under study.

There are many life distributions that can be used to model reliability data. Examples are exponential, gamma, Weibull, lognormal distributions among others. These distributions are exhibited by systems based on their failure modes and/or empirical success with failure data, Udoh (2018). The Gamma function is a generalization of the factorial function to non-integral values and was introduced by a Swiss Mathematician Leonhard Euler in the 18^{th} century. It is suitable for modeling data with different types of hazard function with increasing and decreasing bathtub shape and unimodal failure rates which makes it particularly useful for estimating individual hazard function, (Agarwa and Kalla, 1996 and Mead, Nasar and Dey, 2018). It characterizes the life distribution of many engineering systems such as time to failure of an equipment and load levels for telecommunication services. Therefore, it is one of the most applied statistical distribution in the area of reliability. Its other applications include; meteorology (rainfall) and insurance claims and default in business for which the variables are always positive and the observations is skewed, (Stephenson et al, 1999 and Saraless, 2009). It has increasing as well as decreasing failure rate depending on the shape parameter which gives an extra edge over exponential distribution which has only constant failure rate, Gauss, Edwin and Giovana (2011).

It is noteworthy that the manner in which a system is operated can affect its reliability level. For instance, the amount of preventive maintenance permitted, if it is permitted at all and the degree to which the system operator can participate in correcting failures must be specified. Therefore, the objective of this study is to model the failure distribution of a small welding machine in terms of probability distribution in order to obtain its mathematical structure. The study would also obtain major reliability indices of the machine which would be used in assessing the operating condition of the machine for a possible maintenance plan. This would add to good maintenance practices to avoid untimely breakdown of the machine and loss of production time.

1.1 Description and Working of a Small Welding Machine

A small welding machine is a mix of mechanical and electrical components used in fabrication or sculptural process that joins materials, usually metals or thermoplastics by causing fusion, which is distinct from lower temperature metal joining techniques. It's adaptable to low voltage compare to other welding machines and its failure mode is sudden and complete. Its major failure mechanism is burnt-coil and switches due to overheat and could be repaired by replacement of the
affected components.

1.2 The Basic Assumptions of Study:

- The failure that occurs at any time, t is independent and continuous.
- The failure of one component of the system causes the failure of the entire system.
- The failures in the system occurs at random.

2. Methodology

The Data: The data for this work is the inter-failure times (in hours) of a locally fabricated small welding machine from artisan welding shop in Uyo, Nigeria for the period 2010-2016.

2.1 Goodness-of-Fit Test for Gamma Distribution

The chi-square test would be used to investigate whether the distribution of failure of the small welding machine follows the expected gamma distribution. This was aimed at determining the appropriate mathematical structure for the failure distribution of the machine. The expected frequencies at given intervals of time, t is given by;

$$E_i = N \times \int_0^t \frac{1}{\beta^\alpha \Gamma(\alpha)} t^{\alpha-1} e^{-\frac{t}{\beta}}$$

The observed and pooled expected frequencies of failure of small welding machine at given intervals of time, t is shown in Table 1. Since $\chi_0^2 < \chi_2^2(0.05)$, we conclude that the distribution of failures follows the gamma distribution. Easyfit (5.6) software was used to aid the goodness-of-fit test and estimation of parameters of the gamma distribution.

Table 1: Observed and Pooled Expected Frequencies

INTERVALS OF TIME t	O_i	E_i
0< t < 1200	12	11.2
1200 < t < 2400	5	7.2
2400 < t < 6000	8	6.6
Total	25	25

2.2 The Two-Parameter Gamma Probability Distributions

The failure density and the distribution functions of the machine characterized by the gamma distribution would be obtained by the functional forms in (2) and (3). These would also be used to derive the reliability and hazard functions in subsequent section. Also, (7) and (8) would be used to obtain the shape and scale parameters estimates of the probability distribution.

2.2.1 The Cumulative Distribution of the Two-Parameter Gamma Distribution

The probability distribution function of a two-parameter gamma distribution is given as;

$$f(x;\alpha,\beta)=\frac{1}{\beta^{\alpha}\Gamma(\alpha)}x^{\alpha-1}e^{-\frac{x}{\beta}} \quad ; \quad 0<x<\infty \tag{1}$$

where $\alpha=$ shape parameter and $\beta=$ scale parameter.

Let $f(u)=\dfrac{1}{\beta^{\alpha}\Gamma(\alpha)}u^{\alpha-1}e^{-\frac{u}{\beta}}$; $0<\mathrm{u}<\mathrm{x}$

Then, $F(x)=\int_0^x f(u)du$

Let $v=\dfrac{u}{\beta}$ and $\dfrac{du}{dv}=\beta$

$$F(x)=\frac{1}{\beta^{\alpha}\Gamma(\alpha)}\int_0^{\frac{x}{\beta}}(\beta v)^{\alpha-1}e^{-\left(\frac{\beta v}{\beta}\right)}\beta dv=\frac{1}{\Gamma(\alpha)}\int_0^{\frac{x}{\beta}}v^{\alpha-1}e^{-v}dv;$$

Where $\int_0^{\frac{x}{\beta}}v^{\alpha-1}e^{-v}dv=\Gamma\left(\alpha,\frac{x}{\beta}\right)$ is an incomplete gamma function.

Therefore, $F(x)=P(X\le x)=\dfrac{\Gamma\left(\alpha,\frac{x}{\beta}\right)}{\Gamma(\alpha)}$ (2)

2.2.2 The Gamma Probability Density Function

Consider a distribution function $F(x)$ of waiting times until the α th Poisson event given the rate of change, $\frac{1}{\beta}$;

$$F(x)=P(X\le x)=1-P(X>x)$$

$$=1-\frac{\Gamma\left(\alpha,\frac{x}{\beta}\right)}{\Gamma(\alpha)}=1-e^{-\frac{x}{\beta}}\sum_{k=0}^{\alpha-1}\frac{\left(\frac{x}{\beta}\right)^k}{k!}$$

The corresponding probability function $P(x)$ of waiting time until the α th Poisson event is then obtained by differentiating $F(x)$.

Let $p(x)=F'(x)$, $\mathrm{U}=\mathrm{e}^{-\frac{x}{\beta}}$ and $\mathrm{V}=\sum_{k=0}^{\alpha-1}\dfrac{\left(\frac{x}{\beta}\right)^k}{k!}$

$$p(x)=\frac{1}{\beta}e^{-\frac{x}{\beta}}\left\{1-\sum_{k=1}^{\alpha-1}\left[\frac{\left(\frac{x}{\beta}\right)^{k-1}}{(k-1)!}-\frac{\left(\frac{x}{\beta}\right)^{k}}{k!}\right]\right\}$$

By using Wolfram's method, Papoulis (1991);

$$\sum_{k=1}^{\alpha-1}\left[\frac{\left(\frac{x}{\beta}\right)^{k-1}}{(k-1)!}-\frac{\left(\frac{x}{\beta}\right)^{k}}{k!}\right]=1-\frac{\left(\frac{x}{\beta}\right)^{\alpha-1}}{(\alpha-1)!}$$

$$\therefore P(x)=\frac{1}{\beta}e^{-\frac{x}{\beta}}\frac{\left(\frac{x}{\beta}\right)^{\alpha-1}}{(\alpha-1)!}=\frac{1}{\beta^{\alpha}\Gamma(\alpha)}x^{\alpha-1}e^{-\frac{x}{\beta}};\quad 0<\mathrm{x}<\infty \qquad (3)$$

2.2.3 The Mean and Variance of Two-Parameter Gamma Distribution

$$E(x)=\int_0^{\infty}xf(x)dx$$

$$=\int_0^{\infty}x\frac{1}{\beta^{\alpha}\Gamma(\alpha)}x^{\alpha-1}e^{-\frac{x}{\beta}}dx \quad =\frac{1}{\beta^{\alpha}\Gamma(\alpha)}\int_0^{\infty}x^{(\alpha+1)-1}e^{-\frac{x}{\beta}}dx$$

By definition;

$$\beta^{(\alpha+1)}\Gamma(\alpha+1)=\int x^{(\alpha+1)-1}e^{-\frac{x}{\beta}}dx$$

$$=\frac{1}{\beta^{\alpha}\Gamma(\alpha)}\Gamma(\alpha+1)\beta^{\alpha+1}$$

but $\Gamma(\alpha+1)=\alpha\Gamma(\alpha)$

$$\therefore E(x)=\alpha\beta \qquad (4)$$

Similarly,

$$E(x^2)=\alpha^2\beta^2+\alpha\beta^2$$

And $Var(x)=\alpha\beta^2$ (5)

2.2.3.1 Estimate of Parameters by Method of Moments:

The first and second moments about the origin from sample data could be obtained as;

$$\bar{x}=\alpha\beta \qquad (6)$$

but $\frac{1}{n}\sum_{i=1}^{n}x_i^2=(\alpha\beta)^2+\alpha\beta^2$

Where;

$$\hat{\alpha}=\frac{\bar{x}^2}{\frac{1}{n}\sum_{i=1}^{n}x_i-\bar{x}^2}=\frac{n\bar{x}^2}{\sum_{i=1}^{n}\left(x_i^{\,2}-\bar{x}\right)^2} \qquad (7)$$

and

$$\hat{\beta}=\frac{\bar{x}}{\frac{\bar{x}^2}{s^2}}=\frac{s^2}{\bar{x}} \qquad (8)$$

$\hat{\beta}$ has a relationship with coefficient of variation, $cv(x)$ given as; $\hat{\beta}=s\ cv(x)$

2.2.4 Reliability Function of Gamma distribution

Reliability, R(t) is defined as the probability that a machine will perform its prescribed duty for a given time when operated correctly in a specified environment without failure[15]. It would be used in this work to examine the operating condition of the machine.

$$R(t)=1-P(T\le t)=1-F(t)=1-\int_0^t f(t)dt$$

For Gamma distribution with parameters α and β;

$$R(t)=1-\int_0^t \frac{1}{\beta^\alpha\Gamma(\alpha)}t^{\alpha-1}e^{-\frac{x}{\beta}}dt$$

But $F(t)=\dfrac{\Gamma\left(\alpha,\frac{t}{\beta}\right)}{\Gamma(\alpha)}$

Hence, $R(t)=1-\dfrac{\Gamma\left(\alpha,\frac{t}{\beta}\right)}{\Gamma(\alpha)}$

But $\Gamma\left(\alpha,\dfrac{t}{\beta}\right)=\int_0^{\frac{t}{\beta}}v^{\alpha-1}e^{-s}dv = \Gamma(\alpha)\left[1-e^{-\frac{t}{\beta}}\sum_{k=0}^{\alpha-1}\frac{\left(t/\beta\right)^k}{k!}\right]$

$$R(t)=1-\left[\frac{\Gamma(\alpha)\left[1-e^{-\frac{t}{\beta}}\sum_{k=0}^{\alpha-1}\frac{\left(t/\beta\right)^k}{k!}\right]}{\Gamma(\alpha)}\right]$$

$$\therefore R(t)=e^{-\frac{t}{\beta}}\sum_{k=0}^{\alpha-1}\frac{\left(t/\beta\right)^k}{k!} \qquad (9)$$

2.2.5 Failure Rate or Hazard Function, $h(t)$ of Gamma Distribution:

The failure rate during a given interval of time, $t=[t_1,t_2]$ shows the probability that a failure per unit time occurs in the interval (t_1,t_2) conditioned on the event that no failure has occurred at or before time, t_1. This means that $T>t_1$. It is another major component in reliability analysis which shows the deteriorating behaviour of the machine. The failure rate can be defined as follows:

$$h(t)=\frac{R(t_1)-R(t_2)}{(t_2-t_1)R(t_1)}=\frac{F(t_2)-F(t_1)}{(t_2-t_1)R(t_1)}$$

Taking the limit of the failure rate at the interval $(t,\Delta t+t)$ as Δt approaches zero, where $t=t_1$ and $(t+\Delta t)=t_2$ gives the hazard function; $h(t)$.

$$h(t)=\underset{\Delta t\to 0}{Lim}\frac{R(t_1)-R(t+\Delta t)}{\Delta t\times R(t)}=\underset{\Delta t\to 0}{Lim}\frac{F(t+\Delta t)-F(t)}{\Delta t}\times\frac{1}{R(t)}$$

But $\lim_{\Delta t \to 0} h(t) = \lim_{\Delta t \to 0} \frac{F(t+\Delta t) - F(t)}{\Delta t} = f(t)$

$$\therefore h(t) = \frac{f(t)}{R(t)}$$

Hence,

$$h(t) = \frac{\frac{t^{\alpha-1} e^{-\frac{t}{\beta}}}{\beta^{\alpha}\Gamma(\alpha)}}{e^{-\frac{t}{\beta}} \sum_{k=0}^{\alpha-1} \frac{\left(t/\beta \right)^k}{k!}} = \frac{\frac{t^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)}}{\sum_{k=0}^{\alpha-1} \frac{\left(t/\beta \right)^k}{k!}} = \frac{t^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha) \sum_{k=0}^{\alpha-1} \frac{\left(t/\beta \right)^k}{k!}} \qquad (10)$$

2.2.6 Reliability Indices of the Gamma Failure Function

Four reliability indices would be used in this work as indicators of the state of health of the machine. They provide information on how long the machine should be used before repair, when to go for repair and the inter failure time, among others.

2.2.6.1 The Mean Time to Failures (MTTF): This would be used to obtain the expected length of time which failure would occur in a machine. It is given by;

$$MTTF = \int_0^t T f(t) dt = \int_0^t T R(t) dt$$

$$= \int_0^t t \frac{1}{\Gamma(\alpha)\beta^{\alpha}} t^{\alpha-1} e^{-\frac{t}{\beta}} dt = \alpha\beta \qquad (11)$$

2.2.6.2 The mean time to repair/replace (MTTR): This would be used to obtain the average time required to replace a failed component or device. It is given by;

$$MTTR = \frac{E}{I} \qquad (12)$$

where E is the total downtime of a machine given by $\sum_{i=1}^{N} E_i$ and I is the total number of failure in the machine.

2.2.6.3 Availability factor: This would be used to obtain the percentage of time that a system remains in a working condition. It is given by;

$$A_f = \frac{MTTF}{MTTF + MTTR} \times 100\% \qquad (13)$$

2.2.6.4 Maintainability factor: This would be used to determine the ease with which the equipment or system can be repaired or maintained. It is given by;

$$M_f = \frac{MTTR}{MTTR + MTTF} \times 100\% \qquad (14)$$

3.0 Results

3.1 Evaluation of Gamma parameters of small welding machine

Numerical estimates of the mean inter-failure times, shape and scale parameters of the machine given in (6), (7) and (8) yielded the results in Table 2;

Table 2: Parameters estimates of small welding machine

Statistic	$\bar{x}$	$\hat{\alpha}$	$\hat{\beta}$

Estimates	1860.72	1.3	1385.9

3.2 Estimates of reliability indices

The reliability measures were obtained respectively from (11), (12), (13) and (14) as shown in Table 3.

Table 3: Values of Reliability measures of small welding machine

(MTTF)	MTBF	MTTR	A_f	M_f
1802	3393	84	95.5%	4.5%

3.5 Evaluation of reliability functions of the small welding machine

The estimated parameters of gamma distribution from the empirical data; $\alpha = 1.3$ and $\beta = 1385.9$ were used to obtain the failure density function, $f(t)$ in Figure 4, the failure distribution function, $F(t)$ in Figure 3, the reliability function, $R(t)$ in Figure 1 and the failure rate, $h(t)$ in Figure 2, for values of $\boldsymbol{t}$ in equations (2), (3), (9) and (10) and the graphs are shown in Figures 1, 2, 3 and 4.

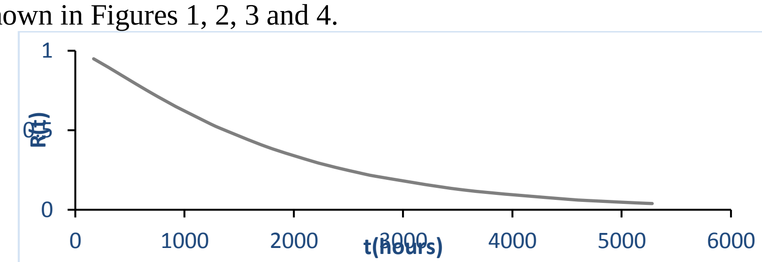

Fig. 1: Graph of Reliability function against time of small welding machine

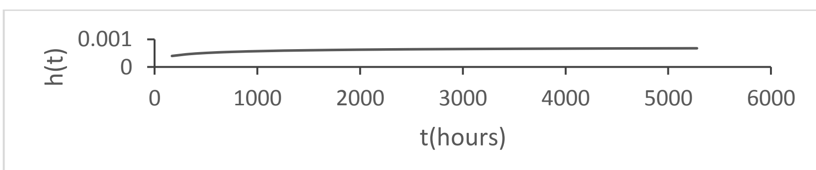

Fig. 2: Graph of Hazard function against time of small welding machine

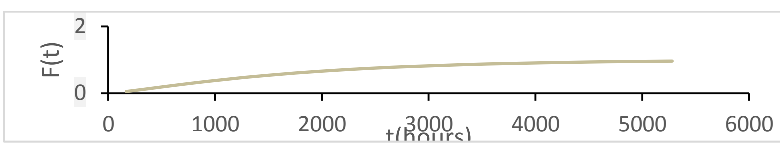

Fig. 3: Graph of Cumulative distribution against time of small welding machine

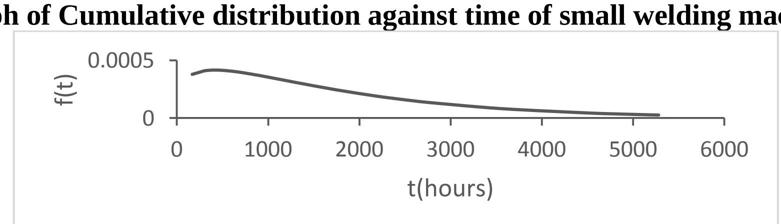

Fig. 4: Graph of Density function against time of small welding machine

3.6 Discussion of result

3.6.1 Probability functions:

Figure 1 shows that the reliability of a small welding machine reduces gradually from 0.98 to 0.290 within the time interval 0 to 2500 hours. Its reliability reduces rapidly within the time interval of 2500 hours to 5400 hours which shows a sudden decrease tending to failure while Figure 2 shows an increasing hazard rate of wear-out failure during the burn-in period and a constant hazard rate of random failures at the useful life span of the machine. The failure rate increases from the value, 0.0004 to 0.0006 in the first 1800 hours of operation and maintains a near constant failure rate from 2000 to 5500 hours at hazard value of 0.00065. Figure 3 shows that the distribution of failures increases from point 0 to 0.68 within the time interval of 0 to 2500 hours and increases continually at a rapid rate within 2500 hours to 5300 hours from point 0.68 to 0.9 portending danger which calls for checks. Finally, Figure 4 is the failure density function which shows that the likelihood of occurrence of failures in a small welding machine increased sharply from point 0.00037 to 0.00043 within 1 to 500 hours of operation and started reducing gradually as time increases. Figure 4 exhibits the behaviour of a typical gamma density function and assures modeling efficacy.

3.6.2. Estimates of the reliability indices:

The mean time to failure of a small welding machine, MTTF=1802, implies that the expected time of failure of the machine is 1802 hours which equals 75 days before failure occurs while the mean time to replace, MTTR=83.88 hours implies that the machine is restored to normal working condition within 4 days on the average after failure. The availability factor, A_f =95.55% implies that the small welding machine is in a working state for about 96% of the time while the maintainability factor, M_f= 4.5% implies that about 5% out of 100% availability of the small welding machine is used for repair and other maintenance actions.

It is worth noting that probabilistic modelling and reliability analysis of equipment are case-specific study analogous to medical checks of human in the hospital because even two equipment with same specification but used under different conditions would have different values of parameters estimates and reliability indices due to usage and maintenance. Based on this development, different analytical and statistical probabilistic or simulation modeling approaches so adopted in specific studies (Karlen, 2012, Pourgolmohamad et al, 2018, Zhu et al, 2013, Mathisen, Ronold andSigurdsson, 2004, Khaleel and Simonen, 2009 and Agarwa and Kalla, 1996) are only appropriate to the physics of the failure mechanism as well as the mode of failure that is imminent in the equipment under study and are therefore not comparable. Consequently, in those different studies of failure modeling and reliability analysis of equipment, specific results were obtained on the operating condition of equipment/material, identified causes of failure and parameters estimates which were used for maintenance action and future design specification.

3.6.3 Proposed maintenance policy of small welding machine

Maintenance policy is a set of administrative, technical and managerial action to apply during the life cycle of a machine used to guide maintenance management and decision making towards retaining certain operational conditions of a machine or dedicated to restoring the machine to said condition. Therefore, the proposed maintenance policy states that *‘’the small welding machine which has 96% availability level should be optimally operated for T≤1802 hours before preventive maintenance actions aimed at reducing the probability of wear and tear* *in the machine with failure maintenance duration of an average of 84 hours, performed at any time, t<T, if it fails at T''.*

4 Conclusion

The gamma function provides a good-fit for modeling the failure distribution of a locally fabricated welding machine and is so recommended for similar equipment. The reliability indices show that the small welding machine is in good working condition and consequently, a maintenance policy aggregating the reliability measures has been formulated in this work to provide a guide for planned maintenance as well as optimum utilization of the machine. The use of probability functions in this work to describe failure behaviour of the machine and provide pointers to dangers on the operating life of the machine is an added performance assessment technique. These results would help in small welding machine design and fabrication.

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