Abacus (Mathematics Science Series) Vol. 44, No 1, Aug. 2019

MATHEMATICAL MODEL ON THE DYNAMICS OF DOMESTIC VIOLENCE

by

Mohammed, Ibrahim Adamu & Musa, Samuel

Department of Mathematics and Computer, School of Sciences, College of Education Billiri, Gombe State,

Nigeria & Department of Mathematics, School of Physical Sciences, Modibbo Adama University of Technology Yola, Adamawa State, Nigeria

alhajiibrahim0011@gmail.com, smusa@mautech.edu.ng

Keywords: mathematical, domestic violence, violent, victim, reproduction number, stability

Abstract

In this research, we proposed a mathematical model on the dynamics of domestic violence. The model was developed using a set of six ordinary differential equations each for a compartment; Potentially Violent individuals (S), *Violent* (V), *Recovered Violent* (R), *Susceptible Victims* (S_V), *Victims* (V_V) *and the Recovered Victims* (R_V). *The domestic violence free and endemic equilibria were obtained and stability analyses have been carried out using Routh-Hurwitz criterion. We obtained the basic reproduction number,* R_0 *which determines whether domestic violence can be eradicated or not. The analytical results revealed that, both the domestic violence-free and endemic equilibria are stable which means the violence can be eliminated and even when it exists, the society can still survive. The result of the numerical experiments agrees with that of analytical.*

1. Introduction

Domestic violence is a crisis within a family in which one individual abuses the other. Domestic Violence (also named domestic abuse, battering or family violence) is a pattern of behaviour which involves abuse by one person against another in a domestic setting, such as in marriage, or cohabitation". Domestic violence can take place in heterosexual relationships, and can involve violence against children in the family (Dutton and Donald, 2006).

"Domestic Violence can be criminal and includes physical assault (hitting, pushing, shoving, etc.), sexual abuse (unwanted or forced sexual activity), and stalking. Although emotional, psychological and financial abuse are not criminal behaviors, they are forms of abuse and can lead to criminal violence.The violence takes many forms and can happen all the time or once in a while" (Bartlett, Rhode and Grossman, 2013).

According to Alodpo, Yusuf and Arulogun (2011) victims can be of any age, sex, race, culture, religion, education, employment or marital status. Although both men and women can be abused, most victims are women. Children in homes where there is domestic violence are more likely to be abused and/or neglected. Most children in these homes know about the violence. Even if a child is not physically harmed, they may have emotional and behavior problems.

Men sometimes are also victims of domestic violence. Though, that one is not as popular as violence on women and children perhaps this is because most men who suffer this type of violence are often ashamed to mention it for fear of being seen as weaklings. Domestic violence happens mostly within intimate relationships and it includes emotional, sexual, physical, psychological, spiritual abuse.

Level of education and socio-economic state are the most potent predictors of domestic violence, the higher the socio-economic status of a family, the lower the incidence of Domestic Violence Duoto and Jodar (2013) and Olujide, Oluremi and Sussan (2011).

According to Gwazane (2011), among the factors associated with domestic violence is poverty. It was found that wealth has an inverse relationship with domestic violence, meaning the richer the couples the lesser the number of cases of domestic violence.

Women that witnessed physical violence are more likely to have tolerant attitudes towards domestic violence and women with tolerant attitudes are more likely to have reported spousal abuse. It was found that, an increasing proportion of women in the community with tolerant attitudes was significantly positively associated with spousal sexual and emotional abuse, but not significantly associated with spousal physical abuse. In addition, an increasing proportion of men in the community with tolerantattitudes and an increasing proportion of women who had witnessed physical violence in the community was significantly positively associated with spousal physical abuse, but not significantly associated with spousal sexual and emotional abuse (Olalekan, Tahereh and Stephen, 2010).

Domestic violence is now an epidemic affecting individuals in every community, regardless of age, economic status, sexual orientation, gender, race, religion, or nationality hence, the need to ascertain its dynamics and tackle it in earnest. A lot of researches have been conducted on domestic violence, but none has approached the dynamics in terms of distinct layers (for violent individuals and that of victims). For examples, Jose (2010), Lynn (2027), Mwai (2006) and Otoo, Sebil and Amposah (2014) considered the interaction between violent individuals and victims in same layer.

The model developed in this research consists of three compartments in violent individuals and also three compartments in victims. The approach adopted is similar to that of vector and host (as in the case of mosquitoes and human populations).

2. Model Formulation

To study the dynamics of domestic violence in two interacting populations of violent individuals (the vector), and victims (the host), we formulate a model which subdivides the population of violent persons into: potentially violent, $S(t)$, violent, $V(t)$ and recovered violent, $R(t)$ while that of victims is subdivided into: susceptible victim, $S_V(t)$, victim, $V_V(t)$, and recovered victim, $R_V(t)$.

Variable	Description
$S(t)$	Potentially violent (individuals) at time (t)
$V(t)$	Violent individuals at time (t)
$R(t)$	Recovered/Removed Violent individuals at time (t)
$S_V(t)$	Susceptible victims at time (t)
$V_V(t)$	Victims (individuals) at time (t)
$R_V(t)$	Recovered/Removed Victims at time (t)

Table 1: Description of state variables

Parameter	Description
Λ	Recruitment rate into violent population
Λ_V	Recruitment rate into victims' population
β	Rate at which potentially violent becomes violent
β_V	Rate at which susceptible victim becomes Victim
γ	Rate at which violent becomes recovered by reformation
γ_V	Rate at which victim becomes recovered
θ	Rate at which potential violent becomes recovered
θ_V	Rate at which potential victim becomes recovered
α	Probability that a contact between violent individual and a victim results into domestic violence
φ	Violence induced death rate
μ	Natural mortality rate

Table 2: Description of model parameters

The classes of potentially violent and Susceptible Victim population are increased by recruiting individuals at rates Λ and Λ_V respectively and get decreased by natural death μ. The Potentially Violent individuals become Violent as they change behaviour due to interaction with Domestic Violent individuals at a rate, β. The Violent individuals become recovered as they get reformed via enlightenment, counselling or enforced law at the rate, γ.

The Potential Victims become Victims as they get contact with Violent individuals at a rate, $\alpha\beta V$. The Victim individuals become recovered as they get separated from the Violent people as a result of divorce (in case of couples) or growth (in case of children) at a rate, γ_V. Both the potentially violent and susceptible victim populations get decreased by moving to recovered/removed classes at rates θ and θ_V respectively. In each subpopulation, individuals suffer natural death at a rate μ. However, violent individuals may murder the victims as a result of victimisation at a rate φ.

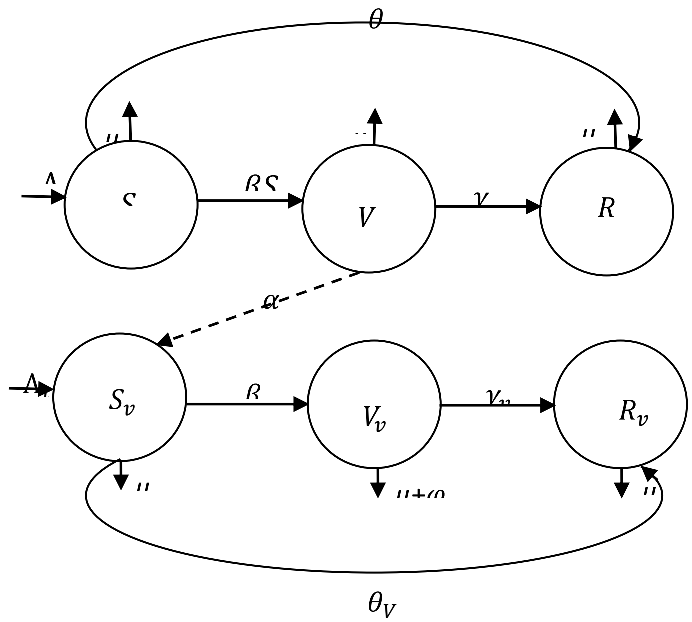

Fig. 1: The compartmental model of domestic violence transmission

2.2 Model Equations

$$\frac{dS}{dt} = \Lambda - (\beta V + \mu + \theta)S \quad 1$$

$$\frac{dV}{dt} = \beta SV - (\gamma + \mu)V \quad 2$$

$$\frac{dV_v}{dt} = \beta_v S_v - (\gamma_v + \mu + \varphi)V_v \quad 3$$

$$\frac{dS_v}{dt} = \Lambda_v - (\mu + \beta_v \alpha V + \theta_v)S_v \quad 4$$

$$\frac{dR_v}{dt} = \gamma_v V_v + \theta_v S_v - \mu R_v \quad 5$$

$$\frac{dR}{dt} = \gamma V + \theta S - \mu R \quad 6$$

Together with the initial conditions:

$$S(0) = S_0, V(0) = V_0, R(0) = R_0, S_V(0) = S_{V_0}, V_V(0) = V_{V_0}, R_V(0) = R_{V_0} \quad (7)$$

3.0 Existence and Stability of Equilibrium Points

3.1. Domestic-violence Free Equilibrium

We are interested in population dynamics of the system in the absence of domestic violence. That is, we want to study how the population changes when there is no domestic violence.

To obtain the domestic-violence free equilibrium, we equate the right hand side of the model Eq. (9) to (14) be zero. That is:

$$\Lambda-(\beta V+\mu+\theta)S=0 \quad (8)$$
$$\beta SV-(\gamma+\mu)V=0 \quad (9)$$
$$\gamma V+\theta S-\mu R=0 \quad (10)$$
$$\Lambda_v-(\mu+\beta_v\alpha V+\theta_v)S_v=0 \quad (11)$$
$$\beta_v S_v-(\gamma_v+\mu+\varphi)V_v=0 \quad (12)$$
$$\gamma_v V_v+\theta_v S_v-\mu R_v=0 \quad (13)$$

At domestic violence free, we assume the absence of domestic violence. Hence, $V=0$ and $V_V=0$.

Now solving Eq. (8), Eq. (10), Eq. (11) and Eq. (13) yield $S=\frac{\Lambda}{(\mu+\theta)}$, $R=\frac{\theta\Lambda}{\mu(\mu+\theta)}$, $S_V=\frac{\Lambda_V}{(\mu+\theta_V)}$ and $R_V=\frac{\theta_V\Lambda_V}{\mu(\mu+\theta_V)}$ respectively. Thus we obtain the domestic violence free equilibrium, E_0 as:

$$E_0=\left(S^*,V^*,R^*,S_V{}^*,V_V{}^*,R_V{}^*\right)=\left(\frac{\Lambda}{(\mu+\theta)},0,R=\frac{\theta\Lambda}{\mu(\mu+\theta)},S_V=\frac{\Lambda_V}{(\mu+\theta_V)},0,R_V=\frac{\theta_V\Lambda_V}{\mu(\mu+\theta_V)}\right) \quad 14)$$

3.2 Endemic Equilibrium

The total population dynamics may be altered when a violent individual is introduced into the population.

For endemic equilibrium, there is an existence of domestic violence hence $V\neq 0$ and $V_V\neq 0$

Solving Eq. (8) to Eq. (13) we obtain:

$$S^*=\frac{\gamma+\mu}{\beta},$$
$$V^*=\frac{\beta\Lambda-\mu(\gamma+\mu)-\theta(\gamma+\mu)}{\beta(\gamma+\mu)},$$
$$R^*=\frac{\gamma\beta\Lambda-\mu\gamma(\gamma+\mu)-\theta\gamma(\gamma+\mu)+\theta(\gamma+\mu)^2}{\mu\beta(\gamma+\mu)}$$

$$S_V{}^*=\frac{\Lambda_V\beta(\gamma+\mu)}{\beta_V\alpha\beta\Lambda+\mu(\gamma+\mu)-\theta\beta_V\alpha(\gamma+\mu)-\mu\beta_V\alpha(\gamma+\mu)+\theta_V\beta(\gamma+\mu)}$$

$$V_V^*=\frac{\beta_V\Lambda_V\beta(\gamma+\mu)}{(\gamma_V+\mu+\varphi)\left[\beta_V\alpha\beta\Lambda-\mu\beta_V\alpha(\gamma+\mu)-\theta\beta_V\alpha(\gamma+\mu)+\mu\beta(\gamma+\mu)+\theta_V(\gamma+\mu)\right]} \text{ and}$$

$$R_V^* = \frac{\gamma_V \beta_V \Lambda_V \beta(\gamma+\mu)}{\mu(\gamma_V + \mu + \varphi)\left[\beta_V \alpha\beta\Lambda - \mu\beta_V\alpha(\gamma+\mu) - \theta\beta_V\alpha(\gamma+\mu) + \mu\beta(\gamma+\mu) + \theta_V(\gamma+\mu)\right]}$$

$$+\frac{\theta_V \Lambda_V \beta(\gamma+\mu)}{\mu\beta_V\alpha\beta\Lambda - \mu\beta_V\alpha(\gamma+\mu) - \theta\beta_V\alpha(\gamma+\mu) + \mu\beta(\gamma+\mu) + \theta_V(\gamma+\mu)}$$

Now, the Domestic-Violence Endemic Equilibrium gives

$$E_0 = \left(S^*, V^*, R^*, S_V{}^*, V_V{}^*, R_V{}^*\right) = \left(a, b, c, d, e, f\right) \tag{15}$$

where

$$a = \frac{\gamma+\mu}{\beta}, \tag{16}$$

$$b = \frac{\beta\Lambda - \mu(\gamma+\mu) - \theta(\gamma+\mu)}{\beta(\gamma+\mu)}, \tag{17}$$

$$c = \frac{\gamma\beta\Lambda - \mu\gamma(\gamma+\mu) - \theta\gamma(\gamma+\mu) + \theta(\gamma+\mu)^2}{\mu\beta(\gamma+\mu)}, \tag{18}$$

$$d = \frac{\Lambda_V \beta(\gamma+\mu)}{\beta_V\alpha\beta\Lambda + \mu(\gamma+\mu) - \theta\beta_V\alpha(\gamma+\mu) - \mu\beta_V\alpha(\gamma+\mu) + \theta_V\beta(\gamma+\mu)}, \tag{19}$$

$$e = \frac{\beta_V \Lambda_V \beta(\gamma+\mu)}{(\gamma_V + \mu + \varphi)\left[\beta_V\alpha\beta\Lambda - \mu\beta_V\alpha(\gamma+\mu) - \theta\beta_V\alpha(\gamma+\mu) + \mu\beta(\gamma+\mu) + \theta_V(\gamma+\mu)\right]}, \tag{20}$$

$$\left.\begin{aligned} f &= \frac{\gamma_V \beta_V \Lambda_V \beta(\gamma+\mu)}{\mu(\gamma_V + \mu + \varphi)\left[\beta_V\alpha\beta\Lambda - \mu\beta_V\alpha(\gamma+\mu) - \theta\beta_V\alpha(\gamma+\mu) + \mu\beta(\gamma+\mu) + \theta_V(\gamma+\mu)\right]} \\ &+ \frac{\theta_V \Lambda_V \beta(\gamma+\mu)}{\mu\beta_V\alpha\beta\Lambda - \mu\beta_V\alpha(\gamma+\mu) - \theta\beta_V\alpha(\gamma+\mu) + \mu\beta(\gamma+\mu) + \theta_V(\gamma+\mu)} \end{aligned}\right\} \tag{21}$$

3.3 Basic Reproduction Number

The Basic Reproduction Number, R_o in this context, is measure of the potentiality for Domestic Violence spread in a given population. It is therefore a threshold for stability of a domestic-violence free equilibrium and is related to the peak and final size of a given epidemic.

We apply the Next Generation Matrix technique to obtain the Basic Reproduction number, R_o, by considering the infected compartments of the system (1) to (6), that is Eq.(2) and Eq. (5).

Let F_i be the rate of appearance of Domestic Violence in the i compartment and V_i be the rate of transfer of individuals out of i, given the domestic violence free equilibrium, then R_0 is the spectral radius, largest eigen value of the next generation matrix donated by:

$$G = FV^{-1}$$

Let

$$F_i(x) = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} \beta SV \\ \beta_V S_V \end{bmatrix}$$

Evaluating the Jacobian matrix of $F(x)$

$$\left[\frac{\partial F_i}{\partial t_i}\right] = \begin{bmatrix} \frac{\partial F_1}{\partial V} & \frac{\partial F1}{\partial V_V} \\ \frac{\partial F_2}{\partial V} & \frac{\partial F_2}{\partial V_V} \end{bmatrix}$$

Hence, at the domestic-violence free equilibrium, E_o, we obtain

$$F = \begin{bmatrix} \frac{\beta\Lambda}{(\mu+\theta)} & 0 \\ 0 & 0 \end{bmatrix}$$

But $V_i(x) = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} (\gamma+\mu)V \\ (\gamma_V+\mu+\varphi)V_V \end{bmatrix}$

Equating the Jacobian matrix of $V(x)$

$$\left[\frac{\partial V_i}{\partial t_i}\right] = \begin{bmatrix} \frac{\partial F_1}{\partial V} & \frac{\partial F1}{\partial V_V} \\ \frac{\partial F_2}{\partial V} & \frac{\partial F_2}{\partial V_V} \end{bmatrix} = \begin{bmatrix} (\gamma+\mu) & 0 \\ 0 & (\gamma_V+\mu+\varphi) \end{bmatrix}$$

Now,

$$V^{-1} = \begin{bmatrix} \frac{1}{(\gamma+\mu)} & 0 \\ 0 & \frac{1}{(\gamma_V+\mu+\varphi)} \end{bmatrix}$$

Thus the Basic Reproduction Number R_0 , is obtained as

$$R_0 = \frac{\beta\Lambda}{(\mu+\theta)(\mu+\gamma)} \tag{22}$$

3.4 Stability Analysis of the Model

3.4.1 Local Stability of the Domestic Violence Free Equilibrium

Theorem 1:

The disease free equilibrium point, E_0 is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.

Proof: To check for the local stability of the free equilibrium of the model, we need to obtain the Jacobian Matrix, $J(E)$ of the model equations.

Thus,

$$J(E) = J_{(S,V,R,S_V,V_V,R_V)} = \begin{bmatrix} \frac{\partial F_1}{\partial S} & \frac{\partial F_1}{\partial V} & \frac{\partial F_1}{\partial R} & \frac{\partial F_1}{\partial S_V} & \frac{\partial F_1}{\partial V_V} & \frac{\partial F_1}{\partial R_V} \\ \frac{\partial F_2}{\partial S} & \frac{\partial F_2}{\partial V} & \frac{\partial F_2}{\partial R} & \frac{\partial F_2}{\partial S_V} & \frac{\partial F_2}{\partial V_V} & \frac{\partial F_2}{\partial R_V} \\ \frac{\partial F_3}{\partial S} & \frac{\partial F_3}{\partial V} & \frac{\partial F_3}{\partial R} & \frac{\partial F_3}{\partial S_V} & \frac{\partial F_3}{\partial V_V} & \frac{\partial F_3}{\partial R_V} \\ \frac{\partial F_4}{\partial S} & \frac{\partial F_4}{\partial V} & \frac{\partial F_4}{\partial R} & \frac{\partial F_4}{\partial S_V} & \frac{\partial F_4}{\partial V_V} & \frac{\partial F_4}{\partial R_V} \\ \frac{\partial F_5}{\partial S} & \frac{\partial F_5}{\partial V} & \frac{\partial F_5}{\partial R} & \frac{\partial F_5}{\partial S_V} & \frac{\partial F_5}{\partial V_V} & \frac{\partial F_5}{\partial R_V} \\ \frac{\partial F_6}{\partial S} & \frac{\partial F_6}{\partial V} & \frac{\partial F_6}{\partial R} & \frac{\partial F_6}{\partial S_V} & \frac{\partial F_6}{\partial V_V} & \frac{\partial F_6}{\partial R_V} \end{bmatrix}$$

At Domestic Violence Free Equilibrium (DVFE) state,

$$J_{(DVFE)} = J_{\left(\frac{\Lambda}{(\mu+\theta)},0,\frac{\theta\Lambda}{\mu(\mu+\theta)},\frac{\Lambda_V}{(\mu+\theta_V)},0,\frac{\theta_V\Lambda_V}{\mu(\mu+\theta_V)}\right)}$$

$$J(E) = \begin{bmatrix} -(\beta V+\mu+\theta) & -\beta S & 0 & 0 & 0 & 0 \\ \beta V & -(\mu+\gamma) & 0 & 0 & 0 & 0 \\ \theta & \gamma & -\mu & 0 & 0 & 0 \\ 0 & \beta_v \alpha S_v & 0 & -(\mu+\beta_v\alpha V+\theta_v) & 0 & 0 \\ 0 & 0 & 0 & \beta_v & -(\mu+\varphi+\gamma_v) & 0 \\ 0 & 0 & 0 & \theta_v & \gamma_v & -\mu \end{bmatrix} \quad (23)$$

To determine the stability of DVFE point, we find the characteristic equation of (23),

Therefore,

$$\left|J_{DVFE} - \lambda I\right| = 0$$

$$\begin{vmatrix} -(\mu+\theta)-\lambda & \frac{-\beta\Lambda}{(\mu+\theta)} & 0 & 0 & 0 & 0 \\ 0 & -(\mu+\gamma)-\lambda & 0 & 0 & 0 & 0 \\ \theta & \gamma & -\mu-\lambda & 0 & 0 & 0 \\ 0 & \frac{\beta_v\alpha\Lambda_v}{(\mu+\theta_v)} & 0 & -(\mu+\theta_v)-\lambda & 0 & 0 \\ 0 & 0 & 0 & \beta_v & -(\mu+\varphi+\gamma_v)-\lambda & 0 \\ 0 & 0 & 0 & \theta_v & \gamma_v & -\mu-\lambda \end{vmatrix} = 0 \quad (24$$

Solving Eq. (24)

$$\lambda_1 = -\mu,\ \lambda_2 = -\mu,\ \lambda_3 = -(\mu+\varphi+\gamma_V),\ \lambda_4 = -(\mu+\theta_V),\ \lambda_5 = -(\mu+\theta),\ \lambda_6 = -(\mu+\gamma) \quad (25)$$

Since all the values of $\lambda_i < 0$ for $i = 1,2,3,4,5,6$ when $R_0 < 1$ we conclude that that the domestic violence free equilibrium state is locally asymptotically stable.

3.4.2 Local Stability of the Domestic Violence Endemic Equilibrium

Theorem 2:

The domestic violence endemic equilibrium point, Ee is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$

Proof:

At the Domestic Violence Endemic Equilibrium (DVEE) state,

$$J_{DVEE} = J_{(a,b,c,d,e,f)}$$

To determine the stability of DVEE point, we find the characteristic equation of Eq. (26), $\left|J_{DVEE} - \lambda I\right| = 0$

Therefore,

$$\begin{vmatrix} -(\beta b+\mu+\theta)-\lambda & -\beta a & 0 & 0 & 0 & 0 \\ \beta b & -(\mu+\gamma)-\lambda & 0 & 0 & 0 & 0 \\ \theta & \gamma & -\mu-\lambda & 0 & 0 & 0 \\ 0 & \beta_v \alpha d & 0 & -(\mu+\beta_v \alpha b+\theta_v)-\lambda & 0 & 0 \\ 0 & 0 & 0 & \beta_v & -(\mu+\varphi+\gamma_v)-\lambda & 0 \\ 0 & 0 & 0 & \theta_v & \gamma_v & -\mu-\lambda \end{vmatrix} = 0 \qquad (27)$$

It is clear that the third column of Eq. (27) contains only the diagonal terms so that we have

$$(-\mu-\lambda)\begin{vmatrix} -(\beta b+\mu+\theta)-\lambda & -\beta a & 0 & 0 & 0 \\ \beta b & -(\mu+\gamma)-\lambda & 0 & 0 & 0 \\ 0 & \beta_v \alpha d & -(\mu+\beta_v \alpha b+\theta_v)-\lambda & 0 & 0 \\ 0 & 0 & \beta_v & -(\mu+\varphi+\gamma_v)-\lambda & 0 \\ 0 & 0 & \theta_v & \gamma_v & -\mu-\lambda \end{vmatrix} = 0 \qquad (28)$$

In the same way, Eq. (28) is reduced to:

Hence,

$$(-\mu-\lambda)(-\mu-\lambda)(-\mu-\varphi-\gamma_v-\lambda)(-\mu-\beta_v \alpha b-\theta_v-\lambda) = 0$$

Implying that

$$\lambda_1 = -\mu, \qquad \lambda_2 = -\mu, \qquad \lambda_3 = -(\mu+\varphi+\gamma_v), \qquad \lambda_4 = -(\mu+\beta_v \alpha b+\theta_v) \qquad (29)$$

And then

$$\begin{vmatrix} -(\beta b+\mu+\theta)-\lambda & -\beta a \\ \beta b & -(\mu+\gamma)-\lambda \end{vmatrix} = 0 \qquad (30)$$

Solving the determinant of Eq. (30), we obtain the characteristic equation

$$(-\beta b-\mu-\theta-\lambda)(-\mu-\gamma-\lambda)+\beta^2 ab = 0 \qquad (31)$$

Expanding and simplifying the Eq. (31)

$$\lambda^2+(\beta b+2\mu+\theta+\gamma)\lambda+\beta b\mu+\beta b\gamma+\mu\gamma+\mu^2+\theta\mu+\theta\gamma+\beta^2 ab = 0 \qquad (32)$$

Eq. (35) can be written in the form

$$A_2\lambda^2+A_1\lambda+A_0 = 0 \qquad (33)$$

Where,

$$\left.\begin{aligned} A_2 &= 1 \\ A_1 &= \beta b + 2\mu + \theta + \gamma \\ A_0 &= \beta b\mu + \beta b\gamma + \mu\gamma + \mu^2 + \theta\mu + \theta\gamma + \beta^2 ab \end{aligned}\right\} \quad (34)$$

But

$$a = \frac{\gamma + \mu}{\beta} \quad \text{and} \quad b = \frac{\beta\Lambda - \mu(\gamma + \mu) - \theta(\gamma + \mu)}{\beta(\gamma + \mu)}$$

Therefore, substituting for a and b into Eq. (34) and simplifying, we get

$$A_0 = 2[\beta\Lambda - \mu(\mu + \gamma) - \theta(\mu + \gamma)] + \mu\gamma + \mu^2 + \theta\mu + \theta\gamma$$

This can be expressed as

$$A_0 = \mu^2 + \mu\gamma + \theta\mu + \theta\gamma + 2\left(\frac{\beta\Lambda(\gamma+\mu)(\theta+\mu) - \mu(\gamma+\mu)^2(\theta+\mu) - \theta\mu(\gamma+\mu)^2(\theta+\mu)}{(\gamma+\mu)(\theta+\mu) - \beta\Lambda}\right)(1 - R_0) \quad (35)$$

Applying Routh-Hurwitz criterion which states that all roots of the quadratic Eq. (36) have negative real parts if and only if the coefficients A_i are positive and the determinants, of the matrices, $H_i > 0$ for $i = 1, 2$. Therefore, from Eq. (34), we see that $A_1 > 0$, and $A_2 > 0$ $A_2 > 0$. That is,

$$H_1 = A_1 = \beta b + 2\mu + \theta + \gamma > 0$$

$$H_2 = \begin{vmatrix} A_1 & 0 \\ 1 & A_0 \end{vmatrix} > 0$$

Therefore, both the eigen values of the quadratic Eq. (36) have negative real parts, it implying that $\lambda_5 < 0$ and $\lambda_6 < 0$. Hence, since all the values of $\lambda_i < 0$ for $i = 1, 2, 3, 4, 5, 6$ when $R_0 < 1$ we conclude that that the domestic violence endemic equilibrium state is locally asymptotically stable.

However, if $R_0 > 1$, and by Descartes rule of signs, there is exactly one sign change in the sequence, A_2, A_1, A_0 of the coefficients of the quadratic Eq. (36) this means that, there exists one eigen value with positive real part; then we conclude that the domestic violence endemic equilibrium point will be unstable.

4. Computational Results

We performed some numerical experiments using ode45 function from MATLAB R2015a to study the behaviour of the system. The initial condition for each plot and parameter values are presented in table 3. Experiment one to eleven present the graphs generated from numerical simulations carried out on the model equations.

Variable/Parameter	Values	Source
Λ	0.29000	Estimated from N.B.S
Λ_V	0.6000	Estimated from N.B.S
β	0.0040	Social Welfare Gombe (2018)
β_V	0.0032	Social Welfare Gombe (2018)

γ	0.0166	Social Welfare Gombe (2018)
γ_V	0.0430	Social Welfare Gombe (2018)
θ	0.00066	Estimated from N.B.S and G.B.S
θ_V	0.0014	Estimated from N.B.S and G.B.S
μ	0.0124	CIA World Fact Book (2018)
α	0.6	EstimatedSocial Welfare Gombe (2018)
φ	0.003	Social Welfare Gombe (2018)
$S(0)$	800	Social Welfare Gombe (2018)
$V(0)$	350	Social Welfare Gombe (2018)
$R(0)$	80	Social Welfare Gombe (2018)
$S_V(0)$	1000	Social Welfare Gombe (2018)
$V_V(0)$	520	Social Welfare Gombe (2018)
$R_V(0)$	200	Social Welfare Gombe (2018)

Table 3: Variables and parameters values used for computational results

Experiment One

In this experiment, we studied the behaviour of potentially violent subpopulation where we kept all the variables and parameters (presented in table 4.6.1) constant except β (rate at which potentially violent individual becomes violent). We simulated for values of $\beta = 0.0001, 0.0002,$and 0.0004 and the results are respectively presented in fig.2, fig.3, and fig.4.

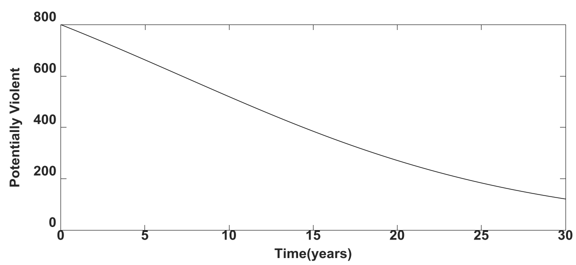

Fig. 1: Result for Potentially violent subpopulation, for $\beta = 0.00001$

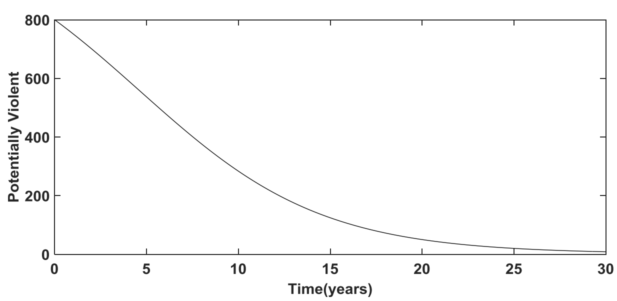

Fig. 2: Result for Potentially Violent subpopulation with $\beta = 0.0002$

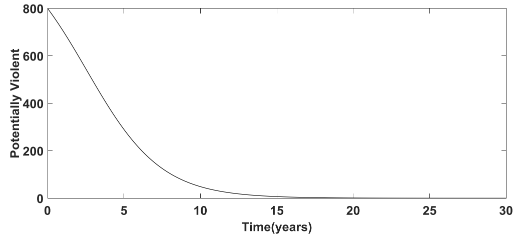

Fig. 3: Result for Potentially Violent subpopulation with $\beta = 0.0004$

Experiment Two

The behaviour of violent subpopulation was studied. All the variables and parameters (presented in table 3) were kept constant except β and γ (rate at which potentially violent becomes violent and rate at which violent becomes recovered). We simulated for $\beta = 0.004$, $\gamma = 0.0166$, separately. Then $\gamma = 0.26$ when $\beta = 0.004$. The results are respectively presented in fig.5, fig.6 and fig.7.

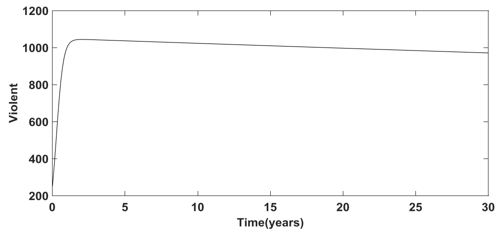

Fig.5: Result for Violent Population, $\beta = 0.004$

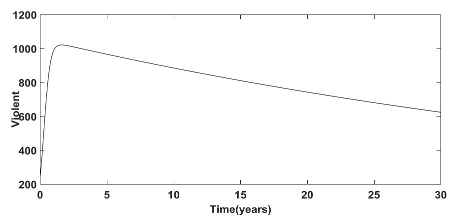

Fig.6: Result for Violent Population $\gamma = 0.0166$

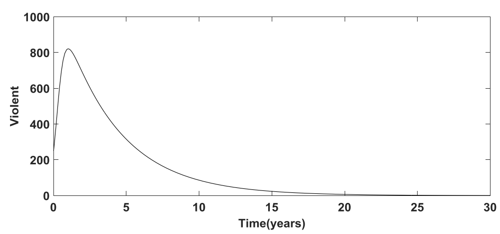

Fig.7: Result for Violent Population, $\beta = 0.004$, $\gamma = 0.26$

Experiment Three

We studied the behaviour of recovered violent subpopulation where we kept all the variables and parameters (presented in table 3) except γ. We simulated for $\gamma = 0.000166$, 0.00096, and 0.00166. . The results are respectively presented in fig.8, fig.9 and fig.10.

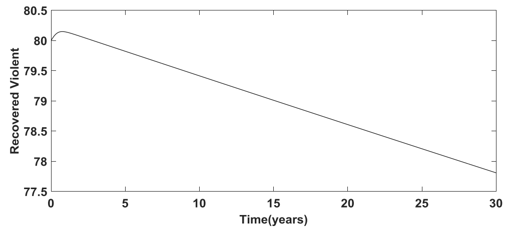

Fig.8: Result for Recovered Violent population, for: $\gamma = 0.000166$

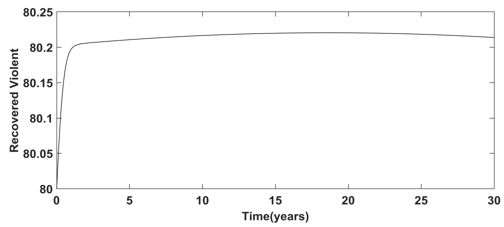

Fig.9: Result for Recovered Violent population, for $\gamma = 0.00096$

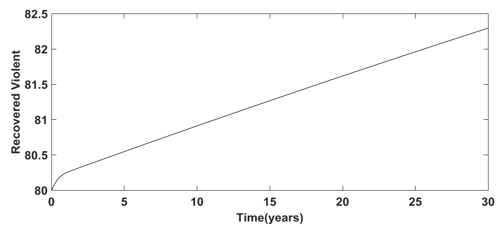

Fig.10: Result for Recovered Violent population, for $\gamma = 0.00166$

Experiment Four

The behaviour of susceptible victim subpopulation was studied. All the variables and parameters (presented in table 3) were kept constant except α . We simulated for $\alpha = 0.03$, 0.13 and 0.6. The results are respectively presented in fig.11, 12 and fig.13.

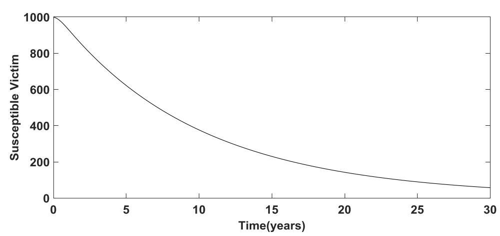

Fig.11: Result for Susceptible Victim population when $\alpha = 0.03$

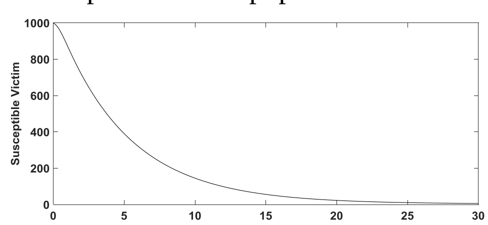

Fig.12: Result for Susceptible Victim population when $\alpha = 0.13$

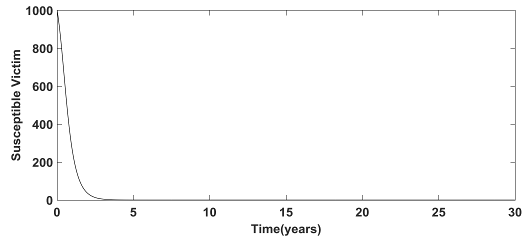

Fig.13: Result for Susceptible Victim population when $\alpha = 0.6$

Experiment Five

We studied the behaviour of victim subpopulation where we kept all the variables and parameters (presented in table 4.6.1) constant except γ_V and φ. We used different values of γ_V and φ. Firstly: $\gamma_V = 0.00043$, 0.23 and $\varphi = 0.03$, Separately. We also simulated at same instance: $\gamma_V = 0.23$, $\varphi = 0.3$. The results are respectively presented in fig.14 to fig.17.

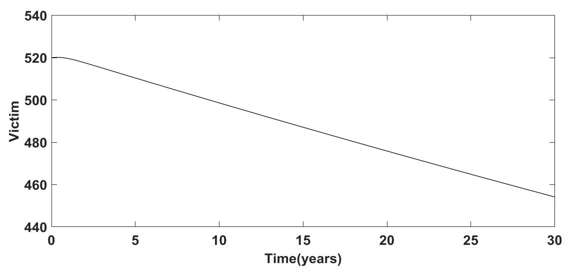

Fig.14: Result for Victim subpopulation for: $\gamma_V = 0.00043$

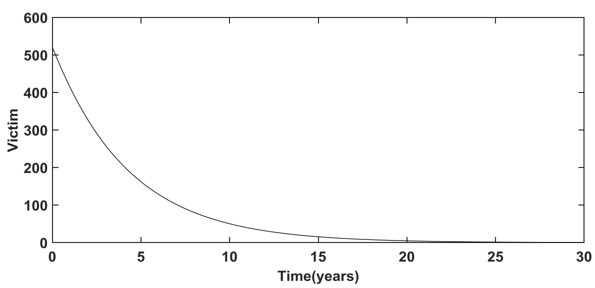

Fig.15: Result for Victim population, for: $\gamma_V = 0.23$

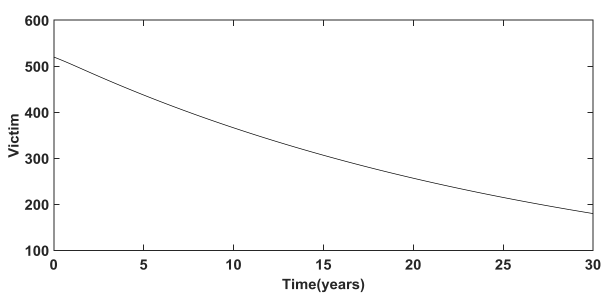

Fig.16: Result for Victim population, for: $\varphi = 0.03$

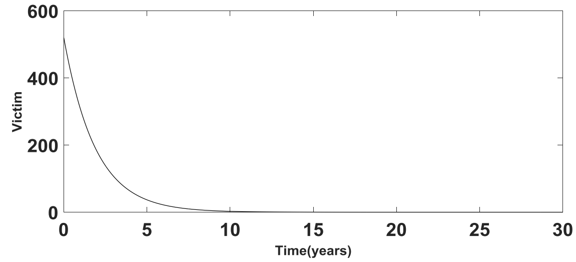

Fig.17: Result for Victim population, for: $\gamma_V = 0.23$ while $\varphi = 0.3$

Experiment Six

Here, the behaviour of recovered victim subpopulation was studied. All the variables and parameters (presented in table 3) were kept constant except γ_V. We simulated for $\gamma_V = 0.00023$, 0.00043, 0.00063 and 0.0063. The results are respectively presented in fig.18 to fig.21.

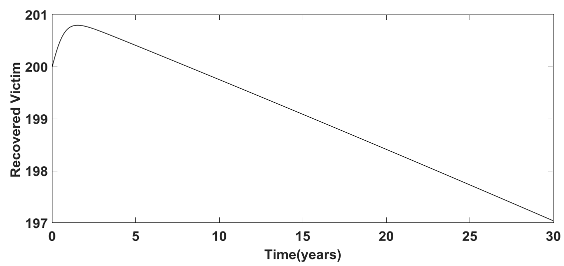

Fig.18: Result for recovered victim population when $\gamma_V = 0.00023$

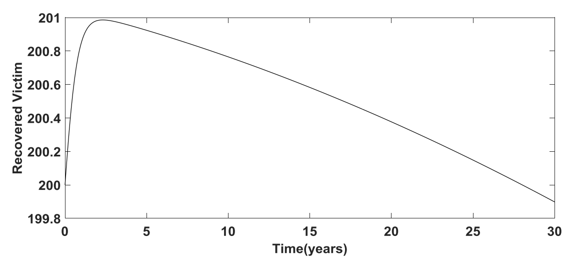

Fig.19: Result for Victim population $\gamma_V = 0.00043$

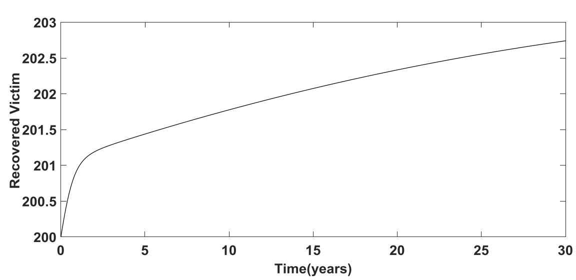

Figure 20: Result for Victim population $\gamma_V = 0.00063$,

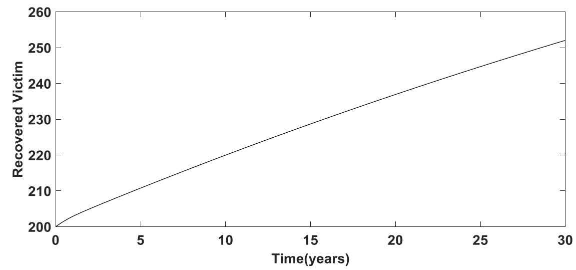

Fig.21: Result for Victim population $\gamma_V = 0.0063$

Experiment Seven

In this experiment, we studied the behaviour of combined subpopulations of potentially violent and the violent individuals where we kept constant all the variables and parameters (presented in table 3) except β. We simulated for $\beta = 0.004$, 0.0001 and 0.00004. The results are respectively presented in fig. 22, fig. 23 and fig24.

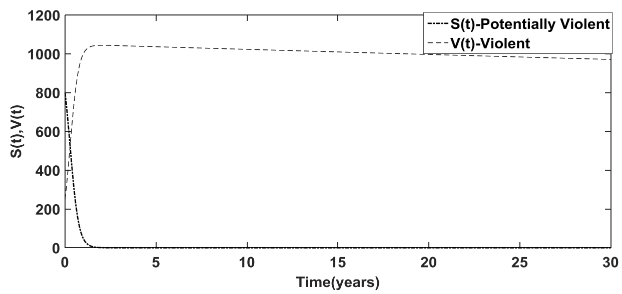

Fig.22: The behaviour of the violent and potentially violent as they interact, when $\beta = 0.004$

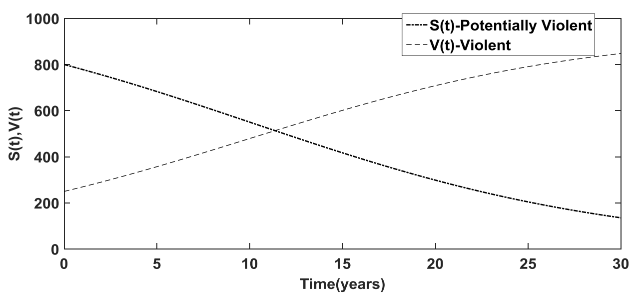

Fig.23: The result for the violent and potentially violent as they interact, when $\beta = 0.0001$

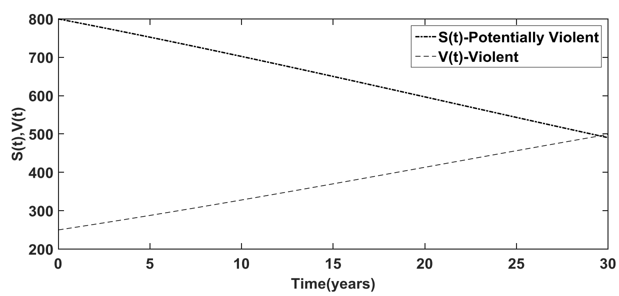

Fig.24: The result for the violent and potentially violent as they interact, when $\beta = 0.00004$

Experiment Eight

In this experiment, we studied the behaviour of violent and susceptible victim subpopulations combined together. All the variables and parameters (presented in table 4.6.1) were kept constant except α (Probability that a contact between violent individual and a victim results into domestic violence) and γ (rate at which violent individual become recovered). We simulated for $\alpha = 0.6$, 0.06, $\gamma = 0.066,\ 0.266$. The results are respectively presented in fig.25 to fig.28.

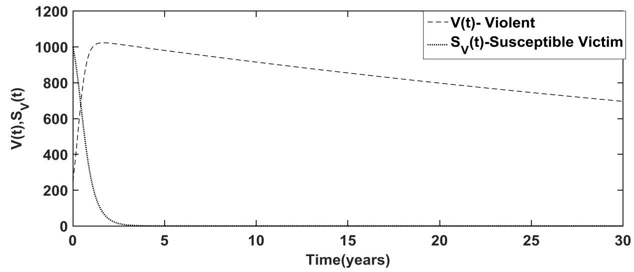

Fig.25: The behaviour of the violent and susceptible victim as they interact, when $\alpha = 0.6$

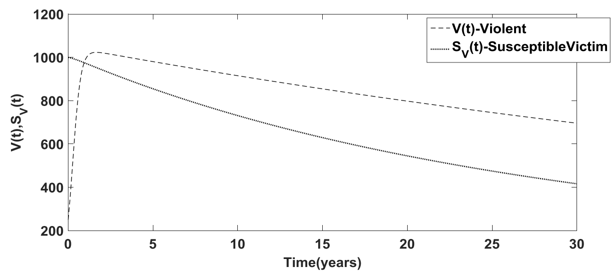

Fig.26: behaviour of the violent and susceptible victim as they interact, when $\alpha = 0.06$

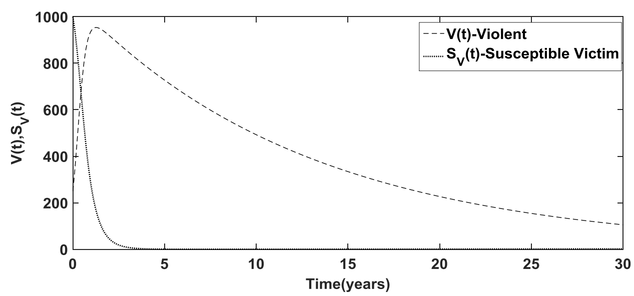

Fig.27: Behaviour of the violent and susceptible victim as they interact, when $\gamma = 0.066$

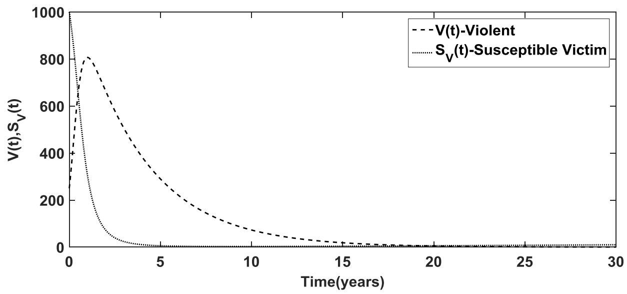

Fig.28: Behaviour of the violent and susceptible victim as they interact, when $\gamma = 0.266$, and $\alpha = 0.006$

Experiment Nine

In this experiment, we studied the behaviour of the entire population without altering any of the parameter values of table 3

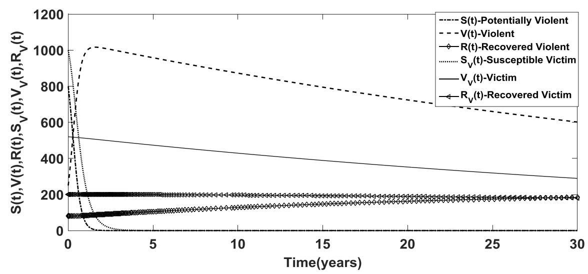

Fig.29: The behaviour of the entire population when all the variables and parameters of table 4.6.1 remained constant.

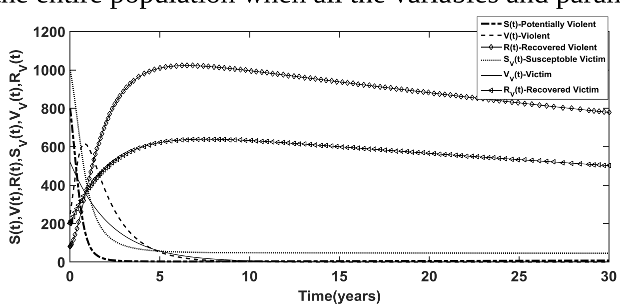

Fig.30: The behaviour of the entire population when all the variables and parameters of table 4.6.1 remained constant except γ and γ_V. Here, $\gamma = 0.66$ while $\gamma_V = 0.43$

4.2.0 Discussion of Results

The modified model consists of a six-dimensional system of ordinary differential equations. The domestic violence free equilibrium was established for the system eq.9 to eq.14 and was given by eq.15.

Reproduction number R_0 was obtained using next generation matrix. We have also established the local stability of the domestic violence free equilibrium of the model using the

Routh-Hurwitz condition for stability. We observed that all the eigen values are negative, that is, $\lambda_i < 0$ for $i = 1,2,3,4,5,6$ when $R_0 < 1$. This implies that the domestic violence free equilibrium is locally asymptotically stable.

The domestic violence endemic equilibrium was also established. The local stability of the endemic equilibrium was established using the Routh-Hurwitz condition. It is observed that all the eigen values are negative, that is, $\lambda_i < 0$ for $i = 1,2,3,4,5,6$ when $R_0 < 1$. This means that, the domestic violence endemic equilibrium is locally asymptotically stable.

The computational results are in line with that of analytical. This is clear when we consider the following:

In experiment one; the result indicates a gradual decrease in the subpopulation of potentially violent from fig.2, to fig.4. In fig.2, the subpopulation decreases from 800 to about 120 in 30 years when β (rate at which potentially violent becomes violent)=0.0001, but as we increase as we increase β to 0.0002, as seen in fig.3, the subpopulation reduces to less than 20. As we further increase β to 0.0004, fig.4 reveals how the subpopulation will go to extinction in 15 years. This is because violent individuals influenced them (as the two interact) making them violent.

In experiment two, we simulated for different values of β and γ. It is observed that as β increases, the violent subpopulation grows rapidly while it decreases with the increase in γ. In fig. 7, $\beta = 0.004$ that is, $R_0 = 3.65 > 1$ (meaning that the violent individual will be increasing) and the graph also indicates an exponential growth in the violent subpopulation. Therefore, analytical and computational results agree with each other. Fig. 6 indicates a gradual decline in the violent subpopulation as $\gamma = 0.0166$. Comparing with the analytical results, it means $R_0 = 3.01$. This also vindicates a gradual decline in the subpopulation. On the other hand, fig.7 reveals that when $\beta = 0.004$ and γ is increased to 0.26, the subpopulation goes to extinction. This re result goes hand in hand with the analytical results as $R_0 = 0.326 < 1$ which means the violence stops.

Experiment seven vindicates the behaviour of combined subpopulations of potentially violent and violent as β decreases. Fig.22 shows how violent subpopulation raises sharply (when $\beta = 0.004$) before it stabilises as it reaches 1000. This agrees with our analytical result as $R_0 = 3.063$ which means that violence will continue. It also indicates a drop in the potentially violent subpopulation which subsequently goes to extension before 3 years as they are converted to violent. Fig. 23 indicates a gradual drop in the subpopulation of potentially violent while that of violent increases slowly. This is when β (the rate at which potentially violent becomes violent) decreases to 0.0001 hence the potentially violent raises. It is in line with our analytical result as $R_0 = 0.0786 < 1$. Fig. 24 vindicates a raise in subpopulation of potentially violent and a fall in that of violent when β decreases to 0.00004. This is true as proven by analytical result, we found R_0 to be 0.031 if evaluated for $\beta = 0.00004$.

The results of experiment eight show the behaviour of violent and susceptible victim subpopulations as they interact. Fig. 25 indicates an exponential growth in violent subpopulation before it drops gradually. This is when $\alpha = 0.6$. For the susceptible victim subpopulation, it drastically falls and dies out before five years as they are being converted to real victims by interacting with violent individuals. This goes hand in hand with the analytical result which we obtained the domestic violence-endemic equilibrium to be stable. In fig. 26, α is decreased to 0.006 which leads susceptible victim subpopulation to rise. This tells us the less the interaction between violent and susceptible victim, the less victims of domestic violence as susceptible ones will not be converted to victims. On the other hand, fig.27 indicates a gradual fall in the violent subpopulation while that of susceptible victim decays sharply until extinction. This is when $\gamma = 0.066$ meaning that, as the rate of conversion from violent to recovered violent increased, the victim population decreased and subsequently domestic violence is minimised. This supports our analytical results because at that value of γ, ($\gamma = 0.066$), our $R_0 = 1.13$. Although R_0 here is slightly greater than one, but the violent subpopulation is declining and this translates the stability we found for the endemic equilibrium. Fig. 28, $\gamma = 0.266$ and $\gamma = 0.006$ (rate of conversion from violent to recovered violent increases while the probability that interaction between susceptible victim and violent decreases), the two subpopulations die completely (though in different rates). This confirms our domestic violence-free equilibrium and our $R_0 = 0.136 < 1$.

Experiment nine vindicates the behaviour of the entire population. Fig.29 indicates a rapid growth in violent subpopulation before it starts dropping slowly as a result of recovery. This is when all the parameters of table 4.1 remained constant. The subpopulations of potentially victim and susceptible victim decay as they get converted to real violent and victim respectively. Recovered violent and victim subpopulations move at constant phase below that of victim subpopulation. This does not contradict our analytical results as $R_0 = 3.063 > 1$. In contrary to fig.29, fig.30 indicates a rapid growth in the subpopulations of recovered violent and recovered victim while that of potentially violent, violent and victim die out. For the susceptible victim, it decays sharply and later stabilises below 10% of its initial value. This is achieved when we increase γ and γ_V to 0.66 and 0.43 respectively. The result agrees with that of analytic. When we evaluate R_0 at this values, we get it to be $0.13 < 1$.

5. Conclusion

Analytical studies were carried out, using linearized stability. The domestic violence-free and endemic equilibria points were obtained and our results reveal that both the equilibria points of the system are locally asymptotically stable if $R_0 < 1$ (in both cases). We conclude that, domestic violence can be eradicated from the society as we found the domestic violence-free equilibrium to be stable. The society can still survive even when there is domestic violence as the endemic equilibrium found to be stable. The result of the numerical experiment carried out indicates that, when proper measures of changing attitudes of violent persons are taken it can significantly eradicate domestic violence from the society.

5.1 Contribution to Knowledge

1. The stability analysis was carried out.
2. The domestic violence free and endemic equilibrium points were obtained.
3. Domestic violence will spread if $R_0 > 1$ and will go extinct if $R_0 < 1$.

5.2 Suggestion for Further Research

In the course of carrying out this work, the following are found worthy of conducting research:

1. A similar research that incorporates factors responsible for domestic violence should be conducted.
2. A model that will consider victims of domestic violence as potential violent should be developed.

References

Alodpo, O., Yusuf O. B. & Arulogun O. S., (2011), *Factors Influencing Gender Based Violence among Men and Women in selected states in Nigeria*. U.S.A National Centre for Biotechnology Information. https://www.ncbi.nlm.nih.gov/pumbed/22571109

Bartlett, K. T., Rhode, D. L., & Grossman, J. L. (2013), *Gender and Law: Theory, Doctrine, Commentary.* Aspen Publishers. ISBN 978-1454817659

Duoto R. & Jodar L., (2013), *MathematicalModelling of the Spread of Divorce in Spain*. https://research gate.net/publication/257296402

Dutton H. and Donald G., (2006). *Rethinking Domestic Violence. Vancouver*, Canada: UBC Press. ISBN 9781282741072.

Gwazane, M. (2011), *A Study to Determine Factors Associated with Domestic Violence among Concordant and Discordant Couples in Zimbabwe*. Babulawyo, Zimbabwe: International Journal of Humanities and Social Science. https//www.ijhssnet.com

Jose, K. S. (2010), *A Mathematical Model of Sentimental Dynamics Accounting for Marital Dissolution.* https://doi.org/10.1371/journal.pone.0009881

Lynn F. K. (2007), *Domestic Violence and Vagal Reactivityto Peer Provocation.* Washington. Journal of Biological Psychology. Vol.74 (2), pp.154-164

Mwai D. M. (2006), *Application of Logistic Regression in Identifying Key Determinants of Domestic Violence in Kenya*. Nairobi

Olalekan, A.U., Stephen, L. & Tahereh, M. (2010), *Sex Disparities in Attitudes Towards Intimates partner violence Against Women in Sub-Saharan Africa: A socio-Ecological Analysis* ISSN:1471-2458 http://www.biomedcentral.com

Olujide A. A., Oluremi H., Abimbola O. & Sussan O. A. (2011), *Gender and Behaviour.* Ile-Ife, Nigeria: Ife Centre for Psychological Studies and Services. Vol.9,No 2

Otoo, D., Sebil, C. & Amponsah, S. K (2014), *Mathematical Modelling of Domestic Violence and Its Trends at Tamale.* Tamale:Ghana, Journal of Asian Scientific Research. https://www.aessweb.com/journals/5003