AN INVENTORY MODEL FOR DETERIORATING ITEMS WITH GENERALISED EXPONENTIAL INCREASING DEMAND AND CONSTANT DETERIORATION WITH LINEAR TIME-VARYING HOLDING COST

by

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Abstract
This paper considers an Economic Order Quantity (EOQ) model with a generalised exponential increasing demand and where shortages are not allowed. The holding cost is a linear function of time and deterioration is assumed to be a constant. Application of the model is illustrated with the help of a numerical example. Sensitivity analysis is carried out on the optimal solution with respect to the parameters of the system to see the effect of various parameter changes.

1.0 Introduction

The demand of a product may decrease with time due to the introduction of a new product which is either technically superior or otherwise more attractive and cheaper than the old one. The demand of the latest product, in this case, will increase with time. Many food products, chemicals and electronic components fall into this category. Generally, deteriorating items refer to the items that become spoiled, damaged, decayed, devaluated, evaporated and so on, through time, Wee (1993). The proposed model in this paper is for deteriorating items which have a time-dependent generalised exponential increasing demand rate and a linearly time-varying holding cost. Items such as electronic equipment, computer chips, mobile phones, fashion apparels and so on, have exponential increasing demand rate when they have just been introduced to the market. They may deteriorate due to spoilage, damage, fading and so on.

A large number of researchers are involved in developing inventory models for items deteriorating with time. Among them include Whitin (1957) who derived a model of fashion goods deteriorating at the end of the storage period. Ghare and Schrader (1963) developed a deteriorating inventory model by assuming that the rate of deterioration of units is a constant. Also in the literature include Deb and Chaudhuri (1986) and Bahari-Kashni (1989) who derived Inventory models with time dependent deteriorate rate. A survey of literature on inventory models for deteriorating items was given by Raafat (1991). Lakdere and Mohammed (1996) presented an exact solution for the inventory replenishment problem with shortages, in which items are deteriorating at a constant rate. The demand rates are increasing with time over a known and finite planning horizon. Some recent works include Shah and Shah (2000) who gave a survey of literature on inventory models for deteriorating items. Mehta and Shah (2003) developed a lot-size inventory model for deteriorating items with exponentially increasing demand by allowing complete backlogging. Also Kumar et al (2012) developed a two- warehouse inventory model with partial backordering and weibull distribution deterioration. Sing and Pattnayak (2013) presented an Economic Order Quantity model for deteriorating item with time-dependent quadratic demand and variable deterioration, under pe rmissible delay in payment. Another related article is the one by Dash et al (2014) in which they developed an inventory model for deteriorating items having a time- dependent exponential declining demand rate and time-varying holding cost as a linear function of time. Rajan and Uthayakumar (2015) studied a two-warehouse inventory model with exponentially increase trend in demand involving different deterioration rates under permissible delay in payment. Aliyu and Sani (2016) in their paper,developed an inventory model for deteriorating items with generalised exponential decreasing demand and linear time-varying holding cost. The rate of deterioration is considered to be constant and shortages are not allowed.

In this paper, an economic order quantity (EOQ) model with a generalised exponential increasing demand and a constant deterioration rate is considered. The holding cost is linear and no shortages are allowed. The paper has a special relationship with Aliyu and Sani (2016) in which an inventory model for deteriorating items with generalised exponential decreasing demand and linear time-varying holding cost was considered. In that paper the deterioration was also constant. Thus, the difference between this paper and Aliyu and Sani (2016) is that in this paper, the demand is generalised exponential increasing while in Aliyu and Sani (2016) the demand was generalised exponential decreasing.

2.0 ASSUMPTIONS AND NOTATION

In formulating the mathematical model, the following notation and assumptions are employed.

2.1 Assumptions

i. The inventory system considers a single item only.
ii. The demand rate is deterministic and is a generalised exponential increasing function of time.
iii. The deterioration rate is considered to be constant.
iv. Lead time is zero.
v. There are no shortages.
vi. The inventory system is considered over an infinite time horizon.

Notation

Z_0: The fixed ordering cost per order
$I(t)$: The inventory at any time $t, 0 \leq t \leq T$
$D(t)$: The exponential demand rate, where $D(t) = ke^{h+\lambda t}, k > 0, \lambda > 0, h > 0$, are all constants.
r: The constant deterioration rate of an item where $(0 < r < 1)$.
$h_0(t)$: Linear time-varying holding cost per unit time where $h_0(t) = s_1 + s_2 t$, $s_1 \geq 0$, $s_2 > 0$. s_1 is a fixed holding cost, as such it is ≥ 0, i.e. $s_1 \geq 0$ and s_2 is the rate at which the holding cost is changing with time, we assume $s_2 > 0$.
A_c: The cost of each deteriorated unit.
T: The length of the ordering cycle.
I_0: Initial stock.
TC: The total cost per unit time.
T^*: The optimal length of the cycle.
I_o^*: The economic order quantity

TC^*: The minimum total cost per unit time.

The figure below shows the demand levels with various values of h.

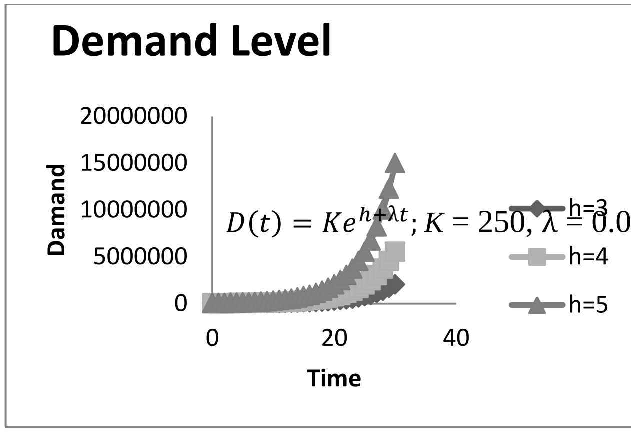

Fig. 1: Graphical Representation of various demand levels having different h values

3.0 MATHEMATICAL MODEL AND ANALYSIS

Using the assumptions above, a typical cycle for the variation of inventory level with time is shown in Figure 2 below.

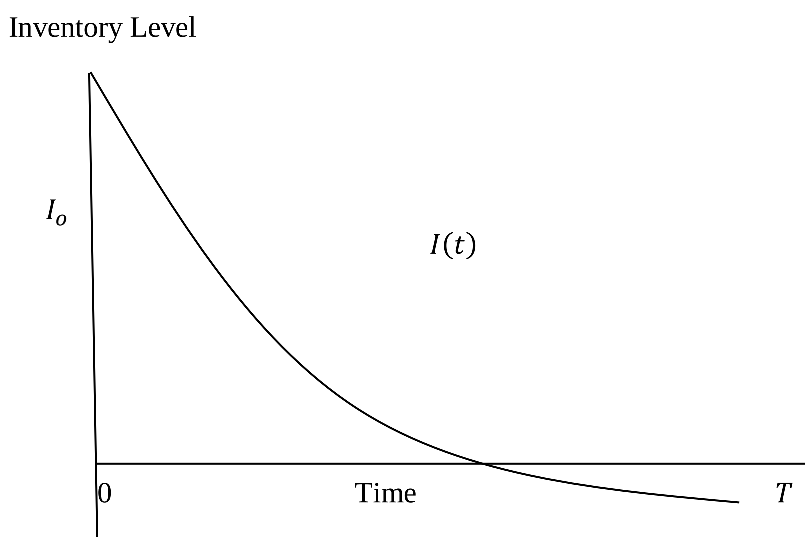

Figure 2: Graphical representation of the inventory system

From the figure above, one can see the inventory level as it gradually decreases from initial stage due to both demand and deterioration. The differential equation which describes the state of inventory levels at any time, t, represented as I(t) in the interval $[0, T]$, is given by;

$$\frac{dI(t)}{dt} + rI(t) = -D(t), \ 0 \leq t \leq T \tag{1}$$

where $D(t) = Ke^{h+\lambda t}$

To obtain the solution of equation (1) with boundary conditions $I(0) = I_0$ and $I(T) = 0$ we proceed as follows:

$\frac{dI(t)}{dt} + rI(t) = -Ke^{h+\lambda t}$

The integrating factor, IF $= e^{\int r dt} = e^{rt}$

so that

$$I(t)e^{rt} = -K\int_0^T e^{h+\lambda t}.e^{rt}dt$$
$$= \frac{-K}{\lambda+r}e^{h+\lambda t+rt} + C$$

$\therefore I(t) = \frac{-K}{\lambda+r}e^{h+\lambda t} + Ce^{-rt}$ (2)

Substituting the boundary condition $I(t) = 0$ when $t = T$ in equation (2), we get;

$I(T) = 0 = \frac{-K}{\lambda+r}e^{h+\lambda T} + Ce^{-rT}$

$$\Longrightarrow \frac{K}{\lambda+r}e^{h+\lambda T} = Ce^{-rT} \text{ or } C = \frac{K}{\lambda+r}e^{h+\lambda T}.e^{rT} = \frac{K}{\lambda+r}e^{(\lambda+r)T+h}$$

We substitute C in (2) to obtain

$$I(t) = \frac{-K}{\lambda+r}e^{h+\lambda t} + \frac{K}{\lambda+r}e^{(\lambda+r)T}.e^{h}.e^{-rt}$$
$$= \frac{Ke^h}{\lambda+r}\left[e^{(\lambda+r)T-rt} - e^{\lambda t}\right], 0 \le t \le T \quad (3)$$

Putting the boundary condition $I(0) = I_0$ in equation (3), we obtain the initial order quantity as follows:

$$I(0) = I_0 = \frac{Ke^h}{\lambda+r}\left[e^{(\lambda+r)T-r(0)} - e^{\lambda(0)}\right]$$
$$= \frac{Ke^h}{\lambda+r}\left[e^{(\lambda+r)T} - 1\right] \quad (4)$$

The total demand during the cycle period $[0, T]$ is

$$\int_0^T D(t)dt = \int_0^T Ke^{h+\lambda t}dt = \frac{K}{\lambda}\left[e^{h+\lambda t}\right]_0^T$$
$$= \frac{K}{\lambda}\left[e^{h+\lambda T} - e^h\right]$$
$$= \frac{Ke^h}{\lambda}\left[e^{\lambda T} - 1\right] \quad (5)$$

The number of deteriorated units is equal to the initial order quantity minus the total demand during the cycle period$[0, T]$. This is therefore given as

$$I_0 - \int_0^T D(t)dt$$

$$= \frac{Ke^h}{\lambda+r}\left[e^{(\lambda+r)T} - 1\right] - \frac{Ke^h}{\lambda}\left[e^{\lambda T} - 1\right]$$
$$= Ke^h\left[\frac{1}{\lambda+r}\left\{e^{(\lambda+r)T} - 1\right\} - \frac{1}{\lambda}\left\{e^{\lambda T} - 1\right\}\right]$$
$$= Ke^h\frac{\left[\lambda\left(e^{(\lambda+r)T}-1\right)-(\lambda+r)\left(e^{\lambda T}-1\right)\right]}{\lambda(\lambda+r)}$$

$$= \frac{Ke^h}{(\lambda+r)\lambda}\left[\lambda e^{(\lambda+r)T} - \lambda e^{\lambda T} - re^{\lambda T} + r\right] \qquad (6)$$

The deterioration cost (DC) for the cycle $[0, T]$ is $A_C \times$ (the number of deteriorated units)

$$= \frac{A_C Ke^h}{(\lambda+r)\lambda}\left[\lambda e^{(\lambda+r)T} - \lambda e^{\lambda T} - re^{\lambda T} + r\right] \qquad (7)$$

The total inventory holding cost (IHC) for the cycle $[0, T]$ is

$$= \int_0^T (s_1 + s_2 t) I(t) dt$$

$$= \int_0^T (s_1 + s_2 t)\left[\frac{Ke^h}{(\lambda+r)}\left\{e^{(\lambda+r)T-rt} - e^{\lambda t}\right\}\mathrm{dt}\right]$$

$$= \frac{Ke^h}{\lambda+r}\left[\int_0^T (s_1 + s_2 t)\left\{e^{(\lambda+r)T-rt} - e^{\lambda t}\right\}dt\right] \qquad (8)$$

To solve equation (8), we can split the square bracket into two parts; we represent the first part by A and the second part by B, i.e.

$\int_0^T (s_1 + s_2 t) e^{(\lambda+r)T-rt} dt$ (A) and $\int_0^T (s_1 + s_2 t) e^{\lambda t} dt$ (B)

After solving (A) and (B), the final solution of equation (8) will be

$\int_0^T (s_1 + s_2 t) I(t) dt = \frac{ke^h}{\lambda+r}$[Solution of A– Solution of B].

Solution of A

$$A = \int_0^T (s_1 + s_2 t) e^{(\lambda+r)T-rt} dt$$

Solving it using integration by parts gives,

$$\int_0^T (s_1 + s_2 t) e^{(\lambda+r)T-rt} dt = (s_1 + s_2 t)\left(-\frac{1}{r}e^{(\lambda+r)T-rt}\right)\Big|_0^T - \int_0^T -\frac{1}{r}e^{(\lambda+r)T-rt} s_2 dt$$

$$= -\frac{(s_1 + s_2 T)}{r}e^{(\lambda+r)T-rT} - \left(-\frac{s_1}{r}e^{(\lambda+r)T}\right) - \frac{s_2}{r^2}\left[e^{(\lambda+r)T-rT} - e^{(\lambda+r)T}\right]$$

$$= -\frac{(s_1+s_2T)}{r}e^{\lambda T} + \frac{s_1}{r}e^{(\lambda+r)T} - \frac{s_2}{r^2}e^{\lambda T} + \frac{s_2}{r^2}e^{(\lambda+r)T} \qquad (x)$$

Solution of B

$$B = \int_0^T (s_1 + s_2 t) e^{\lambda t} dt$$

$$= (s_1 + s_2 t)\left(\frac{1}{\lambda}e^{\lambda t}\right)\Big|_0^T - \frac{s_2}{\lambda}\int_0^T e^{\lambda t} dt$$

$$= \frac{(s_1+s_2T)}{\lambda}e^{\lambda T} - \frac{s_1}{\lambda} - \frac{s_2}{\lambda^2}e^{\lambda T} + \frac{s_2}{\lambda^2} \qquad (xx)$$

$\therefore \int_0^T (s_1 + s_2 t) I(t) dt = \frac{ke^h}{\lambda+r}$ [Solution of A – Solution of B].

$$= \frac{ke^h}{\lambda+r}\left[-\frac{(s_1+s_2T)}{r}e^{\lambda T} + \frac{s_1}{r}e^{(\lambda+r)T} - \frac{s_2}{r^2}e^{\lambda T} + \frac{s_2}{r^2}e^{(\lambda+r)T} - \frac{(s_1+s_2T)}{\lambda}e^{\lambda T} + \frac{s_1}{\lambda} + \frac{s_2}{\lambda^2}e^{\lambda T} - \frac{s_2}{\lambda^2}\ \}\right]$$

$$= \frac{ke^h}{\lambda+r}\left[-\frac{(s_1+s_2T)}{r}e^{\lambda T} + \frac{s_1}{r}e^{(\lambda+r)T} - \frac{s_2}{r^2}e^{\lambda T} + \frac{s_2}{r^2}e^{(\lambda+r)T} - \frac{((s_1+s_2T))}{\lambda}e^{\lambda T} + \frac{s_1}{\lambda}\right.$$

$$\left. + \frac{s_2}{\lambda^2}e^{\lambda T} - \frac{s_2}{\lambda^2}\right]$$

(9)

$$= \frac{ke^h}{\lambda+r}\Big[-\frac{(s_1+s_2T)}{r\lambda}(\lambda e^{\lambda T}+re^{\lambda T})+\frac{s_1}{\lambda r}\big(\lambda e^{(\lambda+r)T}+r\big)+\frac{s_2}{\lambda^2 r^2}\{\lambda^2 e^{(\lambda+r)T}-\lambda^2 e^{\lambda T}+r^2e^{\lambda T}-r^2\}\Big]$$

$$= \frac{ke^h}{(\lambda+r)r\lambda}\Big[-(s_1+s_2T)\big(\lambda e^{\lambda T}+re^{\lambda T}\big)+s_1\big(\lambda e^{(\lambda+r)T}+r\big)+\frac{s_2}{r\lambda}(\lambda^2e^{(\lambda+r)T}-\lambda^2e^{\lambda T}+r^2e^{\lambda T}-r^2)\Big] \quad (10)$$

The total Variable Cost = Ordering Cost (OC) + Deterioration Cost (DC) + Inventory Holding Cost (IHC).

The total Variable Cost per unit time $TC(T)$ is therefore

$$TC(T)=\frac{\text{Total variable cost}}{\text{T}}=\frac{Z_0}{T}+\frac{A_CKe^h}{(\lambda+r)\lambda T}\big[\lambda e^{(\lambda+r)T}-\lambda e^{\lambda T}-re^{rT}+r\big]$$

$$+\frac{ke^h}{(\lambda+r)r\lambda T}\Big[-(s_1+s_2T)\big(\lambda e^{\lambda T}+re^{\lambda T}\big)+s_1\big(\lambda e^{(\lambda+r)T}+r\big)+\frac{s_2}{r\lambda}(\lambda^2e^{(\lambda+r)T}-\lambda^2e^{\lambda T}+r^2e^{\lambda T}-r^2)\Big] \quad (11)$$

$$=\frac{Z_0}{T}+\frac{A_Cke^h}{(\lambda+r)\lambda}\left[\frac{\lambda e^{(\lambda+r)T}}{T}-\frac{\lambda e^{\lambda T}}{T}-\frac{re^{\lambda T}}{T}+\frac{r}{T}\right]+\frac{ke^h}{(\lambda+r)r\lambda}\left[\frac{-(s_1+s_2T)\big(\lambda e^{\lambda T}+re^{\lambda T}\big)}{T}+\frac{s_1\big(\lambda e^{(\lambda+r)T}+r\big)}{T}+\frac{s_2}{r\lambda}\left(\frac{\lambda^2e^{(\lambda+r)T}}{T}-\frac{\lambda^2e^{\lambda T}}{T}+\frac{r^2e^{\lambda T}}{T}-\frac{r^2}{T}\right)\right] \quad (12)$$

Our aim is to find the minimum variable cost per unit time. The necessary and sufficient conditions to minimize TC(T) are

$\frac{dTC(T)}{dT}=0$ and $\frac{d^2TC(T)}{dT^2}>0.$

Now we differentiate equation (12) with respect to T, as follows:

$$\frac{dTC(T)}{dT}=-\frac{Z_0}{T^2}+\frac{A_Cke^h}{\lambda(r+\lambda)}\left[\frac{\lambda(r+\lambda)e^{(r+\lambda)T}}{T}-\frac{\lambda e^{(r+\lambda)T}}{T^2}-\frac{\lambda^2e^{\lambda T}}{T}+\frac{\lambda e^{\lambda T}}{T^2}-\frac{r\lambda e^{\lambda T}}{T}+\frac{re^{\lambda T}}{T^2}-\frac{r}{T^2}\right]$$
$$+\frac{ke^h}{(r+\lambda)r\lambda}\left[\frac{-s_2\big(re^{\lambda T}+\lambda e^{\lambda T}\big)}{T}-\frac{(s_1+s_2T)\big(r\lambda e^{\lambda T}+\lambda^2e^{\lambda T}\big)}{T}+\frac{(s_1+s_2T)\big(\lambda e^{\lambda T}+re^{\lambda T}\big)}{T^2}+\frac{s_1\lambda(r+\lambda)e^{(r+\lambda)T}}{T}-\frac{s_1\big(\lambda e^{(r+\lambda)T}+r\big)}{T^2}+\frac{s_2}{\lambda r}\left[\frac{\lambda^2(r+\lambda)e^{(r+\lambda)T}}{T}-\frac{\lambda^2e^{(r+\lambda)T}}{T^2}-\frac{\lambda^3e^{\lambda T}}{T}+\frac{\lambda^2e^{\lambda T}}{T^2}+\frac{r^2\lambda e^{\lambda T}}{T}-\frac{r^2e^{\lambda T}}{T^2}+\frac{r^2}{T^2}\right]\right] \quad (13)$$

We now equate equation (13) to zero and simplify by multiplying with $[-T^2\lambda^2r^2(\lambda + r)]$ on both sides so as to determine the T that minimizes the variable cost per unit time as follows:

$$Z_0\,\lambda^2r^2(r+\lambda) - A_Cke^h\lambda^2r^2\,T\,(r+\lambda)e^{(r+\lambda)T} + A_Cke^h\lambda^2r^2e^{(r+\lambda)T} + A_Cke^hT\lambda^3r^2e^{\lambda T} - A_Cke^h\lambda^2r^2\,e^{\lambda T} + A_Cke^hT\lambda^2r^3e^{\lambda T} - A_Cke^h\lambda r^3e^{\lambda T} + A_Cke^h\lambda r^3 + ke^hs_2T\lambda r\left(\lambda e^{\lambda T} + re^{\lambda T}\right) + ke^hT\lambda r\,(s_1+s_2T)\left(r\lambda e^{\lambda T} + \lambda^2e^{\lambda T}\right) - ke^h\lambda r\,(s_1+s_2T)\left(\lambda e^{\lambda T} + re^{\lambda T}\right) - ke^h\,Ts_1\lambda^2r\,(r+\lambda)e^{(r+\lambda)T} + ke^h\lambda rs_1\left(\lambda e^{(\lambda+r)T} + r\right) - ke^hs_2T\lambda^2(r+\lambda)e^{(r+\lambda)T} + ke^hs_2\lambda^2e^{(r+\lambda)T} + ke^hs_2T\lambda^3e^{\lambda T} - ke^hs_2\lambda^2\,e^{\lambda T} - ke^hs_2T\lambda r^2e^{\lambda T} + ke^hs_2r^2e^{\lambda T} - ke^hs_2r^2 = 0 \quad (14)$$

The value of T obtained, gives the minimum cost provided it satisfies the following condition

$$\frac{d^2TC(T)}{dT^2} > 0.$$

Putting the values of Z_0, K, λ, r, s_1, s_2 A_c, and h into equation (14), gives the T value which provides the minimum cost, with the proviso above. The example below satisfies the condition above and so it gives the T value providing the minimum cost.

4 .0 NUMERICAL EXAMPLE

Example 1

Let Z_0 = 500 , K = 250, λ = 0.02, r = 0.8, s_1 = 0.5, s_2 = 0.2, A_c = 1, and h = 0.9,
Substituting the above parameters into equation (14) and solving, we obtain T^* = 0.846575 (309 days). On substitution of the optimal value T^*in equations (12) and (4), we obtain the minimum total cost per unit time as TC^* = 1043.077 and economic order quantity I_0^* = 751.4465. Note that the T^* value satisfies $\frac{d^2TC(T)}{dT^2} > 0$, as already mentioned.

5.0 SENSITIVITY ANALYSIS

We have performed sensitivity analysis on example1 through changing each of the parameters Z_0, K, λ, r, s_1, s_2 , A_c and h by 50%, 25%, 5%, 2%, -2% -5% -25% -50%, and keeping the remaining parameters at their original values. The corresponding changes in the cycle time, total cost per unit and the economic order quantity are exhibited in Table2.

Example 2

Using the same values as in example 1, with h changed to 1.5 the solutions are T^*= 0.665753 (243 days), TC^* = 1359.292 and I_0^* = 992.2459.

Example 3

Using the same values as in example 1, with h changed to 2.5, the solutions, are T^* = 0.435616 (159 days), TC^* = 2146.673, and I_0^* = 1594.605.
A summary of the results for the above examples are shown in table 1 below:

Table 1: Summary of the Results of Examples 1, 2 &3

h	T^*	TC^*	I_0^*
0.9	0.846575 (309 days)	1043.077	751.4465
1.5	0.665753 (243 days)	1359.292	992.2459
2.5	0.435616 (159 days)	2146.673	1594.605

As we can observe from Table1, as the value of h increases TC^* and I_0^* increase while T^* decreases as it is expected. This is due to the fact that whenever the demand increases the economic order quantity also increases, hence the total variable cost, TC^* also increases. This will make the cycle period, T^*, to decrease as a result of higher demand.

Table 2: Sensitivity Analysis on example1 to see changes in the values of T^*, TC^* and I_0^* as other parameters change.

Parameter	% change in parameter	T^*		TC^*	I_0^*
Z_0	50	0.991781	(362days)	1314.557	941.2847
	25	0.923288	(337 days)	1184.085	848.9196
	5	0.863014	(315 days)	1072.288	771.8205
	2	0.854795	(312 days)	1054.828	761.5992
	0	**0.846575**	**(309** days)	**1043.077**	**751.4465**
	-2	0.841096	(307 days)	1031.232	744.7159
	-5	0.830137	(303 days)	1013.287	731.3452
	-25	0.756164	(276 days)	887.3251	644.1685
	-50	0.641096	(234 days)	708.6542	518.6465
K	50	0.720548	(263 days)	1246.387	906.0652
	25	0.775342	(283 days)	1149.946	832.8307
	5	0.832877	(304 days)	1065.457	771.4105
	2	0.841096	(307 days)	1052.095	759.6102
	0	**0.846575**	**(309** days)	**1043.077**	**751.4465**
	-2	0.854795	(312 days)	1033.965	746.3672
	-5	0.865753	(316 days)	1020.119	736.4808
	-25	0.947945	(346 days)	921.448	661.1813
	-50	1.10411	(403 days)	776.535	552.2283

Table 2(continued): Sensitivity Analysis on example1 to see changes in the values of $\mathbf{T^*}$, TC^* and $\mathbf{I_0^*}$ as other parameters change.

Parameter	% change in parameter	T^*		TC^*	I_0^*
λ	50	0.843836	(308 days)	1045.723	751.6053
	25	0.846575	(309 days)	1044.401	753.222
	5	0.846575	(309 days)	1043.342	751.8012
	2	0.846575	(309 days)	1043.183	751.5883
	0	**0.846575**	**(309** days)	**1043.077**	**751.4465**
	-2	0.846575	(309 days)	1042.972	751.3046
	-5	0.849315	(310 days)	1042.813	754.4658
	-25	0.849315	(310 days)	1041.75	753.0385
	-50	0.852055	(311 days)	1040.422	754.6148
r	.50	0.709589	(259 days)	1225.676	693.8631
	25	0.769863	(281 days)	1136.299	719.1793
	5	0.830137	(303 days)	1062.059	745.0136
	2	0.841096	(307 days)	1050.691	750.3249
	0	**0.846575**	**(309** days)	**1043.077**	**751.4465**
	-2	0.854795	(312 days)	1035.433	755.8154
	-5	0.865753	(316 days)	1023.913	760.4258
	-25	0.945205	(345 days)	945.3265	790.2825
	-50	1.073973	(392 days)	842.1979	834.4918
s_1	50	0.794521	(290 days)	1123.371	688.7109
	25	0.819178	(299 days)	1084.047	718.0941
	5	0.841096	(307 days)	1051.412	744.7159
	2	0.846575	(309 days)	1046.421	751.4465
	0	**0.846575**	**(309** days)	**1043.077**	**751.4465**
	-2	0.849315	(310 days)	1039.722	754.8231
	-5	0.854795	(312 days)	1034.667	761.5992
	-25	0.879452	(321 days)	1000.238	792.4711
	-50	0.915068	(334 days)	955.2556	838.1804

Table 2(continued): Sensitivity Analysis on example1 to see changes in the values of $\mathbf{T^*}$, TC^* and $\mathbf{I_0^*}$ as other parameters change.

Parameter	% change in parameter	T^*		TC^*	I_0^*
s_2	50	0.838356	(306 days)	1051.87	741.362
	25	0.841096	(307 days)	1047.504	744.7159
	5	0.846575	(309 days)	1043.967	751.4465
	2	0.846575	(309 days)	1043.433	751.4465
	0	**0.846575**	**(309** days)	**1043.077**	**751.4465**
	-2	0.849315	(310 days)	1042.721	754.8231
	-5	0.849315	(310 days)	1042.184	754.8231
	-25	0.852055	(311 days)	1038.587	758.2073
	-50	0.860274	(314 days)	1034.029	768.4057

A_c	50	0.767123	(280 days)	1168.64	656.7523
	25	0.80274	(293 days)	1107.828	698.4394
	5	0.838356	(306 days)	1056.378	741.362
	2	0.843836	(308 days)	1048.42	748.0774
	0	**0.846575**	(**309** days)	**1043.077**	**751.4465**
	-2	0.852055	(311 days)	1037.703	758.2073
	-5	0.857534	(313 days)	1029.583	764.9986
	-25	0.90137	(329 days)	973.5271	820.4417
	-50	0.964384	(352 days)	897.9496	903.715
h	50	0.709589	(259 days)	1271.291	928.3329
	25	0.775342	(283 days)	1150.885	834.3783
	5	0.832877	(304 days)	1063.697	768.4922
	2	0.841096	(307 days)	1051.27	758.2422
	0	**0.846575**	(**309** days)	**1043.077**	**751.4465**
	-2	0.854795	(312 days)	1034.955	748.013
	-5	0.863014	(315 days)	1022.905	737.8585
	-25	0.926027	(338 days)	946.5135	680.7474
	-50	1.008219	(368 days)	859.9893	614.8234

DISCUSSION OF RESULTS

After carrying out the sensitivity analysis in Table 2, the following observations can be made from the Table.

1) With increase in the value of the parameter Z_0, the values of T^*, TC^* and I_0^* all increase. This is expected since when ordering cost increases then the model will try to reduce more orders which will result in increase in both T^* and I_0^*. Also TC^* increases because of increase in the stockholding cost. The increase in the values is also moderate so the decision variables are moderately sensitive to changes in Z_0.
2) With increase in the value of parameter K, T^* decreases while $TC^* and\ I_0^*$ increase. This is also expected because when K increases, the demand will increase which will make the optimal total cost and the economic order quantity increase and hence the cycle period will decrease because of the higher demand. The increase /decrease in the values of the decision variables are moderate so they are moderately sensitive to changes in K.
3) With increase in the value of parameter λ,T^* decreases while TC^* and I_0^* increase. When λ increases, it causes the demand to be high which will result in making T^* to decrease. As a result, I_0^* increases and TC^* will increase due to stockholding cost. The increase/decrease in the values is moderate so the decision variables are moderately sensitive to changes in λ.
4) With increase in the value of parameter r, T^* and I_0^* decrease while TC^* increases. This is probably because when r increases, the deterioration increases which makes the model to reduce T^* so as to reduce deterioration. Hence both T^* and I_0^* decrease while TC^* increases probably due to cost of deterioration. The decrease/increase in the values is low so the decision variables are not very sensitive to changes in r.
5) With increase in the value of parameter s_1, T^* and I_0^* decrease while TC^* increases. This is also expected since whenever s_1 increases the stockholding cost increases, so the model reduces T^* and I_0^*. The TC^* increases probably due to ordering cost. The

increase/decrease in the values is moderate so the decision variables are moderately sensitive to changes in s_1.

6) With increase in the value of parameter s_2, T^* and I_0^* decrease while TC^* increases. This is expected as in the case above, since when s_2 increases the stockholding cost increases and so the model reduces T^* and I_0^*. The TC^* increases probably due to ordering cost. The increase/decrease in the values is low so the decision variables are not very sensitive to changes in s_2.
7) With increase in the value of parameter A_c, T^* and I_0^* decrease while TC^* increases. This is also expected since when A_c increases the stockholding cost increases so the model reduces T^* and I_0^*. The TC^* increases probably due to deterioration and ordering costs. The increase/decrease in the values is moderate so the decision variables are moderately sensitive to changes in A_c.
8) With increase in the value of parameter h, TC^* and I_0^* increase while T^* decreases as it is expected. This is due to the fact that whenever the demand increases the economic order quantity also increases, hence the total variable cost, TC^* also increases. The cycle period decreases as a result of higher demand. The increase/decrease in the decision variables is moderate so they are moderately sensitive to changes in h.

6.0 CONCLUSION

The model developed in this paper assumes a generalised exponential increasing demand which is different from that in Aliyu and Sani (2016) where in their paper they considered a generalised exponential decreasing demand. The model determines the optimal order quantity to be ordered as well as the corresponding cycle period and optimal cost per unit time. A numerical example has been solved to show the application of the model. Later, a sensitivity analysis is carried out to see the effect of changes in the parameter values. The analysis shows that T^*, TC^* and I_0^* are more sensitive to changes in the parameters, Z_0, K, s_1, A_c, λ, s_2 and h. However, T^*, TC^* and I_0^* are not very sensitive to changes in the parameter *r*.

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