SIMULATION ANALYSIS OF A MIGRATION RATE IN THE DYNAMICS OF FISHERY WITH PREY RESERVE AND HARVESTING

by

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Abstract

The aim of this study is to evaluate the variation of the migration rate of the prey species in the free fishing zone of the prey biomass in the free fishing zone using a computational approach that is based on the ODE45 Runge-Kutta Numerical scheme. The results of the study are fully presented and discussed in this paper.

Introduction

Following the model formulation embodied in the work of Khamis, et al., (2011). We continue in this paper with a simulation analysis of a migration rate in the dynamics of fisheries with prey reserve and harvesting to evaluate the variation of the migration rate of the prey species in the free fishing zone of the prey biomass in the free fishing zone.

Food demands are rapidly increasing more than ever and this high demand coupled with aggressive fishing techniques is having a disastrous impact on fish populations. Many marine fisheries show a concerning decline in their abundance to the extent of difficult, or even impossible recovery,Alsharawi and Rhouma (2009). However, the biological diversity of marine habitats is threatened by humans' activities. The increasing study of realistic mathematical models in theoretical ecology is a very good reflection of their use in helping to understand the dynamic processes involved and in making practical prediction, Haque (2009).

Migratory species have received considerable attention among fish biologist and resource managers due to their long-distance movements which often involve spectacular numbers of individuals, Lucas and Baras (2001) as well their importance in many of the largest riverine fisheries around the world, Allan et al.,(2005). Although the significance of migratory species to commercial and recreational fisheries has long been realized, over the past two decades there is increasing recognition that migratory species can be major ecological drivers shaping the structure and function of freshwater ecosystem via a host of direct and indirect mechanisms as consumers, ecosystem engineers, modulators of biogeochemical processes, and transport vector, Greathouse et al., (2006). Appreciation of the fundamental ecological roles of migratory species has, in part been an outcome of a growing literature linking the role of species and ecosystem function and the notion that the loss of key species can have widespread consequences in ecosystems, Hooper et al., (2005).

MODEL FORMULATION

From Khamis,etal., (2011), a deterministic system of three-dimensional nonlinear ordinary differential equations is used to model the population dynamics of pre-predator fishery habitat in an aquatic ecosystem, where there is a two-patch environment regarded as homogenous. The proposed model is based on the logistic form of the Lotka-Volterra predator-prey equations in two patches, with some modifications to incorporate immigration of the prey species from one zone to the other and harvesting. The model considers two populations, the fishes (preys) and the predictor (marine mammals), while fishes are subdivided into two patches, the predator is common in both patches. We consider the prey in patch 1 denoted by $N_1(t)$ to be free for fishing and preys in patch 2 denoted by $N_2(t)$ as prey refuge which constitutes a reserve area and no fishing is permitted in that area. The predator population (density P(t)) has no barrier between the two patches.

Harvesting and predation are both processes in which members of a population are removed by an external agency, sometimes for population management, but more often for the benefit of the harvester, whether in the wild or in a managed environment, Kribs-Zaleta (2009).

Table 1: Description of variables and parameters of the model

Variable	Description
1. $N_1(t)$	Mass density of prey biomass in the free fishing zone at any time t.
2. $N_2(t)$	Mass density of prey biomass in the reserve zone at any time t.
3. P(t)	Total mass density of predator biomass at any time t in both areas.
4. K_1	Carrying capacity of the prey in the free zone.
5. K_2	Carrying capacity of the prey in the reserve zone.
6. E	Effort applied to harvest the prey species in the free fishing zone.
7. r_1	Intrinsic growth rate of prey species in the free fishing zone.
8. r_2	Intrinsic growth rate of prey special in the reserve zone.
9. β_1	Migration rate of prey species in the free zone.
10. β_2	Migration rate of prey species in the reserve zone.
11. m_1	Prey mortality rate due to predation in the free zone.
12. m_2	Prey mortality rate due to predation in the reserve zone.
13. $\propto_1$	Rate of predation on preys in the free fishing zone.
14. $\propto_2$	Rate of predation on preys in the reserve zone.
15. q	Catchability coefficient.
16. σ	Intra-specific competition coefficient.
17. d	Death rate of the predator.

Prey migrate between zone 1 and 2 randomly at different rates β_1 and β_2, respectively. Predators are assumed to be everywhere throughout the whole patches with different rates. In the absence of preys, the predators die at the rate d due to lack of food. Since the model depicts animal population dynamics, all variables and parameters defined in Table 1 are assumed positive.

Table 2: Initial values for state variables and default parameters for the model.

Variable / Parameter	Value
$N_1(0)$	50
$N_2(0)$	50
P(0)	45
K_1	110
K_2	100
E	2
r_1	3
r_2	1.5
β_1	0.5
β_2	0.4
m_1	0.3

m_2	0.2
$\propto_1$	0.03
$\propto_2$	0.02
q	0.01
σ	0.05
d	0.6

THE MODEL

Following Khamis et al., (2011), we have considered the equations of the model dynamics given by the following nonlinear system of ordinary differential equations.

$$\frac{dN_1}{dt} = r_1N_1\left(1 - \frac{N_1}{K_1}\right) + \beta_2N_2 - \beta_1N_1 - m_1N_1P - qEN_1 \quad \text{……….} (1)$$

$$\frac{dN_2}{dt} = r_2N_2\left(1 - \frac{N_2}{K_2}\right) - \beta_2N_2 + \beta_1N_1 - m_2N_2P \quad \text{………}(2)$$

$$\frac{dP}{dt} = P(-d - \sigma P + \propto_1 N_1 + \propto_2 N_2) \quad \text{………}(3)$$

withinitial conditions, $N_1(0) = N_{10} > 0$, $N_2(0) = N_{20} > 0$, $P(0) = P_o > 0$

METHOD OF ANALYSIS

For the purpose of this study, we have used the ODE45 to simulate the proposed model equations and utilize this method to calculate the extent of biodiversity gain and biodiversity loss that is time dependent due to a variation of the migration rate of the prey species of the free fishing zone of the prey biomass of the prey species of the free fishing zone.

RESULTS AND DISCUSSION

We present the following results that we have obtained using ODE45 numerical scheme.

Table 3: Accessing the impact of biodiversity gain due to a decrease in the migration rate β_1 = 0.05 using ODE45 numerical scheme with initial condition (50 50 45).

Month	**N_1**	**$N_{1m}(\beta_1)$**	**BG(%)**
0	50.0000	50.0000	0
1	0.9789	1.0436	6.6119
2	2.2234	2.7939	25.6587
3	10.8324	16.1731	49.3027
4	38.0014	50.6558	33.2997
5	35.9462	40.3773	12.3271
6	17.3200	24.4792	41.3346
7	15.9800	24.5241	53.4673
8	20.9127	29.0467	38.8947
9	23.7667	30.2511	27.2837
10	21.8648	28.8250	31.8330
11	20.2886	28.2246	39.1159
12	20.6347	28.5761	38.4855
13	21.3910	28.8523	34.8806
14	21.4470	28.7943	34.2582
15	21.1297	28.6912	35.7861
16	21.0316	28.7127	36.5219
17	21.1465	28.7403	35.9102
18	21.2044	28.7329	35.5049

19	21.1732	28.7252	35.6678
20	21.1388	28.7328	35.9244
21	21.1520	28.7364	35.8565
22	21.1696	28.7247	35.6886
23	21.1612	28.7297	35.7658

Table 3 is quantifying the impact of decreasing the migration rate of the prey species in the free fishing zone of the prey biomass of the free fishing zone which led to biodiversity gain.

Table 4: Accessing the impact of biodiversity loss due to an increase in the migration rate $\square_1$ = 0.9 using ODE45 numerical scheme with initial condition (50 5045).

Month	$\mathbf{N_1}$	$\mathbf{N_{2m}(\square_1)}$	**BL(%)**
0	50.0000	50.0000	0
1	0.9789	0.6509	33.5040
2	2.2234	1.2607	43.2988
3	10.8324	5.4758	49.4497
4	38.0014	22.5632	40.6254
5	35.9462	30.9075	14.0174
6	17.3200	11.5446	33.3454
7	15.9800	8.7029	45.5390
8	20.9127	12.3352	41.0161
9	23.7667	16.6762	29.8336
10	21.8648	15.6148	28.5847
11	20.2886	12.9707	36.0691
12	20.6347	12.7774	38.0783
13	21.3910	13.9241	34.9067
14	21.4470	14.4622	32.5675
15	21.1297	14.0040	33.7236
16	21.0316	13.5968	35.3503
17	21.1465	13.7033	35.1986
18	21.2044	13.9360	34.2775
19	21.1732	13.9576	34.0789
20	21.1388	13.8404	34.5259
21	21.1520	13.8021	34.7479
22	21.1696	13.8484	34.5834
23	21.1612	13.8800	34.4081

Table 4 is quantifying the impact of increasing the migration rate of the prey species in the free fishing zone of the prey biomass of the free fishing zone which led to biodiversity loss.
N_1 data represents the magnitude of the prey biomass in the free fishing zone when all the model parameters are fixed. Similarly, N_{1m} data specify the magnitude of the prey biomass in the free fishing zone when only the migration rate of prey species in the free zone is varied while all other parameter values are fixed.

From Table 3 above, the N_{1m} data is increasing monotonically. It is clear from this analysis that the variation of the migration rate of the prey species in the free zone is associated with a dominant biodiversity gain in which the highest value has been predicted to occur at the 7th month while the first month recorded the smallest quantify value.

From Table 4, the N_{1m} data is decreasing which implies that the analysis of the variation of the migration rate of the prey species in the free zone is predicting biodiversity loss in which the third month shows a high levelof vulnerability while the fifth month show the least level of susceptibility.

In all the reviewed and cited mathematical literatures on other aspect of modeling the dynamics of fisheries with prey reserve and harvesting, we haven't seen where the effect of the migration rate of prey species in the free zone on the prey biomass in the free fishing zone was studied. Therefore, our present analysis can be considered as pioneering analysis which has produced a set of novel results that we have not seen elsewhere. These results are empirical in the sense that biodiversity gain and biodiversity loss are differently quantified on the bases of the monthly unit of time.

In summary, decreasing the migration rate of prey species in free fishing zone tends to predict biodiversity gain whereas increasing the same parameter value tends to predict biodiversity loss.

CONCLUSION

On the basis of these analyses, we discovered that by decreasing the migration rate of the prey species in the free zone, we have found a dominant occurrence of biodiversity gain that can provide better insight for the ecological functioning of ecological services. Another result which we have observed due to an increase of the migration rate of the prey species in the free zone, the prey biomass dominantly tends to be vulnerable to the occurrence of biodiversity loss.

Therefore, the policy implication of the numerical simulation contribution is that the stakeholders in-charge of monitory and managing fisheries within the Niger Delta region should make extra effort to mitigate against the severe loss of biodiversity in combination with a strategic cutting-edge data analysis to sustain the observed biodiversity which is time dependent.

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