CHEYBYSHEV POLYNOMIAL TECHNIQUE FOR OPTIMIZING STOCK ALLOCATION

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Abstract
The dire need for optimum distribution of goods and services from manufacturers to satisfy demand by consumers through warehouses has made it necessary to develop a mathematical model that enhance stock allocation .Goods should be at the right place at the appropriate time and at a minimal cost .This work considered the different ways of stock allocation and identified some of their weaknesses. The feature of the Chebyshev polynomial were outline and the Chebyshev polynomial approximation was use to develop the new allocation technique for stock allocation The Chebyshev polynomial approximation technique is show to optimize returns, minimizes cost and have a wide area of applicability. The costs of observed and expected allocation were compared and the approximate (expected) prove more effective and time efficient. The Chebyshev polynomial allocation improves on some of the short comings of earlier methods. Some relevant theorems were stated and illustrated were included while a theorem asserting the minimal cost of the new technique is added key\words: polynomial approximation, stork allocation, Chebyshev polynomial approximation technique, optimal allocation, continuous function, mathematical model.

Keywords: *Polynomial approximation, Stock allocation, Chebyshev polynomial, Optimal allocation, Approximation technique, Mathematical model.*

1. The Background of the Study

Stock can be defined as good being held for feature use or sale. The ability to keep goods in a warehouse to make it available for sale or future use is called stock. The need to manage stocks requires control and the essence of stocking is to meet up with demand. Control is a process by which events are made to conform to a plan. Demand is dynamic and hence the pertinence to keep goods in stock. The act of maintaining stock has its associated costs likewise the act of not keeping stock. The later makes the manufacturer /distribute lose customers' goods will thereby incurring shortage cost while the former may increases holding costs. It is therefore often necessary to stock physical goods in order to satisfy demand over a specified time period in a way that the firm minimizes cost and maximizes profit. The act of stocking goods to satisfy future demand gives rise to the need to design an efficient allocation technique so as to minimize cost and maximizes profit numerous establishment have incurred great losses due to inefficient allocation of goods and resources. Decision regarding quantity allocation and the time at which it is allotted may be based on the minimization of appropriate cost function which balances the total cost resulting from over-stock and stock-out Chikwendu and Emenonye (2013). The major objective of the stock allocation model is to find and obtain a stock level that minimizes the sum of the shortage cost, the holding cost and other associated costs. To solve the stock allocation problem, a mathematical model would be developed as a basis for strategic organization decision.

This research work is aimed at looking into stock allocation system in a manufacturing company with widespread distribution outlets and developing a mathematical model using Chebyshev polynomial approximation. This is to ensure optimal allocation for the organization. In this work, Chebydshev polynomial approximation technique is used to obtain a new (approximate) allocation that optimizes stock allocation. The Chebyshev polynomial is a recursive technique for approximating polynomial function. The work used polynomial as the basic means of approximation. Flanney (2007) states that the degree of the Chebyshev polynomial function is chosen in such a way as to minimize the worst-case error.

2. Motivation

The quest for optimum of goods to the wholesaler, retailer and consumer has called for the development of a mathematical model that enhances steady and efficient allocation according to Sniedovich (2006) the growing global economy has caused dramatic shift in the stock management in the twenty-first century. Kutanoglu (2007) asserts that the cost associated with goods is very large, hence the need to reduce storage costs by avoiding unnecessary large stock. It is therefore clear that maintaining stock through product allocation is necessary for any company dealing with physical products. Kopecky (2007) affirm the use of polynomial in functional approximation through the weistress theorem which states that if C[a, b] is the set of all continuous functions on [a, b] then for all [a, b] there polynomial p that approximates any continuous function. The forgoing amount others forecloses the need for a good stock allocation model is to solve allocation problem. This work wishes to provide model that is stable, time efficient and unifies several condition of stock allocation model. The work would develop a stock allocation model with better approximation properties. It would also present a solution with a wide area of applicability, find the allocation that optimizes return and obtain or more cost effective method

3. Review of Related Literature

Good most is move from their area of manufacture to the point of consumers. These goods would be stored in ware house near consumers. These products therefore stock in warehouses to meet the timely demand of customers, hence the need for stock management. Kang and Gershwin (2005) Point out that there is a conflict between the need to give a good service and the need to economize in stock holding. The more the stock held the easier it is to have required items readily available on demand. On the other hand, the more the stocks held the greater the holding cost. Burbridge (1998) defined stock allocation as the operation of continuously arranging receipts and issues to ensure that stock balances are adequate to support the current rate of consumption with due regard to economy. Most of the work so far done focused on one or two warehouse stock allocation with the economic order quantity (EOQ). Some of the works are: optimal inventory model for items with imperfect quality, Cardenas-Barron (2000), Economic ordering quantity model for items with imperfect quality single period inventory model with two level of storage, two warehouse inventory model with imperfect quality Tien-Yu Lin (2011) , A two level of storage, two warehouse inventory for deteriorating items with stock dependent rate and holding cost Wilke (2010), Deterministic inventory model for determining items with capacity constraint and backlogging rate Erhan (2007), a generalized economic order quantity model with deteriorating items and time varying demand Balki and Tudj (2008), Dynamic programming in minimum risk paths of stock allocation Milovanic and Mastrion (2008). A hybrid technique for optimum stock allocation Chikwendu and Emenonye (2017) to mention but a few.

Researchers have indeed made reasonable contributions to this area but most of them have been on a single warehouse. The researcher's area cut across several disciplines such as management, economics, engineering and sciences. The point to the importance and wide applicability of this concept. Erhan (2007) posits that customer service has become an important dimension of competition along with price and quality. In order to retain a company's current customers and to acquire new customers, prompt service is always considered for which the first requirement is to have service parts readily available Chikwendu and Emenonye (2013). The company therefore faces a problem of determining optimal storing level of goods.

The Economic Order Quantity (EOQ) is another method vastly used in stock allocation. This is the order quantity that optimizes the trade-off between holding inventory and the cost associated with and minimizing total inventory cost. It is the order that minimizes the total holding costs and ordering costs. According to Mihaila and Mihaila (2002), EOQ applies when demand for a product is constant over the year and each new order is delivered in full when inventory reaches zero. It is characterized by a fixed cost for each order placed regardless of the number of units ordered. The underlying assumption of the EOQ model without which it cannot work to its optimal potential are; the cost of the ordering remains constant; the demand rate for the year is known and evenly spread throughout the year; the purchase price is constant for every item; the optimal plan is calculated for only one product and there is no delay in the replenishment of the stock, and the order is delivered in the quantity that was demanded, (in whole batch). These underlying assumptions are the key to the economic order quantity model, and these assumptions help the companies to understand the shortcomings they are incurring in the application of this model. Some of the demerits of the EOQ model are:

It does not take into consideration seasonal products and many products have seasonal sales pattern, it is designed for stable situations mainly which may not be the case always, it is limited to the assumption of one product business only, it assumes the steady demand of products and immediate availability of items and it assumes fixed cost of inventory units, ordering charges and holding charges.

3.1 Chebyshev Approximation

According to Raul (2015), the weierstrass theorem states that if f is a continuous function on an interval [a, b] and is given, there exists a polynomial $p(x)$ such that

$$sup_{x\in c[a,b]}|f(x) - p(x)| \leq \in \quad (1)$$

The weierstrass theorem therefore implies that "any continuous real-valued function f defined on a bounded interval [a, b] of the real line can be approximated to any degree of accuracy using a polynomial" i.e. for any $\in > 0$, there is a polynomial p such that

$$||f - p||\infty \equiv \sup_{a\leq x \leq b}|f(x) - p(x)| < \in . \quad (2)$$

Kopecky (2007) states that if C[a, b] is the set of all continuous functions on [a, b], then for all $f \in C[a, b]$ and $\in > 0$, the exists a polynomial p for which

$sup|f(x) - p(x)| \leq \in$ (3)
a$\leq x \leq b$

In order words, there exists a polynomial that approximates any continuous function over a compact domain arbitrarily well. Pressman and Sethi (2004) therefore concluded that it is possible to use polynomial approximation to obtain an optimal stock allocation if the allocation is shown as a continuous function. They stated the chebyshev approximation as follows: if Pn is the collection of all polynomial whose degree is at most n and f be a continuous function on the interval [a, b] the polynomial P is said to be the best approximation to f from P_n if $p \in P_n$
And
$\max|f(x) - p(x)| \leq \max|f(x) - q(x)| \ \forall q \in Pn$ (4)
$x \in [a,b] x \in [a,b]$

Kopecky (2010) defined the chebyshev polynomials by a three term recursion;
$T_0(x) = 1, T_1(x) = x, and\ Tn + 1(x) = 2xTn(x) - Tn - 1(x);\ n = 1, 2 \ldots$.
The first two polynomial $T_0(x)$ and $1, T_1(x)$ were obtained by means of Rodrigue's
$Tn(x) = (-\underline{2)^n n! \sqrt{(1-x^2)}}\ d^n\ (1-\underline{x}^2)^{n-1/2}\ n = 0, 1.$ (5)
$(2n)!\ dx^n$

According to Malakoto (2013), the chebyshev polynomial $Tn(x), n \geq 1$, has the following properties that make it plausible for polynomial approximations;
Recursive formula; $2xT_n(x) - T_{n-1}(x)\ n = 1, 2 \ldots$ (6),
The leading coefficient is 2^{n-1} for $n \geq 1 and\ 1\ for\ n = 0$,
Symmetric property $T_n(-x) = (-1)T_n(x)$
$T_n(x)$ has n zeros in $[-1, 1]$ given by
$X_k = cos\left(\frac{2k+1}{n}\pi\right), k = 0, 1, \ldots n - 1$ (7),
Orthogonality property. The Chebyshev polynomial is orthogonal for both continuous and discrete functions,
Minimax property; of $all\ nth$ degree polynomial with leading coficient 1, $2^{1-n}T_n$ has the smallest maximum norm in $[-1,1]$. The value of its maximum norm is 2^{1-n}.

4. Theorems

The following relevant theorems are useful to the topic under discussion. These are theorem that assert the possibility of using polynomial to approximate function, the existence of best approximating polynomial and the uniqueness of the best approximating polynomial are included. A proposition showing the effectiveness of the new technique is added.

Theorem 4.1 *Jensen and Rowland* (1975)
For each $x \in [-1,1]$ and $n = 1, 2, \ldots$ of the chebyshev polynomial,
$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$

Theorem 4.2 Powel and Belcerzek (2009)
If f is continuous on [a, b]then a best approximation to f from Pn exist
Theorem 4.3 *Wilke* (2010)
If $f \in C[a,b]$, then the best chebyshev approximation to f
From p_n is unique.

Proposition4.1 $CPTFOSA$ (2018)
The cost fe incured by the expected allocation is less than the cost fo incured by the observed allocation.

$\sum_{j=1}^{n} C_{i,j}x_{i,j} \ \geq \ \sum_{j=1}^{n} C_{i,j}y_{i,j}$ (8)

Proof of proportion
$Let\ i = 1, 2, \dots, m$ be the products manufactured by the firm.
$Let\ j = 1, 2, \dots, n$ be the warehouses used by the firm.
$Let\ X_{ij}$ be the (observed) quantity of products i held as stock in warehouse j.
$Let\ C_{ij}$ be the cost of holding one unit of product i as stock in waregouse j. we
Assume that $\forall i$ the warehouses are arranged in such a way that $C_{ij} \geq C_{ij} + 1$
Let yij be the (expected) optimal quantity of product i held as stock in warehouse j.
Let f be the cost function
$With\ C_{ij} \geq C_{ij} + 1 (j = 1, 2, \dots. n)\ \forall i$
Let P_{n-1} I be the chebyshev polynomial of order $n - 1$
Let $Ni = \sum_{j=i}^{n} X_{ij}\ and\ D_i = \sum_{j=i}^{n} P_{n-1}(j)$
Then define the normalizer of the observed and expected allocation as

$q_{n-1,}\ i(x) = \frac{Ni}{Di}\ P_{n-1,}\ i(x)$ (9)

So that $\forall i,\ y_{ij} = q_{n-1,}i(j)\ (j = 1, 2, \dots, n)$ where q is the weight that normalizes the allocations.
Let
$f_{i,o} = \sum_{j=1}^{n} C_{i,j}x_{i,j}$ (10)

and
$f_{i,e,} = \sum_{j=1}^{n} C_{i,j}y_{i,j}$ (11)

Where 0 and e represent the observed and approximated stock allocation respectively.
$f_{i,o} - f_{i,e} = \sum_{j=1}^{n} C_{i,j}x_{i,j} - \sum_{j=1}^{n} C_{i,j}y_{i,j\,(12)}$

$f_{i,o} - f_{i,e} = \sum_{j=1}^{n} C_{i,j}(x_{i,j} - y_{i,j})$
$= \sum_{j=1}^{n} C_{i,j}\left[x_{i,j} - \frac{N_i}{D_i}P_{n-1,i}(j)\right] \geq \ C_{i,o}(\sum_{j=1}^{n} x_{i,j} - N_i)$ (13)

$$C_i^*(N_I - N_i) = 0 \ (14)$$

$Therefore\ f_{i,o} - f_{i,e} \geq 0$
$\Rightarrow\ f_{i,o} \geq f_{i,e}$

5. Model Developments

In this work a model is developed for stock allocation of a multi- warehouse firm. The allocation to the warehouse is modeled using Chebyshev polynomial. The Chebyshev polynomial

approximation is applied on the allocation to produce a technique that optimizes stock allocation. The step taken to obtain the model are stated as follows.
let i, j, x_{ij}, C_{ij}, y, N_{ij} and D_i be as defined above
Calculate the normalize:

$$q_{n-1,}\ i(x) = \frac{Ni}{D}\ P_{n-1,\ i}\ (x) \quad (15)$$

Using the allocation tables:

Table 1: Table of allocations

Warehouse (j)	1	2	. . .	n
Stock(x_{ij})	x_{i1}	x_{i2}	. . .	x_{in}
Cost(c_{ij})	c_{i1}	c_{i2}	. . .	c_{in}

The allocation algorithm is given thus
$\forall i\ (i = 1, 2, \dots, m)$

$y_{i,j} = min(q_{n-1}, i(j), x_{i,j}), j = 1, 2, \dots, n-1$ (16)
$n-1$
$y_{i,n} = Ni - \sum y_{ij}$ (17)
$j\ = 1$
Obtain $f_{i,o}$ and $f_{i,e}$ where $f_{i,o} = \sum_{j=i}^{n}\ C_{i,j}x_{ij}\ f_{i,e} = \sum_{j=i}^{n}\ C_{i,j}y_{ij}$ are respectively as the total stock cost bases on the observed and expected allocations.
Compare the two costs of the allocation

6. Application of the Chebyshev Polynomial Technique For Stock Allocation (CPTFOSA)

6.1 Illustration 1

The allocation to the six warehouses of a firm is given below. The firm ensures the cost of 0.01 per unit for every 50 kilometers of operation and makes a return of 0.05per unit supplied. The distance to warehouse are 198km, 67km 8km, 98km, 133km and 136km to warehouse 1,2 ... . . ,3.

Table 2: Table of allocation

Warehouse(x)	1	2	3	4	5	6
Stockx_{ij}(000 units	130	293	8	556	943	43
Cost(c_{ij})(000 units	.04	.01	.02	.02	.03	.03

6.2 Solution

Table 3: Table of allocation

Warehouse(x)	1	2	3	4	5	6
Stockx_{ij}(000 units	130	293	8	556	943	43
Cost(c_{ij})(000 units	.04	.01	.02	.02	.03	.03

Now re-arrange in descending order of cost i.e $C_{ij} \geq C_{ij+1,j}$

Table 4: Table of allocations

Warehouse(x)	1	2	3	4	5	6	
Stockx_{ij} (000 units)	130	8.	943	43	556	293	1973=N

cost(c_{ij})(000units)	.04	03	.03	.02	.02	.01	

$P_n = 6$ and so the corresponding Chebyshev polynomial is P_{n-1} $i.e$
$P_5(x) = 16x^5 - 29x^3 + 5x$ (18)
The normalize is
$q_5(x) = \frac{1973}{186501}(16x^5 - 29x^3 + 5x)$ (19)
This results to the approximate allocation.

Table 5: Table of allocation

Warehouse (x)	1	2	3	4	5	6	T
Stock x_{ij} (000 units)	130	8	943	43	556	293	1973
Cost (obs) (c_{ij}) (000units)	.04	0.03	0.03	0.02	0.02	0.01	
Stock(approx)(y, j)(00units)	0	4	36	160	503	1273	1974

From the table above, the cost f_o associated with the observed stock and the cost f_e associated with the approximate (expected) stock are:
$.f_0 = 48.64$ and $f_e = 27.17$
$F_0 - f_e = 21.47 > 0$ (20)

Also $y_0 = 98.65$, $y_e = 98.70$
Furthermore, the returns$T$ of the allocation
$T_0 = y_0 - f_0 = 50.01$; and $T_e = y_0 - f_e = 71.53$
$\Rightarrow T_e - T_0 = 21.52$
Now applying the algorithm
$y_{ij} = min(q_{n-1}, i(j), x_{i,j})$ $j = I, 2, \ldots, n-1$ (21)
$n - 1$
$y_{in} = N_i - \sum y_{ij}$ (22)
$j = 1$
To the allocation above to obtain the following
$f_0 = 88.64$
$f_e = 15.89$
$y_o = 98.65$
$y_e = 45\ 50$
Hence $T_0 = y_0 - f_0 = 10.01$ and
$T_e = y_e - f_e = 29.61$.
Clearly $T_e > T_0$ i.e the actual return of the (expected)allocation is higher than that of the observe allocation.

6.3 Illustration 2

Consider the allocation according to the three products x, y, z of a firm with the table of their associate below.

Table 6: Table of allocations

Warehouse	1	2	3	4	5	6	T
Stock$x$(000units)	3	15	50	100	200	350	718
Stock$y$(000units)	30	20	110	170	345	125	800
Stock$z$(000units)	28	23	93	243	150	156	693

Table 7: Table of costs

Warehouse(x)	1	2	3	4	5	6
Cost $x_{i\,j}$	0.5	1.0	0.03	2.oo	3.5	0.15
Cost $y_{i\,j}$	0.5	1.0	o.03	2.00	3.5	0.01
Cost $z_{i\,j}$	0.3	o.93	0.02	1.56	2.43	0.13

This is a polynomial of degree hence

$P_{n-1} = 16x^5 - 20x^3 + 5x$ (23)

Applying the Chebyshev polynomial gives;

$P_5(1) = 1, P_5(2) = 362\ P_5(3) = 3{,}363\ P_5(4) = 15{,}124\ P_5(5) = 47{,}525\ P_6(6) = 120{,}126$

Now apply the normalizer

$q_{5,1}(x) = \frac{718}{186501}(16x^5 - 29x^3 + 5x)$ (24)

$q_{5,2}(x) = \frac{800}{186501}(16x^5 - 29x^3 + 5x)$ (25)

$q_{5,3}(x) = \frac{693}{186501}(16x^5 - 29x^3 + 5x)$ (26)

Applying the algorithm

$y_{i\,j} = \min q_{5,i}(j), x_{i,j}\ j = 1, \ldots\ldots, 5$ (27)

$y_{i,6} = N_i - \sum_{j=1}^{5} y_{ij}$ (28)

on the observed and approximate allocation gives the following table;

Table 8: Table of allocations

Warehouse(x)	1	2	3	4	5	6	T
Cost $x_{i\,j}$(000units)	0	1	13	28	183	463	718
Cost $y_{i\,j}$(000units)	0	2	14	65	204	515	800
Cost $z_{i\,j}$(000units)	0	1	13	56	150	473	693

The total returns of the observed and expected allocations per products are obtained as follows:

$\gamma_{0,1} = 1{,}752.34\ \gamma_{0,2} = 1{,}356.40\ \gamma_{0,3} = 870.52$

$\gamma_{e,1} = 2{,}051.15\ \gamma_{e,2} = 2{,}282.80\ \gamma_{e,3} = 1{,}439.50$

From the result above, it is obvious that the allocation obtained using the Chebyshev polynomial approximation (CPTFOSA) has a higher returns.

7. Discussion Of Result

The Chebyshev technique has been used to solve the allocation problem. The approximation obtained is now compared with the original (observe) allocation.

1. it has been shown that the cost incurred in the original (observe) allocation is higher than that associated with the expected (approximate) allocation. It is proved that the cost incurred in the expected allocation is always smaller to that incurred by the observed.
2. The advantage of the stock allocation by the CPTFOSA is attested to if the unit costs of the stocks are calculate as it show a lower per unit cost.
3. Considering α, the difference between returns and total cost, $\alpha_0 < \alpha_e$.i.e. the total returns of the expected stock allocation id higher than that of the observed allocation.

8. Summary and Conclusion

8.1 Summary

The features of stock allocation, chebyshev polynomial approximation, and the approximation technique for stock allocation are discussed. The model for the new approximation technique is developed. The Chebyshev polynomial approximation has been applied to the allocation to obtain a good approximation. Relevant theorems that assert the possibility of using polynomial for functional approximation, its existence and the uniqueness of the solution are presented while numerical illustrations are included. The model has proved to be.

1. More cost effective
2. Possesses wide area of applicability.
3. It can handle the case of allocation to n- warehouses $(n > 2)$.
4. It ensures an optimal allocation.
5. The technique ensures higher returns

8.2 Conclusion

In recent times, rules have been made by relevant agencies to standardize research, specify products and ensure quality control of products. These among others have rendered some stock allocation/ management conditions obsolete as perfection has been enhanced. There is therefore need to develop a model that addresses many conditions of stock allocation simultaneously. This is to answer the question "how best can stock be managed to minimize losses and which method of allocation would be suitable for solving the problem of different conditions simultaneously and optimize returns? " This work has shown that;

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