MATHEMATICAL MODELLING OF AGRO-CHEMICALS RESIDUE IN RICE PRODUCTION

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Abstract
Food supply and consumption is one among the basic necessities of life. Its availability has been a focal point of concern to the global community and has also necessitated the increased agricultural activities with the cultivation of rice inclusive. This paper studied the application of a mathematical model in analysing the impact of agro-chemical residues in rice production. The numerical simulations of the model confirmed the analytical results and the outcome of the study attest to the fact that agro-chemical residues could have negative impact when consumed through farm products, such as rice grains among others, if not controlled.

Keywords: *Rice, Cultivation, Agro-chemicals, Model.*

1. Introduction

The global interest on agricultural produce currently emphasis on measures of improving food production and ways of satisfying the increasing demand for agricultural produces from the general population as source of daily food (Agaba, 2018; Kumari *et al.*, 2014; Salas *et al.*, 2000). Rice (Oryza Sativa) is considered a member of the grain of grass family. According to Trinkley and Fick (2018), there are two species of rice that are generally cultivated out of the over all 22 species recognizable. These two species they itemized as Oryza glaberrima cultivated in West Africa and Oryza sativa cultivated in the rest of the world.

Aside the elimination of many important minerals during the milling process of rice (Trinkley and Fick, 2018), there could be some deposits of residues generated from the agro-chemicals used in the cultivation processes which are in most cases considered hazardous to human health when consumed along with the produces (Agaba, 2013; Agaba, 2014; Agaba, 2018; Blair *et al.*, 2014; Kumari *et al.*, 2014; Moser *et al.*, 2017; Prank *et al.*, 2016; Webster *et al.*, 2005).

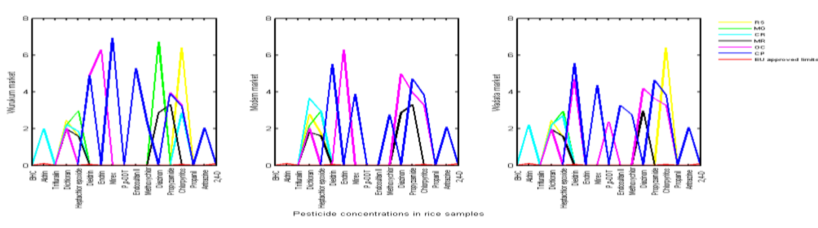

Figure 1: The result showing the various agro-chemical residue in rice produce analysed using three different markets in Makurdi, Benue State, Nigeria, as extracted from Adah and Ado (2017)

A research carried out by Adah and Ado in 2017 across three selected markets in Makurdi, Benue State, Nigeria on pesticide residue in rice produces gave the results shown in Fig. 1 (as extracted from (Adah and Ado, 2017) which affirmed the fact that some agro-chemical residual deposits (more than the required amount accepted by the European Union standard) could be found in some rice produces. These agro-chemical residues could be as a result of the excessive usage of agro-chemicals and they cause great havoc to human health when consumed and the immediate environment (Agaba, 2018; Kumari *et al.*, 2014; Roslund, 2015; Salas *et al.*, 2000; World Health Organization (WHO), 2008).

According to World Health Organisation (2008), the most affected families by pesticides are those who tend to consume food supplied or harvested directly from the farm/fields. These they opined to be as a result of the poor monitoring or control of residue of pesticides in our locally produced foods and in some cases, the imported food stuffs. Scholars, such as (Agaba, 2014; Agaba, 2018; Blair *et al.*, 2014; Kumari *et al.*, 2014; Udeze, 2011), have proven in their researches the existence of agro-chemical in agricultural produces and consequently, emphases on the need to control hazardous impact of agro-chemical residues or waste on human health that are being consumed through various food substances.

Kumari *et al.* (2014) and Agaba (2018) were of the opinion that the dependence on chemical inputs on agricultural activities should be reduced to the barest minimum so as to reduce accordingly the adverse effect of agro-chemicals on cultivated crops. This they believed can be done effectively through the ecofriendly approaches of farming system. The study carried out by Huan *et al.* in 2005 (details in Roslund, 2015) also affirmed the aforementioned assertion.

The increasing demand for rice production, most especially the locally produced rice (home grown rice as a result of the ban on the importation of foreign rice) and the excessive use of agro-chemicals during cultivation process thus necessitate the study of the impact of agro-chemical residues in rice produces. Consequently, this paper evaluates the impact of agro-chemical residues in rice production using a mathematical model derived based on the concept of the South Korea system for rice farm cultivation (as shown in Fig. 2) in comparison with the conventional rice farming system in Nigeria.

Figure 2: South Korea system for rice farm cultivation extracted from Kim *et al.*, 2018

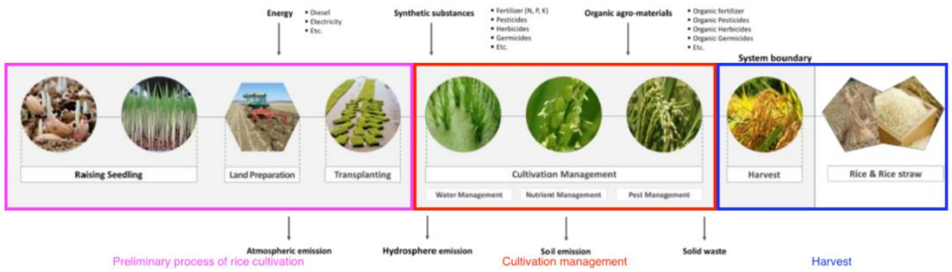

Figure 3: Model diagram: Farming system for rice production showing the summarised three major categories involved in rice farming process, where the purple rectangle represents the preliminary process of rice cultivation, the red rectangle represents the cultivation management and the blue rectangle represents the Harvest

The model derivation applied the principle of the SIV (Susceptable, Infected and Vector) ideology of the epidemic model to help drive home the concept of analysing the effect of agro-chemical residue in rice produce and possibly to deduce measures of reducing the amount of residual deposits to the accepted standard stipulated by the European Union (EU). The paper consists of five sections, beginning with the introduction while the second section considered the model derivation and in the third section, the analyses of the various states of the model were carried out. The numerical simulation and evaluation of the model were covered in the fourth section and finally, the fifth section discussed findings and the conclusion of the study.

2. Model Derivation

Comparing the South Korea system for rice farm cultivation discussed in Kim *et al.* (2018) with the Nigerian system of rice farming (as summarized into three categories shown in Fig. 3) aid the model derivation for analysing the agro-chemical residue in rice production. Mathematically, the model leads to the following system of equations:

$$\begin{aligned}
\frac{dx}{dt} &= r\left(1-\frac{x+y}{k}\right)x-\beta xz+\alpha y,\\
\frac{dy}{dt} &= \beta xz-(\alpha+m)y, \qquad\qquad (1)\\
\frac{dz}{dt} &= \phi_s+\phi_c-(\lambda_h+\lambda_s+\lambda_w)z,
\end{aligned}$$

where x represents the amount of rice cultivated on the piece of land and its assumed to grow logistically with r representing the growth rate of the plant and k the carrying capacity of the cultivated land. Let z denote the level of chemical application or composition sprayed on the farmland (plants) during the cultivation management. It is assumed that the plants were sprayed with synthetic substances and organic agro-materials either for pest or weed management, or to improve solid nutrients. The sprayed plants are considered harmful as a result of the chemical composition until an elapse time interval known as the incubation period or waiting period. Consequently, let α denotes the rate at which the incubated rice plants become healthy after the elapsed duration of time and y representing the population of the harmful rice plants.

The parameter β represents the rate at which the chemical is applied on the plant, indicating the contact rate between the healthy plants and the chemical sprayed, while λ_h, λ_s, λ_w representing the loss rate of hydrosphere emission, soil emission and solid waste respectively. Furthermore, ϕ_s denoting the amount of synthetic substance applied on the farmland and ϕ_c representing the quantity of organic agro-materials used during the cultivation management process. The parameter m represents the loss rate of the harmful plants due to excess chemical application and consequently loss of nutrients.

Consequently, let ϕ denote the amount of synthetic substances and organic agro-materials (the total chemical concentrate) used in the process of pest/weed control or nutrient enhancement then $\phi = \phi_s + \phi_c$ and if λ represents the rate at which the overall chemical applied losses it strength/toxicity through hydrosphere emission, soil emission and solid waste (that is, $\lambda = \lambda_h + \lambda_s + \lambda_w$) then Eq. (1) becomes

$$\begin{aligned} \frac{dx}{dt} &= r\left(1 - \frac{x+y}{k}\right)x - \beta xz + \alpha y, \\ \frac{dy}{dt} &= \beta xz - (\alpha + m)y, \\ \frac{dz}{dt} &= \phi - \lambda z, \end{aligned} \tag{2}$$

having the initial conditions: $x(0) > 0$, $y(0) \ge 0$, $z(0) \ge 0$ defined within the region

$$\varphi = \left\{ (x, y, z) \in R_+^3 : 0 \le x, y \le k, 0 \le z \le \frac{\phi}{\lambda} \right\}.$$

3. Steady States and their Stability

Analysing the model equation (2) gives three steady states. The first is the plant-free steady state E_0 denoting the preparation state where the farmer prepares the seedlings and the land for cultivation process to the point of transplanting the rice seedlings. While the second is the hazard-free steady state E_x which indicates the cultivation of rice at the farmland but awaiting the application of agro-chemicals and/or the recovered grains of rice after the incubation period had elapsed and the applied chemicals had wane off totally leaving no residue or with insignificant amount accepted by European Union Standard. At this stage, the grains are harmless and considered free from agro-chemical residues. The last steady state is the endemic state E_n representing the cultivation management stage where the agro-chemicals had been applied, as means of improving the nutrients so as to enhance growth and its yield, along with other natural phenomenon taking effect.

The plant-free steady state E_0 is therefore obtained as

$$E_0 = [\hat{x}, \hat{y}, \hat{z}] = \left[0,0, \frac{\phi}{\lambda} \right],$$

with the characteristics equation taking the form

$$\left(r-\frac{\beta\phi}{\lambda}-\mu\right)(\alpha+m+\mu)(\lambda+\mu)+\alpha\beta\phi=0\,,$$

which simplifies to

$$\mu^3+P_2\mu^2+P_1\mu+P_0=0$$

where

$$P_2=\frac{1}{\lambda}\left[\lambda(\alpha+m+\lambda)+\beta\phi-r\lambda\right],\; P_1=\lambda(\alpha+m)+\frac{1}{\lambda}(\alpha+m+\lambda)(\beta\phi-r\lambda),\; P_0=\beta\phi m-r\lambda(\alpha+m)$$

.

Consequently, using the Routh-Hurwitz criterion for stability indicates that the steady state is stable provided the conditions $P_2, P_1, P_0>0$ and $P_2P_1>P_0$ are satisfied. It is obvious that if P_0 is a positive value then P_2 and P_1 are also positive values. Hence, we have

$$\beta\phi m-r\lambda(\alpha+m)>0\,,$$

which implies that

$$R_0=\frac{r\lambda(\alpha+m)}{\beta\phi m}<1\,,$$

as the condition for stability since $P_2P_1>P_0$ is also satisfied if $R_0<1$, where R_0 is the threshold value. Hence the following result:

Theorem 1. *The plant-free steady state E_0 of the model equation (2) is stable provided $R_0<1$, undergoes bifurcation when $R_0=1$ and become unstable whenever $R_0>1$.*

The hazard-free steady state is

$$E_x=[\tilde{x},\tilde{y},\tilde{z}]=[k,0,0],$$

and using the Jacobian matrix for this steady state, the characteristics equation is obtained as

$$\left[r\left(1-\frac{2\tilde{x}}{k}\right)-\mu\right](\alpha+m+\mu)(\lambda+\mu)=0\,,$$

which gives

$$(r+\mu)(\alpha+m+\mu)(\lambda+\mu)=0\,,$$

since $\tilde{x}=k$ and therefore generated the following results:

Theorem 2. *The hazard-free steady state E_x of the system of equations (2) is always stable whenever it exists.*

The endemic steady state of the model is obtained as

$$E_n=[\bar{x},\bar{y},\bar{z}],$$

where

$$\bar{x}=\frac{k\lambda(\alpha+m)}{\beta\phi+\lambda(\alpha+m)}\left(1-\frac{\beta\phi m}{r\lambda(\alpha+m)}\right),\; \bar{y}=\frac{\beta\phi\bar{x}}{\lambda(\alpha+m)},\; \bar{z}=\frac{\phi}{\lambda}, \qquad (3)$$

and it is feasible only when $R_0>1$. The stability is determined using the Jacobian matrix (4) generated from the model equation.

$$J_n = \begin{pmatrix} r\left(1-\frac{2\bar{x}+\bar{y}}{k}\right)-\beta\,\bar{z} & -\frac{r\bar{x}}{k}+\alpha & -\beta\,\bar{x} \\ \beta\,\bar{z} & -(\alpha+m) & \beta\,\bar{x} \\ 0 & 0 & -\lambda \end{pmatrix} \qquad (4)$$

The matrix (4) consequently gives the characteristics equation

$$\left[r\left(1-\frac{2\bar{x}+\bar{y}}{k}\right)-\beta\,\bar{z}-\mu\right](\alpha+m+\mu)(\lambda+\mu)+\beta\lambda\bar{z}\left(\alpha-\frac{r\bar{x}}{k}\right)=0,$$

with $\bar{x}$, $\bar{y}$ and $\bar{z}$ as defined in Eq. (3). Hence, the equation simplifies into the form

$$\mu^3+P_a\mu^2+P_b\mu+P_c=0,$$

where

$$P_a=\alpha+m+\lambda+\frac{r\bar{x}}{k}+\frac{\beta\phi\alpha}{\lambda(\alpha+m)},\ P_b=(\alpha+m+\lambda)\left(\frac{r\bar{x}}{k}+\frac{\beta\phi\alpha}{\lambda(\alpha+m)}\right)+\lambda(\alpha+m),$$

$$P_c=\frac{r\bar{x}}{k}\left[\beta\phi+\lambda(\alpha+m)\right].$$

Consequently, for the endemic state to be stable the conditions $P_a, P_b, P_c > 0$ and $P_aP_b > P_c$ must be satisfied. This indicates that if the endemic state exists then it is obvious that $P_a, P_b, P_c > 0$ and it is also true that $P_aP_b > P_c$. Hence the following result:

Theorem 3. *The endemic state* E_n *of the model equation (2) is only feasible when* $R_0 > 1$*, where*

$$R_0=\frac{r\lambda(\alpha+m)}{\beta\phi m},$$

and is always stable whenever it exists.

Furthermore, analysing the threshold value R_0 indicates that for its value less than one (that is, $R_0 < 1$) the system equation has a stable plant-free steady state E_0 which bifurcates at $R_0 = 1$ and transits to an unstable steady state when $R_0 > 1$ at which point the endemic steady state E_n exists and is stable. As the threshold value R_0 tends to infinity, the system transits into an hazard-free steady state E_x which is always stable whenever it exists. Note that $R_0 \to \infty$ as $\lambda \to \infty$, $\beta \to 0$ and/or $\phi \to 0$ which implies that as the application of agro-chemical drops to zero and also with the gradual loss in the strength of the applied chemical concentration over an elapsed time interval, the entire cultivated rice (with the exception of those lost to high toxicity of the agro-chemical used in managing the cultivation process) will eventually become healthy for consumption after harvest. The above result is similar to what was obtained in Agaba (2018).

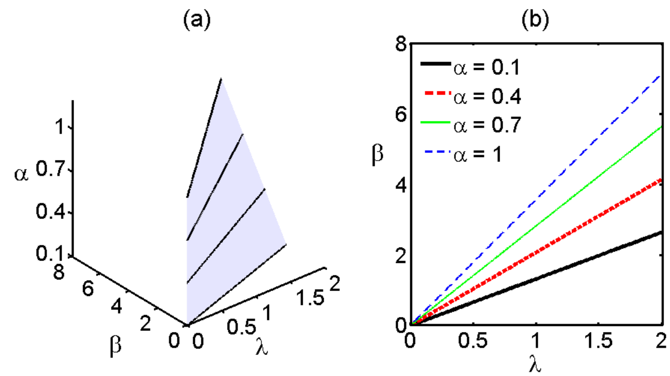

Figure 4: Stability region of the model shown in (a) three dimensional plot and (b) two dimensional plot. The parameter values used are $r = 0.28$, $m = 0.43$, $\phi = 0.26$

4. Numerical Simulation of the Model

The mathematical model was studied numerically to determine the dynamical nature of the model and also the effect of varying some parameter values on the model dynamics. Fig. 4 explains the result for the stability region of the model indicating the respective steady states of the model equation (2) as regards the variation of certain parameters. From the result, it can be deduced that the portion below the lines represent the regions for the stable endemic steady state with $R_0 > 1$. While, the portion above the lines denote the regions representing the plant-free steady state of the system with $R_0 < 1$ indicating that the endemic steady state is not feasibly within this region whereas the plant-free steady state exists and is stable.

The results in Fig. 5 show the dynamics of the model where $R_0 < 1$ and $R_0 > 1$ respectively representing the plant-free and endemic steady states of the model. The behaviour of the model as $R_0 \to \infty$ in conjunction with $\phi \to 0$ is captured in Fig. 6, that is, showing the dynamics of the hazard-free steady state. A general study of the behaviour of the model with respect to the threshold value R_0 is captured in Fig. 7 which shows the transitions between the various steady states. This implies the initial transition from the plant-free steady state E_0 to the endemic steady state E_n and then finally to the hazard-free steady state E_x. The outcome from this numerical simulation affirmed the analytical evaluation earlier carried out using the equation determining the threshold value R_0.

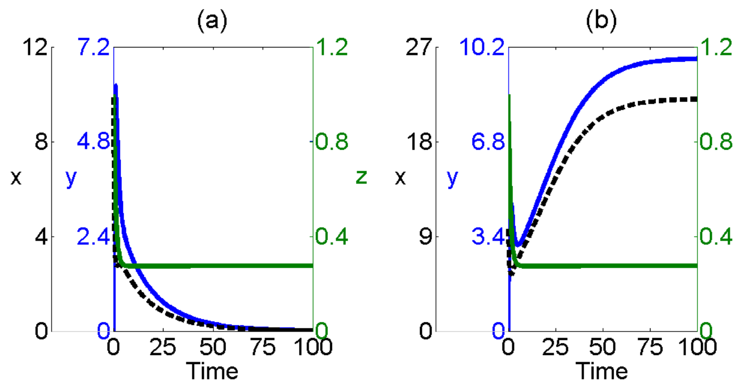

Figure 5: Dynamics of the model equation (2) with (a) $R_0 < 1$ **(** $R_0 = 0.7610$ **) and (b)** $R_0 > 1$ **(** $R_0 = 1.4677$ **). Parameter values are,** $r = 0.28$ **,** $\alpha = 0.52$ **,** $\lambda = 0.95$ **,** $\phi = 0.26$ **,** $m = 0.43$ **,** $k = 100$ **.** $\beta = 2.92$ **and** 1.54 **respectively in (a) and (b)**

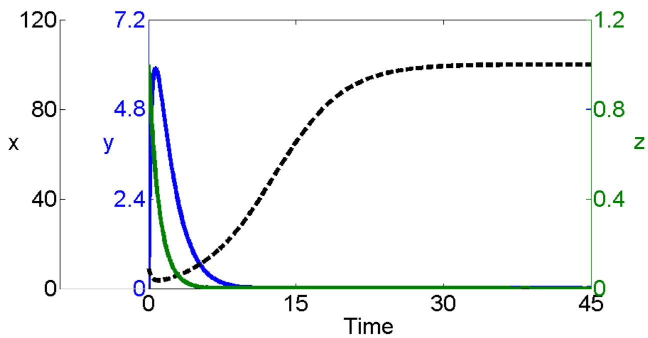

Figure 6: The model dynamics as $R_0 \to \infty$ **. The parameters used are,** $r = 0.28$ **,** $\beta = 1.54$ **,** $\alpha = 0.52$ **,** $\lambda = 0.95$ **,** $\phi = 0$ **,** $m = 0.43$ **,** $k = 100$

The model analyses with various values of the parameter α gave the result in Fig. 8 showing the impact value of the parameter α (that is, the rate at which the incubated grains of rice become healthy again for consumption after the toxicity of the applied agro-chemicals have wane off) on the produce from the cultivation of rice.

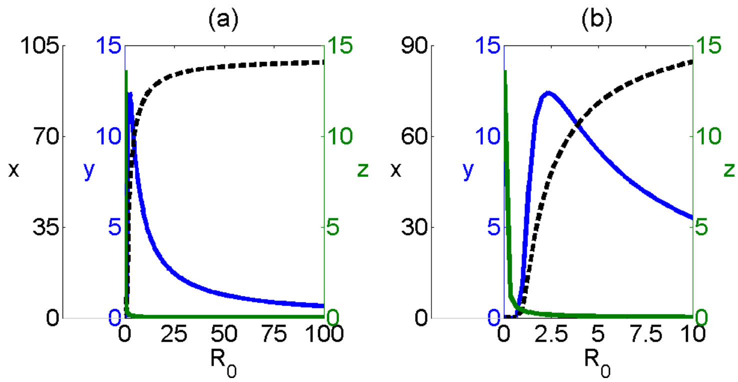

Figure 7: The model dynamics with respect to increasing value of R_0 (a) covering the three steady states (b) a zoom position of (a) showing clearly the result within the interval $[0,10]$. The parameters used are, $r=0.28$, $\beta=1.54$, $\alpha=0.52$, $\phi=0.26$, $m=0.43$, $k=100$ while the value for λ was varied

5. Discussion

The analytical evaluation and numerical simulation carried out on the mathematical model derived in the course of studying the impact of agro-chemical residues in rice produces gave the pictorial representation in Figs. 4 - 8. In Fig. 4, the stability regions were obtained indicating the respective regions for the stable and unstable/not feasible steady states of the model with respect to the various values of R_0, that is, for $R_0 < 1$ and $R_0 > 1$. The results in Figs. 5 and 6 affirmed the result obtained in Fig. 4 whereas the outcome of the model analyses with respect to the various steady states of the Eq. (2) captured in Fig. 7 is a summary of what was obtained in Figs. 5 and 6. While Fig. 8 shows the impact of varying the recovery rate from incubation on the model dynamics.

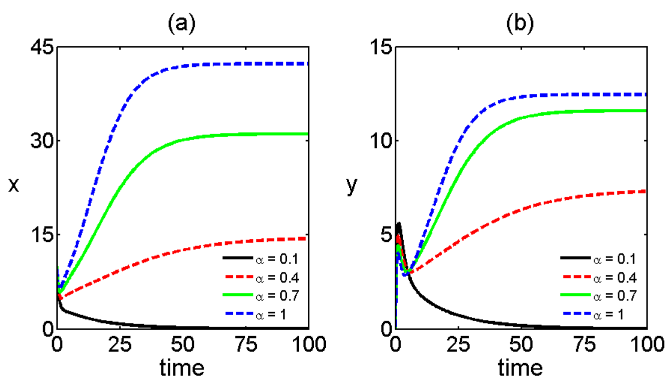

Figure 8: The plot showing the impact of α on the dynamics. Parameter values remained the same as defined in Fig. 6 with the exception of α

The overall outcome signifies that the application of agro-chemicals with high concentration on the farmland after cultivation could have very serious effect both on the grains and on the consumers and farmers (See Fig. 5). But if the strength/toxicity of the chemical is wane off over certain time interval, it is possible to have or recover the grains in the state fit for healthy consumption. This is clearly demonstrated by Figs. 5, 6 and 7. From Fig. 8 it can be deduced that the increment in the recovery rate of harmful grains to healthy ones, α generated a corresponding increment in the total number of healthy grains fit for harvest/consumption. Hence, the higher the recovery rate α, the much healthier grains x to be harvested.

Consequently, the study shows the need to pay adequate attention on cultivation, harvest and consumption of rice produce. Furthermore, the outcome of the study indicate that much attention be directed towards the application of agro-chemicals on agricultural produce to ensure that the proper measures are adhered to. Also, ensuring that the required incubating period be observed in order to aid the total waning off of the toxicity of the agro-chemicals leaving no or minimal residual deposits as stipulated by the European Union. In addition, there is the need for a wake-up call to all monitoring agents/agencies, both Governmental and Non-Governmental, to arise with all enthusiasm creating awareness that will ensure farmers adhere to the accepted standard procedures of agro-chemical application on cultivation processes. When the aforementioned are carried out, the agricultural sector will be enhanced, the economic system and the state of the peoples' health will be greatly improved positively.

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