EFFECTS OF VARIABLE VISCOSITY AND VISCOUS DISSIPATION ON FREE CONVECTIVE COUETTE FLOW IN A VERTICAL CHANNEL: THE HOMOTOPY PERTURBATION METHOD APPROACH

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Abstract
An analysis has been carried out to study the viscous dissipation effect on steady natural convection Couette flow of heat generating fluid in a vertical channel. The Homotopy Perturbation method is used to obtain the expressions for temperature and velocity. The effects of various physical parameter such as variable viscosity parameter, viscous dissipation parameter, thermal buoyancy parameter, heat generation/absorption parameter and Prandtl number to determine the temperature and velocity profiles with the help of graphs. During our investigation, it was found that fluid temperature and velocity increase with an increase of viscous dissipation while a decrease in the fluid viscosity lead to decrease in temperature profile. The comparisons of the present study with Jha and Ajibade (2010) shows an excellent agreement when the variable viscosity and viscous dissipation terms are neglected.

Keywords*: Free convection; heat generation/absorption; viscous dissipation; variable viscosity; Homotopy perturbation method.*

1 Introduction

The study of natural convection flow with viscous dissipation along a vertical plate is receiving considerable attention of many researchers because of its applications in many fields of Engineering, such as nuclear reactors and those dealing with the liquid metals. In addition, understanding of the effects of viscous dissipation is also significant in numerous applications that include reactor safety analysis, metal waste, spent nuclear fuel. Jha and Ajibade (2012) studied the effect of viscous dissipation on natural convection flow between vertical parallel plates with time-periodic boundary condition. The work shows that the fluid temperature increases as a result of dissipation heating within the channel. Kabir et al. (2013) studied the effect of viscous dissipation on magnetohydrodynamics natural convection flow along a vertical wavy surface with heat generation. They concluded their work as the dissipation increases, the temperature profile increases. Dessie and Naikoti (2014) examined magnetohydrodynamics effects on heat transfer over stretching sheet embedded in porous medium with variable viscosity, viscous dissipation and heat source/sink. Their conclusion shows that the effect of viscosity parameter is to increase the velocity profile and the reverse phenomenon is observed in temperature, while an increase in viscous dissipation increase both temperature and velocity profile. Manjunatha and Gireesha (2016) studied effects of variable viscosity and thermal conductivity on magnetohydrodynamics flow and heat transfer of a dusty fluid. They observed that the velocity profile decreases with increasing values of fluid viscosity while the temperature of the fluid and dusty phase increases as the viscous dissipation increases. Fahad et al. (2017) investigated combined effect of viscous dissipation and radiation on unsteady free convective Non-Newtonian fluid along a continuously moving vertically stretched surface with No-Slip phenomena. Their findings showed that velocity increases with an increase of viscous dissipation Ec. Ajibade and Tafida (2019) considered viscous dissipation effect on a steady generalised Couette flow of heat generating/absorbing fluid in a vertical. They reported that fluid temperature and velocity increase with an increase in Eckert number. In another article, Ajibade and Tafida (2019) investigated viscous dissipation effect on steady natural convection Couette flow of heat generating fluid in a vertical channel. The outcome of their study showed that the fluid temperature and velocity increase with the increase in viscous dissipation. Tafida and Ajibade (2019) analyzed the effect of variable viscosity on natural convection flow between vertical parallel plates in the presence of heat generation/absorption. The concluded that viscosity contributes to decrease velocity and hence reduced resistance to flow.

Pantokratoras (2005) studied the effect of viscous dissipation in natural convection along a heated vertical plate. He discovered that the viscous dissipation has a strong influence on the results as it assists the upward flow opposes the downward flow. Mahesha and Subha (2008) examined heat transfer in magnetohydrodynamics viscoelastic fluid flow over a stretching sheet with variable thermal conductivity, non-uniform heat source and radiation. They showed that the temperature profile increases with an increase in Eckert number. Jha and Ajibade (2010) studied unsteady free convective Couette flow of heat generating/absorbing fluid. They concluded that the absence of convection currents does not translate to flow stagnation. Also, a reverse type of fluid flow is achieved in case of external heating of the moving plate. Hazarika and Gopal (2012) analyzed the effects of variable viscosity and thermal conductivity on magnetohydrodynamics flow past a vertical plate. They observed that the velocity profile decreases with the increase of variable viscosity. Bandita (2017) studied the effects of variable viscosity and thermal conductivity on steady magnetohydrodynamics slip flow of micropolar fluid over a vertical plate. He discovered that due to the increase of viscosity, velocity of the fluid decreases while temperature and micro-rotation of the fluid increases.

Ferdousi and Alim (2010) considered effect of heat generation on natural convection flow from a porous vertical plate. They concluded that an increase in the values of heat generation parameter leads to increase both the velocity and the temperature profiles. Mahdy (2010) considered the effects of chemical reaction and heat generation on double-diffusive natural convection heat and mass transfer near a vertical truncated cone in porous media. Afterwards, Siddiqa et al. (2010) studied natural convection flow of a viscous incompressible fluid over a semi-infinite at plate with the effects of exponentially varying temperature dependent viscosity and the internal heat generation. They outcomes showed that the velocity of the fluid increases whilst the temperature decreases within the boundary layer for increasing values of heat generation.

Homotopy Perturbation Method (HPM) was first studied by He (1999) to solve linear, non-linear and couple problems in partial or ordinary form. He (2000) introduced the new method to solve non-linear and boundary value problems (BVP). Hossien et al. (2008) analysed the application of Homotopy perturbation method for solving Gas Dynamics equation. They discovered that Homotopy perturbation method is a powerful and efficient technique in finding exact and approximate solution for nonlinear differential equation. Adamu (2017) considered

parameterized Homotopy perturbation method. He discovered that the new technique is proved to be powerful and efficient. Abou-Zeid (2019) investigated Homotopy perturbation method for magnetohydrodynamics non-Newtonian nanofluid flow through a porous medium in eccentric annuli with peristalsis. He concluded that the temperature increases with the increase of Brinkman number. The obvious advantage of the Homotopy perturbation method in this work is that it can be applied to various nonlinear problems. Moreover, the calculations in the HPM are simple and straightforward. The reliability of the method and the reduction in the size of the computational domain gives this method a wider applicability. Also, Homotopy perturbation method is very efficient and powerful tool to get the exact solution over other approximate analytical methods such as Differential transform method DTM, Adomian decomposition method ADM and Homotopy analysis method HAM.
The aim of this study is to investigate the effects of variable viscosity and viscous dissipation on free convective Couette flow ina vertical channel using the Homotopy Perturbation method. Homotopy perturbation method is an efficient tool for solving coupled and non linear system of differential equations. Due to the nonlinearity and coupling of the governing equations in the present problem, the Homotopy perturbation method shall be engaged to obtain the solutions of the energy and momentum equations.

2 Mathematical Analysis

This study considers a steady natural convection flow of a viscous incompressible viscous fluid in a vertical channel of width h. The flow is assumed to be in the x^* - direction which is taken vertically along one of the plates while $y^* -$ axis is taken normal to it. The second plate is placed h distance away from the first. The temperature of the fluid and one of the channel plates are kept at T_0 while the temperature of the plate $y^* = 0$ is raised or fell to T_w and thereafter maintained constant. Also, the plate $y^* = 0$ moves in its own plane impulsively with a uniform velocity $u^* = U$ while the other plate remains at rest. The flow configuration and coordinates system is shown in Figure 1.

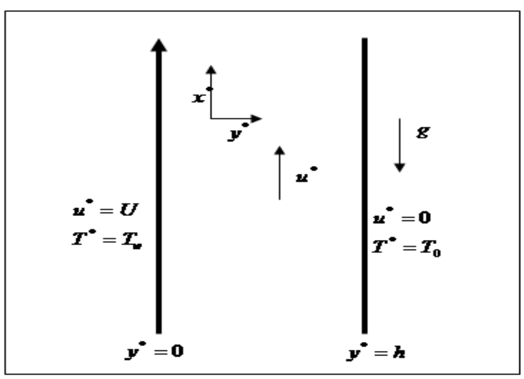


Figure 1: Schematic diagram of the problem

Under the usual assumption of Boussinesq's approximation, the governing dimensional equations of the energy and momentum are:

$$\alpha \frac{d^2T^*}{dy^{*2}} - \frac{Q_0}{\rho c_p}(T^* - T_0) + \frac{\mu^*}{\rho c_p}\left(\frac{du^*}{dy^*}\right)^2 = 0, \tag{1}$$

$$\frac{1}{\rho}\frac{d}{dy^*}\left(\mu^* \frac{du^*}{dy^*}\right) + g\beta(T^* - T_0) = 0. \tag{2}$$

Here T^* and u^* are the dimensional temperature and velocity. x^* and y^* are the dimensional distance perpendicular to the plates, β is the coefficient of thermal expansion, ρ is the density of the fluid, c_p is the specific heat constant pressure, Q_0 is the heat generation/absorption coefficient, μ is the viscosity coefficient and g is the acceleration due to gravity.

Equation (1) is the energy equation: the second term of the equation is the temperature dependent heat generation term while the third term is the viscous dissipation term. In addition, the momentum equation is equation (2) in which the first term shows the variable viscosity effect on velocity of the working fluid. The viscosity of the working fluid is assumed to vary linearly with temperature as follows

$$\mu^* = \mu_0(1 - a(T^* - T_0)).$$

The boundary conditions that satisfy the problem are:

$$u^* = U,\ T^* = T_w \text{ at } y^* = 0,$$

$$u^* = 0,\ T^* = T_0 \text{ at } y^* = h. \tag{3}$$

Due to the nature of the quantities that are given in different dimensions, we introduce some dimensionless quantities that can transform the governing equations and their conditions into dimensionless form. The dimensionless quantities used in equations (1) - (2) and the boundary condition (3) are

$$y = \frac{y^*}{h}, u = \frac{u^*}{U}, T = \frac{T^* - T_0}{T_w - T_0}, S = \frac{Q_0 h^2}{k},$$

$$Ec = \frac{U^2}{c_p(T_w - T_0)}, Gr = \frac{g\beta h^2(T_w - T_0)}{\upsilon U} \tag{4}$$

$$\Pr = \frac{\mu c_p}{k}, \lambda = a(T_w - T_0)$$

By using the dimensionless quantities of equations (1) - (2) and the dimensionless boundary condition equation (3), the governing equations and the boundary conditions are transformed into non-dimensional form as

$$\frac{d^2T}{dy^2} + Ec\,\mathrm{Pr}\left(\frac{du}{dy}\right)^2 - ST = 0, \tag{5}$$

$$(1-\lambda T)\frac{d^2u}{dy^2} - \lambda\frac{du}{dy}.\frac{dT}{dy} + Gr(1-\lambda T)T = 0, \tag{6}$$

And the boundary conditions are

$$\begin{aligned} &u = 1,\ T = 1,\ \text{at } y = 0,\\ &u = 0,\ T = 0,\ \text{at } y = 1. \end{aligned} \tag{7}$$

2.1 Basic Idea of Homotopy Perturbation Method

In order to illustrate the basic ideas Homotopy perturbation method, we consider the following nonlinear differential equation

$$A(u) - f(r) = 0,\ r \in \Omega, \tag{8}$$

With the boundary conditions

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0,\ r \in \Gamma, \tag{9}$$

Where A is a general differential operator, B is a boundary operator, $f(r)$ is known analytical function and Γ is the boundary of the domain Ω, respectively. Generally speaking, the operator A can be divided into two parts which are L and N, where L is linear part and N is nonlinear part. Therefore (8) can be written as:

$$L(u) + N(u) - f(r) = 0, \qquad r \in \Omega \tag{10}$$

By the homotopy techniques, we construct a homotopy as follows

$v(r,p): \Omega \times [0,1] \to R$ which satisfies

$$H(v,p) = (1-p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0, \tag{11}$$

in equation (11), $p \in [0,1]$ is an embedding parameter, while u_0 is an initial approximation of equation (10), which satisfies the boundary conditions. We can assume that the solution of equation (11) can be written as a power series in p:

$$v = v_0 + pv_1 + p^2v_2 + ..., \tag{14}$$

And the best approximation is as follows

$$u = \lim_{p\to 1} v = v_0 + v_1 + v_2 + ..., \tag{15}$$

Applying the Homotopy perturbation technique to solve the governing equations in the present problem, we construct a convex Homotopy on equations. (5) and (6) to get

$$H(T,p) = (1-p)\left(\frac{d^2T}{dy^2}\right) - p\left(\frac{d^2T}{dy^2} + Ec\,\mathrm{Pr}\left(\frac{du}{dy}\right)^2 - ST\right) = 0, \tag{16}$$

$$H(u,p)=(1-p)\left(\frac{d^2u}{dy^2}\right)+p\left(\frac{d^2u}{dy^2}+\lambda T\frac{d^2u}{dy^2}+\lambda\frac{du}{dy}.\frac{dT}{dy}-GrT+\lambda GrT^2\right). \tag{17}$$

Simplify

$$\frac{d^2T}{dy^2}+p\left(\frac{d^2T}{dy^2}+Ec\Pr\left(\frac{du}{dy}\right)^2-ST\right)=0, \tag{18}$$

$$\frac{d^2u}{dy^2}+p\left(\frac{d^2u}{dy^2}+\lambda T\frac{d^2u}{dy^2}+\lambda\frac{du}{dy}.\frac{dT}{dy}-GrT+\lambda GrT^2\right). \tag{19}$$

Assume the solutions of (5) and (6) to be written as

$$T=T_0+pT_1+p^2T_2+...,$$
$$u=u_0+pu_1+p^2u_2+..., \tag{20}$$

Substituting (20) into (18) and (19) and simplifying, we have the following:

$$\frac{d^2T_0}{dy^2}+p\frac{d^2T_1}{dy^2}+p^2\frac{d^2T_2}{dy^2}+....=-p\left(Ec\Pr\left(\frac{du_0}{dy}\right)^2\right)-p^2\left(2Ec\Pr\left(\frac{du_0}{dy}.\frac{du_1}{dy}\right)\right)-...$$
$$+pST_0+p^2ST_1+... \tag{21}$$

$$\frac{d^2u_0}{dy^2}+p\frac{d^2u_1}{dy^2}+p^2\frac{d^2u_2}{dy^2}+....=p\lambda T_0\frac{d^2u_0}{dy^2}+p^2\left(\lambda T_0\frac{d^2u_1}{dy^2}+\lambda T_1\frac{d^2u_0}{dy^2}\right)+...$$
$$+p\lambda\frac{du_0}{dy}.\frac{dT_0}{dy}+p^2\left(\frac{du_0}{dy}.\frac{dT_1}{dy}+\lambda\frac{du_1}{dy}.\frac{dT_0}{dy}\right)+.... \tag{22}$$
$$-pGrT_0-p^2GrT_1-...$$
$$+p\lambda GrT_0^2+p^2(2\lambda GrT_0T_1)+...$$

By comparing the coefficient of p^0, p^1 and p^2 of equation (21) and (22), we have

$$p^0:\frac{d^2T_0}{dy^2}=0 \tag{23}$$

$$p^0:\frac{d^2u_0}{dy^2}=0 \tag{24}$$

$$p^1:\frac{d^2T_1}{dy^2}=-Ec\Pr\left(\frac{du_0}{dy}\right)^2+ST \tag{25}$$

$$p^1:\frac{d^2u_1}{dy^2}=\lambda T_0\frac{d^2u_0}{dy^2}+\lambda\frac{du_0}{dy}.\frac{dT_0}{dy}-GrT_0+\lambda GrT_0^2 \tag{26}$$

$$p^2 : \frac{d^2T_2}{dy^2} = -2Ec\Pr\frac{du_0}{dy}.\frac{du_1}{dy} + ST_1 \tag{27}$$

$$p^2 : \frac{d^2u_2}{dy^2} = \lambda T_0\frac{d^2u_1}{dy^2} + \lambda T_1\frac{d^2u_0}{dy^2} + \lambda\frac{du_0}{dy}.\frac{dT_1}{dy} + \lambda\frac{du_1}{dy}.\frac{dT_0}{dy} - GrT_1 + 2\lambda GrT_0T_1 \tag{28}$$

The boundary conditions are transformed also as

$$u_0(0) = 1, u_1(0) = u_2(0) = u_3(0)... = 0,$$
$$u_0(1) = u_1(1) = u_2(1) = u_3(1)... = 0,$$
$$T_0(0) = 1, T_1(0) = T_2(0) = T_3(0)... = 0, \tag{29}$$
$$T_0(1) = T_1(1) = T_2(1) = T_3(1)... = 0,$$

As the zeroth order of the homotopy gives a linear ordinary differential equation, it is easily solvable without making recourse to initial guess.

Therefore solving (23) and (24) and applying the boundary conditions $T_0(0) = 1$ and $T_0(1) = 0, u_0(0) = 1$, and $u_0(1) = 0$, we obtain (30) and (31) as

$$T_0 = A_1y + A_2 \tag{30}$$

$$u_0 = B_1y + B_2 \tag{31}$$

Solving (25) and (26) and applying the boundary conditions $T_1(0) = 0$ and $T_1(1) = 0$, $u_1(0) = 0$ and $u_1(1) = 0$, we obtain (32) and (33) as

$$T_1 = -\frac{Ec\Pr y^2}{2} + S\left(\frac{y^2}{2} - \frac{y^3}{6}\right) + A_3y + A_4, \tag{32}$$

$$u_1 = \frac{\lambda y^2}{2} + \lambda Gr\left(\frac{y^2}{2} - \frac{y^3}{3} + \frac{y^4}{12}\right) - Gr\left(\frac{y^2}{2} - \frac{y^3}{6}\right) + B_3y + B_4. \tag{33}$$

Solving (27) and (28) and applying the boundary conditions $T_2(0) = 0$ and $T_2(1) = 0$, $u_2(0) = 0$ and $u_2(1) = 0$, we obtain (32) and (33) as

$$\begin{aligned} T_2 = {} & \frac{Ec\Pr\lambda y^3}{3} + 2\lambda Ec\Pr Gr\left(\frac{y^3}{6} - \frac{y^4}{12} + \frac{y^5}{60}\right) - 2Ec\Pr Gr\left(\frac{y^3}{6} - \frac{y^4}{24}\right) \\ & + \frac{Ec\Pr Gry^2}{3} - \frac{Ec\Pr\lambda y^2}{2} - \frac{Ec\Pr\lambda Gry^2}{2} - \frac{Ec\Pr Sy^4}{24} \\ & + S^2\left(\frac{y^4}{24} - \frac{y^5}{120}\right) + \left(\frac{Ec\Pr Sy^3}{12} - \frac{S^3y^3}{18}\right) + A_5y + A_6, \end{aligned} \tag{34}$$

$$\begin{aligned} u_2 = {} & \lambda^2\left(\frac{y^2}{2} - \frac{y^3}{6}\right) + \lambda^2Gr\left(\frac{y^2}{2} - \frac{y^3}{2} + \frac{y^4}{4} - \frac{y^5}{20}\right) - \lambda Gr\left(\frac{y^2}{2} + \frac{y^3}{3} + \frac{y^4}{12}\right) \\ & + \frac{\lambda Ec\Pr y^3}{6} - \frac{\lambda Ec\Pr y^2}{4} - \lambda S\left(\frac{y^3}{6} - \frac{y^4}{24}\right) + \frac{\lambda Sy^2}{6} - \frac{\lambda^2Sy^2}{6} - \frac{\lambda^2y^2}{4} \end{aligned}$$

$$-\lambda^2 Gr\left(\frac{y^3}{6}-\frac{y^4}{24}+\frac{y^5}{60}\right)+\lambda Gr\left(\frac{y^3}{6}-\frac{y^4}{24}\right)-\frac{\lambda Gry^2}{6}+\frac{\lambda^2 Gry^2}{8}+\frac{GrSy^3}{18}$$
$$+\frac{GrEc\Pr y^4}{24}-GrS\left(\frac{y^4}{24}-\frac{y^5}{120}\right)-\frac{GrEc\Pr y^3}{12}-\lambda GrEc\Pr\left(\frac{y^4}{12}-\frac{y^5}{20}\right)$$
$$+2\lambda GrS\left(\frac{y^4}{24}-\frac{y^5}{30}+\frac{y^6}{180}\right)+\lambda GrEc\Pr\left(\frac{y^3}{6}-\frac{y^4}{12}\right)-\frac{2\lambda GrS}{3}\left(\frac{y^3}{6}-\frac{y^4}{12}\right)$$
$$+B_5 y+B_6. \tag{35}$$

Equations (30) - (35) gives the expressions for temperature and velocity as

$$T=T_0+T_1+T_2+..., \tag{36}$$

$$u=u_{.0}+u_1+u_2+.... \tag{37}$$

Where,

$$A_1=B_1=-1,\ A_2=B_2=1,\ A_3=\frac{Ec\Pr}{2}-\frac{S}{3},\ B_3=\frac{Gr}{3}-\frac{\lambda}{2}-\frac{\lambda Gr}{4},$$

$$A_4=B_4=A_6=B_6=0,\ A_5=-\frac{Ec\Pr S}{24}+\frac{S^2}{45}+\frac{\lambda Ec\Pr}{6}+\frac{\lambda Ec\Pr Gr}{20}-\frac{Ec\Pr Gr}{12},$$

$$B_5=-\frac{5\lambda^2}{12}-\frac{9\lambda^2 Gr}{40}+\frac{7\lambda Gr}{24}+\frac{\lambda Ec\Pr}{12}-\frac{\lambda S}{24}+\frac{EcPGr}{24}-\frac{GrS}{45}-\frac{\lambda GrEc\Pr}{20}+\frac{\lambda GrS}{36},$$

To obtain the rate of heat transfer and skin friction at both plates, the expression for temperature and velocity are differentiated with respect to y, that is

$$Nu_0=-1+A_3+A_5 \tag{38}$$

$$Nu_1=-1-Ec\Pr+\frac{S}{2}+A_3-\frac{\lambda Ec\Pr Gr}{2}+\frac{Ec\Pr S}{12}-\frac{S^2}{24}+A_5 \tag{39}$$

$$\tau_0=-1+B_3+B_5 \tag{40}$$

$$\tau_1=-1+\lambda-\frac{Gr}{2}+B_3+\frac{\lambda^2 Gr}{12}-\frac{\lambda^2 S}{3}+\frac{GrS}{24}-\frac{Ec\Pr Gr}{12}+\frac{\lambda Ec\Pr Gr}{12}-\frac{2\lambda GrS}{45}+B_5 \tag{41}$$

We further obtain the mass flux Q by evaluating the integral

$$Q=\int_0^1 u\,dy \tag{42}$$

3 Results and Discussion

The present work analyses the effect of variable viscosity and viscous dissipation on natural convection Couette flow in a vertical channel using Homotopy Perturbation Method. The velocity field, temperature field are presented graphically in Figures 2-11 for various values of Prandtl number $\Pr$, Eckert number Ec, thermal Grashof number Gr, heat generation/absorption parameter S, and variable viscosity λ.

Figures 2 and 3 are display to see the response of temperature and velocity profiles to variation in the values of Prandtl number $\Pr.$ The Prandtl number is the ratio of momentum diffusivity to thermal conductivity. An observation from the figures is that temperature and velocity profile increase with an increase of Prandtl number $\Pr.$ This is due to the physical fact that thermal diffusivity of the working fluid decreases as the Prandtl number increases. It is therefore an impedance to the diffusion of heat generated by the viscous dissipation at every fluid section within the channel. Consequently, there is heat accumulation and hence fluid temperature increases with growing Prandtl number.

Figures 4 and 5 show the influence of viscous dissipation on the fluid temperature and velocity. Eckert number is the ratio of the kinetic energy of the flow to the boundary layer enthalpy difference. It is noticed that fluid temperature and velocity increase with an increase in viscous dissipation. Also, greater viscous dissipative heat causes rise in the temperature as well as velocity and as consequences greater buoyancy force so that fluid velocity increases as Ec increases.

Figures 6 and 7 show representatively the effect of Grashof number Gr on the fluid temperature and velocity within the channel. It is clear that an increase in the thermal buoyancy both temperature and velocity profile increase. This is physically expected since the convection current grows.

The effect of heat generation/absorption S on the fluid temperature and velocity is presented in figures 8 and 9 for fixed values of $\Pr = 0.71, Ec = 0.6, Gr = 5.0$ and $\lambda = -0.3$. It is noticed that as the heat generation $(S < 0)$ increases, fluid temperature and velocity increase while it decreases with increase in heat absorption $(S > 0)$. Increasing the heat generation parameter causes the fluid temperature to increase and it strengthens the convection current within the channel. This is because when heat is observed, the buoyancy forces and thermal diffusivity of the fluid increase which accelerate the flow rate and thereby give rise to an increase in the velocity as well as temperature profile. In addition, it is interesting to note that increasing the heat absorption parameter $(S > 0)$ causes the fluid to cool and the thermal boundary layer becomes thinner, thereby reducing thermal buoyancy effect and hence reduces the velocity distribution of the fluid as shown in Figure 9.

The effects of viscosity variation on the fluid temperature and velocity are plotted in figures 10 and 11 respectively. It is observed that an increase in the fluid viscosity leads to increase in temperature profile as shown in figure 10. On the other hand, decreasing the viscosity contributes a decrease in the temperature of the working fluid. This is attributed to change in the sheared heating that characterizes the increases/decrease in fluid viscosity.

The rate of heat transfer, skin friction and mass flux for different values of Prandtl number, heat generation/absorption, variable viscosity, Eckert number and Grashof number are simulated and presented in Table 1-3. To validate the present problem with the work of Jha and Ajibade (2010), we plot the table 4 to make the comparison the present problem and that of Jha and Ajibade (2010).

The rate of heat transfer on the surface of the boundary plate is simulated and presented in Table 1. It is clearly seen from the table that the rate of heat transfer decreases on the heated plate while it increases on the cold plate as a result of Prandtl number increases. This is due to the temperature increases with the increase in Prandtl number leading to a decrease in the temperature gradient on the heated plate and the opposite trend also discovered on the cold plate. Furthermore, heat absorption leads to increase in heat transfer on the heated wall and this is due to temperature decreases caused by growing heat absorption which consequently leads to increase in the rate of heat transfer on the heated wall. This is physically possible since fluid temperature increase with heat generation and this decrease the temperature gradient on the heated wall while heat transfer to the cold wall increases. In addition, increasing the viscosity contributes a decrease in the heated plate and the reverse trend is observed on the cold plate.

Table 2 represents the skin friction on the surface of boundary plates. A general view of this table indicates that the skin friction is higher when the working fluid is air as compared to that of mercury. This is physically expected since fluid velocity increases with increase in Prandtl number causing an increase in the skin friction. However, growing viscosity contributes a decrease on the heated plate and it increases on the cold plate. The table further shows that increasing viscous dissipation have tendency to increase the skin friction on both plates.

Table 3 reveals the mass flux on both plates. It is observed that the mass flux increases with the increase in heat generation and decreases with the increasing heat absorption. Moreover, mass flux increases with the increasing viscous dissipation. The table further shows that the mass flux decreases with growing viscosity.

4 Validation

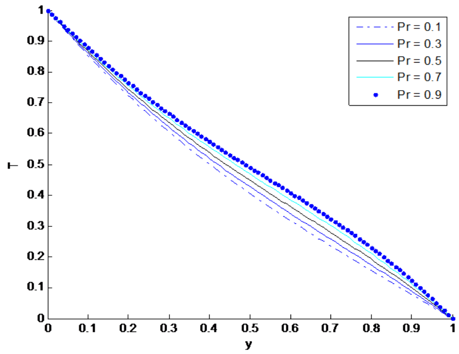
To validate this work, we have compared our results with the existing results of Jha and Ajibade (2010) in the absence of variable viscosity and viscous dissipation. Our results are in good agreement with the existing results (see table 4) which shows that the Homotopy perturbation method is an efficient tool for solving coupled and nonlinear system of differential equations.

5 Conclusions

In this paper investigates the effects of variable viscosity and viscous dissipation on free convective Couette flow in a vertical channel. From the analysis, it is clearly observed that the variable viscosity parameter λ, viscous dissipation parameter Ec, heat generation/absorption parameter S, thermal Grashof number parameter Gr and Prandtl number $\Pr$ have substantial effects on temperature and velocity profile within the boundary layer. The work concluded as:

1. The increasing values of viscous dissipation increase both fluid temperature and velocity.
2. Both temperature and velocity increase as thermal buoyancy parameter increases.
3. Also heat generation increases fluid temperature and velocity increase while it decreases with increase in heat absorption.
4. An increase in the fluid viscosity leads to increase in temperature profile.
5. Temperature and velocity profile increase with the increase of Prandtl number.

When the variable viscosity and viscous dissipation are neglected in this work, there is an excellent agreement with the result of Jha and Ajibade (2010).



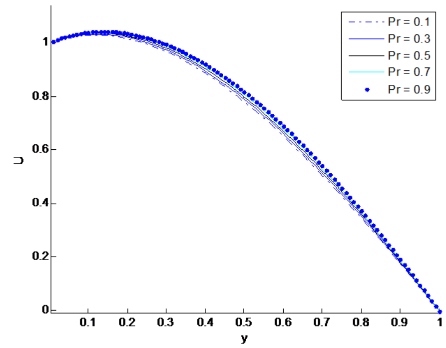
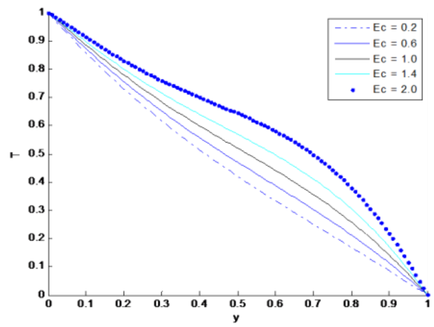


Figure 2: Temperature profile for different values Fig 3: Velocity profile for different values of $Pr(S = 2.0, \mathrm{Ec} = 0.6, \mathrm{Gr} = 5.0, \lambda = -0.3)$ $Pr(S = 2.0, \mathrm{Ec} = 0.6, \mathrm{Gr} = 5.0, \lambda = -0.3)$



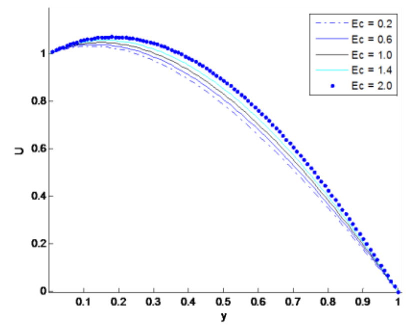
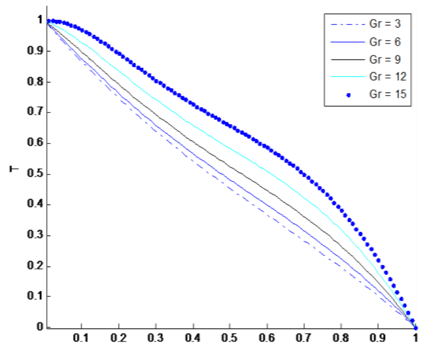


Figure 4: Temperature profile for different values Fig 5: Velocity profile for different values of $Ec(Pr = 0.71, S = 2.0, \mathrm{Gr} = 5.0, \lambda = -0.3)$ $Ec(Pr = 0.71, S = 2.0, \mathrm{Gr} = 5.0, \lambda = -0.3)$



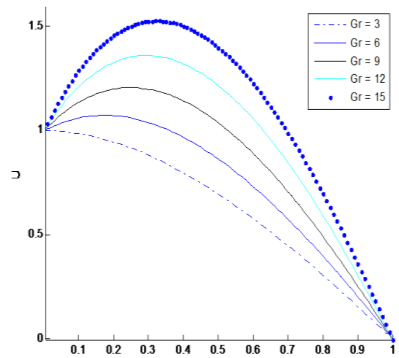


Figure 6: Temperature profile for different values Fig 7: Velocity profile for different values of $Gr(Pr = 0.71, S = 2.0, \mathrm{Ec} = 0.6, \lambda = -0.3)$ $Gr(Pr = 0.71, S = 2.0, \mathrm{Ec} = 0.6, \lambda = -0.3)$

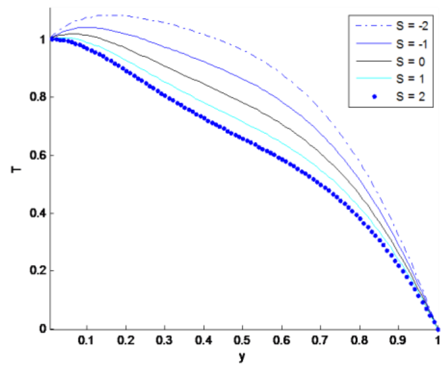


Figure 8: Temperature profile for different values of $S(Pr = 0.71, \mathrm{Ec} = 0.6, \mathrm{Gr} = 5.0, \lambda = -0.3)$

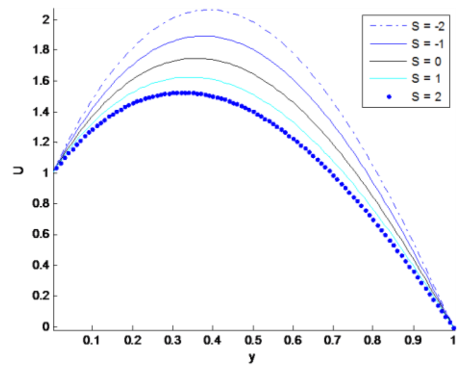


Fig 9: Velocity profile for different values $S(Pr = 0.71, \mathrm{Ec} = 0.6, \mathrm{Gr} = 5.0, \lambda = -0.3)$

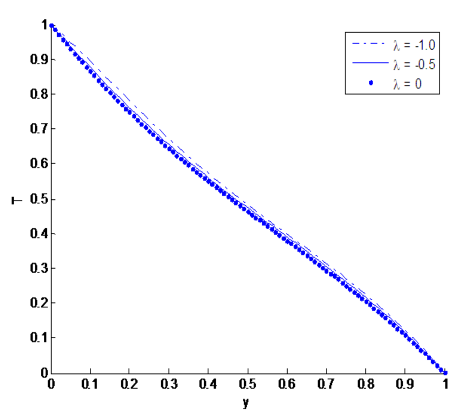


Figure 10: Temperature profile for different values of $\lambda(Pr = 0.71, S = 2.0, \mathrm{Ec} = 0.6, \mathrm{Gr} = 5.0)$

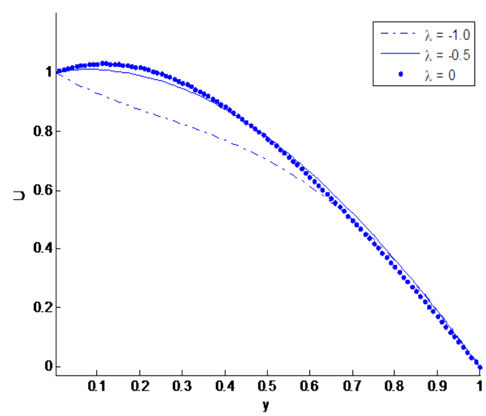


Fig 11: Velocity profile for different values $\lambda(Pr = 0.71, S = 2.0, \mathrm{Ec} = 0.6, \mathrm{Gr} = 5.0)$

Table 1: Estimated numerical values of rate of heat transfer Nu_0 and Nu_1

	S	Nu_0	Nu_1	Nu_0	Nu_1	Nu_0	Nu_1
	$Ec = 0.4, Gr = 5.0$		$Ec = 0.6, Gr = 5.0$			$Ec = 0.6, Gr = 5.0$	
			$\lambda = -0.2$	$\lambda = -0.2$		$\lambda = -0.1$	
	-1.0	0.64371	1.35104	0.64334	1.43351	0.64224	1.53891
	-0.5	0.82741	1.24166	0.82723	1.31839	0.82613	1.42379
$\Pr = 0.044$	0.5	1.16148	1.05206	1.16166	1.11733	1.16056	1.22273
	1.0	1.31184	0.97184	1.31221	1.03138	1.31111	1.13678
	-1.0	0.63261	1.33994	0.62669	1.41686	0.60894	1.50561
	-0.5	0.82186	1.23611	0.81890	1.31007	0.80115	1.39882
$\Pr = 0.71$	0.5	1.16703	1.05761	1.16999	1.12565	1.15224	1.21440
	1.0	1.32294	0.98294	1.32886	1.04803	1.31111	1.13678

Table 2: Estimated numerical values of skin friction τ_0 and τ_1

	S	τ_0	τ_1	τ_0	τ_1	τ_0	τ_1
	$Ec = 0.4, Gr = 5.0$ $\lambda = -0.2$		$Ec = 0.6, Gr = 5.0$ $\lambda = -0.2$			$Ec = 0.6, Gr = 5.0$ $\lambda = -0.1$	
	-1.0	0.88968	2.71996	0.89256	2.80261	0.85362	2.43223
	-0.5	0.81151	2.65463	0.81414	2.73727	0.78697	2.37371
$\Pr = 0.044$	0.5	0.65936	2.52396	0.66189	2.60661	0.65568	2.25669
	1.0	0.58499	2.45863	0.58748	2.54128	0.59086	2.19817
	-1.0	0.97136	2.64271	1.01550	2.68673	0.95623	2.33333
	-0.5	0.89157	2.57737	0.93482	2.62139	0.88843	2.27481
$\Pr = 0.71$	0.5	0.73659	2.44671	0.77828	2.49073	0.75514	2.15778
	1.0	0.66098	2.38137	0.70199	2.42539	0.68944	2.09927

Table 3: Estimated numerical values of mass flux Q

	S	Q	Q	Q
	$Ec = 0.4, Gr = 5.0$ $\lambda = -0.2$	$\lambda = -0.2$	$Ec = 0.6, Gr = 5.0$ $\lambda = -0.1$	$Ec = 0.6, Gr = 5.0$
	-1.0	0.71776	0.70877	0.70585
	-0.5	0.70737	0.69839	0.69747
$\Pr = 0.044$	0.5	0.68663	0.67764	0.68070
	1.0	0.67626	0.66726	0.67232
	-1.0	0.74995	0.75705	0.75080
	-0.5	0.73958	0.74668	0.74242
$\Pr = 0.71$	0.5	0.71882	0.72592	0.72566
	1.0	0.70845	0.71555	0.71227

Table 4: Numerical comparison between the work of Jha and Ajibade (2010) and the present problem

	Jha and Ajibade (2010) $Gr = 9.0, y = 0.5$		Present work $Gr = 9.0, \Pr = 0.71, y = 0.5, Ec = \lambda = 0$	
S	**Temperature**	**Velocity**	**Temperature**	**Velocity**
-1.0	0.56975	1.12772	0.56967	1.12705
-0.5	0.53296	1.09337	0.53296	1.09329
0.5	0.47030	1.03462	0.47029	1.03469
1.0	0.44341	1.00932	0.44335	1.00986

Nomenclature

g -acceleration due to gravity[ms^{-2}]
h-width of the channel [m]
Q_0 - heat generation/absorption coefficient $\left[Kgm^{-1}s^{-3}K^{-1}\right]$
T^*- dimensional fluid temperature $[K]$
T_w^* - channel wall temperature $[K]$
T_0^* - temperature of the ambience $[K]$
T - dimensionless fluid temperature
u^* - dimensional velocity [ms^{-1}]
u-dimensionless velocity
U- dimensional velocity of the moving plate[ms^{-1}]
y^* -co-ordinate perpendicular to the plate [m]
y - dimensionless co-ordinate perpendicular to the plate
β -coefficient of thermal expansion $\left[K^{-1}\right]$
μ -coefficient of viscosity $\left[Kgm^{-1}s^{-1}\right]$
ν-kinematic viscosity [m^{2}s^{-1}]
λ - variable viscosity
S - dimensionless heat generation/absorption parameter
$\Pr$ - Prandtl number
Gr - Grashof number
Ec - Eckert number
c_p - specific heat at constant pressure $\left[m^2s^{-2}K^{-1}\right]$
ρ - density of the fluid $\left[Kgm^{-3}\right]$
α - thermal diffusivity of the fluid $\left[Kgm^{-3}\right]$
p -embedding parameter

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