APPLICATION OF HOMOTOPY PERTURBATION METHOD TO HEAT TRANSFER IN NANOFLUID

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Abstract

Homotopy perturbation method is applied to study heat transfer in Nanofluid in the presence of thermal radiation. The advantage of the HPM is that it eliminates the limitation of perturbation method, hence, it is applicable in the absence of a small parameter. The governing equations are formulated based on already existing model and are transformed to ordinary differential equations using stream function and similarity variables. The resulting dimensionless equations are then solved by Homotopy perturbation method. The effects of some governing parameters on the rate of heat transfer are discussed with the help of a table generated. The rate of heat transfer increases with increase in solid volume fraction parameter, magnetic field parameter and Grashof number parameter for different nanoparticles.

***Keywords**: Nanofluid, Magnetohydrodynamics, Homotopy perturbation method, heat transfer and Radiation.*

Nomenclature

a	=	Constant
g	=	Acceleration due to gravity
k	=	Thermal Conductivity
P_r	=	Prandtl Number
T	=	Fluid Temperature
T_w	=	Surface Temperature
T_∞	=	Free Stream Temperature
u,v	=	Velocity Components
x,y	=	Cartesian Coordinates
f(x)	=	Dimensionless Stream Function
G_r	=	Grash of Number
q_r	=	Heat Flux Radiation
B_o	=	Magnetic Field of Constant Strength
R	=	Radiation Parameter
K_s	=	Rosseland Mean Absorption Coefficient
K	=	Thermal Conductivity Coefficient

Greek Symbols

β	=	Thermal Expansion Coefficient
μ	=	Dynamic Coefficient of Viscosity

$\theta(\eta)$ = Dimensionless Temperature
η = Similarity Variable
ρ = Fluid Density
ψ = Stream Function
σ = Stefan-Bottzman Constant

Introduction

Nanofluid is formed with combination of base fluid and nanoparticles. When nanoparticles such as Aluminium Oxide (Al_2O_3), Copper (Cu), Copper Oxide (CuO), Gold (Au), Silver (Ag), Silica particles , e.t.c are mixed with base fluids such as water, oil, acetone, ethylene, etc., nanofluids are formed. The spectacular capability of nanofluids heat transfer/removal enhancement can properly address the energy demand and emission issues of the present world. Utilization of nanofluids for industrial cooling could result in great energy savings and emission reduction. The discovery of nanofluids in enhancement heat transfer/removal and utilization for industrial cooling has drawn the attention of both scholars and industrialists to make researches into this area, Wang & Choi, (1999) studied the thermal conductivity of nanoparticle fluid mixture containing Al_2O_3 and CuO nanoparticles and showed that the thermal conductivity of nanofluids increased with increasing volume fraction of the nanoparticles Kyu, Do & Jang (2010) examined the effect of nanofluids on the thermal performance of a flat micro heat pipe with rectangular grooved wick: Uddin, Kahn & Ismail, (2012) investigated free convective flow of a nanofluid past a vertical plate with Newtonian heating. Similarity transformation was applied and the resulting equations were solved using the Runge-Kutta-Fehlberg method coupled with shooting technique.

Pourmehran et al (2016) investigated the effects of buoyancy and thermal radiation on a nanofluid over a stretching sheet with an external magnetic field applied. The flow equations were solved using a 4^{th}-order Runge-kutta scheme coupled with Shooting Iteration. Zeinali, Salehi&Noie,(2012) investigated the effect of magnetic field of various strengths on the thermal performance of Silver-Water nanofluid.

Hamad (2011) studied natural convection flow of a nanofluid over linearly sretching sheet in the presence of magnetic field. Oahimire, Bazuaye & Harry (2016), extended the work of Hamad by incorporating radiation term to have a differential equations that model the effect of thermal radiation on heat transfer of a nanofliud and applied Runge-kuttaFehlberg method and shooting technique to obtain solution for analysis. To the best of our knowledge, HPM has not be applied to the extension of Hamad (2011) by Oahimire, Bazuaye & Harry, (2016). In his present study, we applied HPM to study the effects of volume fraction, magnetic field and buoyancy force on the rate of heat transfer of natural convection flow of a nanofluid over linearly sretching sheet in the presence of magnetic field

The Homotopy Perturbation method (HPM) is a technique based on the concept of the Homotopy and perturbation method that was introduced in topology by Dr. Ji-Huan He in 1998. The basic idea can be illustrated by considering the following non-linear differential equation:
$A(u) - f(r) = 0, r \in \cap$ (1)
Subject to the boundary condition:
$B\left(u, \frac{\partial u}{\partial n}\right) = 0, r \in \Gamma$ (2)

Where A is a general operator, B is the boundary operator, $f(r)$ is a known analytic function and Γ is the boundary of the domain $\cap$ and $\frac{\partial u}{\partial n}$ denotes differentiation along the normal drawn outward from $\cap$. A can be divided into two parts which are linear $L(u)$ and non-linear $N(u)$. By the homotopy perturbation method, we construct $V(r,p)$:$\cap \times [0,1] \to R$ which satisfies:

$H(V,p) = (1-p)[L(V) - L(u_0)] + [A(V) - f(r)] = 0$ (3)

where $p\epsilon[0,1]$ is an embedding parameter or homotopy parameter and u_0 is an initial approximation that satisfies the boundary condition or from other part of the original equation. From (3), we have

$H(V,0) = L(V) - L(u_0) = 0$ (4)

$H(V,1) = A(V) - f(r) = 0$ (5)

The changing of p from zero to one is just that of $V(r,p)$ changing from u_0 to $u(r)$ and the process is called deformation in topology.

According to the HPM, we can first use the embedding parameter as small parameter and assume

$$V = V_0 + pV_1 + p^2V_2 + O(p^3)$$

Then setting $p = 1$, we have

$$u = \frac{limit}{p \to 1} = V_0 + V_1 + V_2$$

Which is the approximate solution to equation (1). In most cases, the series solution is a convergent one which leads to exact solution. Banach's fixed point theorem can be applied for studying convergence of the series. In the case of constructing the homotopy in terms of operator that will lead to direct integration method, the operator should be chosen in a way that it should be easy to handle. It is strongly recommended for beginners to choose linear operator but not restricted to linear operator only.

Mathematical Formulation

Consider two-dimensional flow of an incompressible viscous nanofluid past a linearly semi-infinite stretching sheet. Magnetic field of strength B_0 is applied normally to the sheet as shown in the Figure 1.

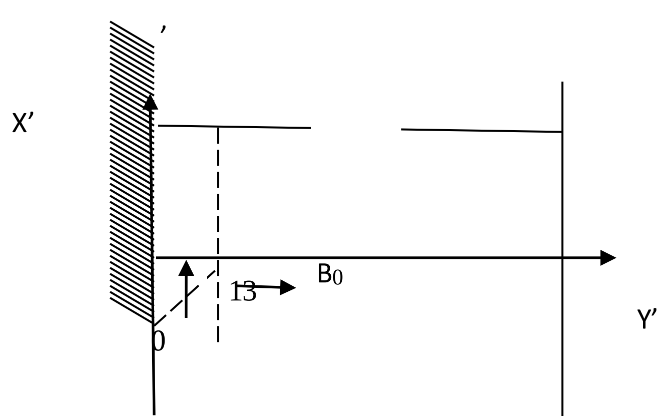

Figure 1: Physical model

A water based nanofluid containing Al_2O_3, Cu, TiO_2 and Ag is used with the assumption that both the fluid and the nanoparticles are in thermal equilibrium. Based on the already existing model of Hamad (2011) extended by Oahimire *et al* (2016), the governing equations are:

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \quad (6)$$

$$\rho_{nf}\left[u'\frac{\partial u'}{\partial x'} + v'\frac{\partial v'}{\partial y'}\right] = \mu_{nf}\frac{\partial^2 u'}{\partial y'^2} - \sigma B_0 u' + g\beta_t (T' - T'_\infty) \quad (7)$$

$$(\rho c_p)_{nf}\left[u'\frac{\partial T'}{\partial x'} + v'\frac{\partial T'}{\partial y'}\right] = K_{nf}\frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} \quad (8)$$

The boundary conditions of the equations are

$$u' = u'_w(x') = ax', v' = 0,\ T' = T'_w \ at\ y' = 0$$

$$u' \to 0, T' \to T'_\infty, y' \to \infty \quad (9)$$

Where q_r is the radiative heat flux, T' is the temperature of the fluid, x' and y' are the coordinates along and perpendicular to the sheet while u' and v' are the velocity components in the x' and y' directions respectively and a is a constant.

The effective density (ρ_{nf}), effective dynamic viscosity (μ_{nf}), heat capacitance $(\rho C_p)_{nf}$ and the effective thermal conductivity (k_{nf}) of the nanofluid, in that order, are given as

$$\rho_{nf} = (1-A)\rho_f + A\rho_s$$

$$\mu_{nf} = \frac{\mu_f}{(1-A)^{2.5}}$$

$$(\rho C_p)_{nf} = (1-A)(\rho C_p)_f + A(\rho C_p)_s \quad (10)$$

$$k_{nf} = k_f\left\{\frac{k_s + 2k_f - 2A(k_f - k_s)}{k_s + 2k_f + 2A(k_f - k_s)}\right\}$$

Where A is the solid volume fraction ($A \neq 1$), μ_f is the dynamic viscosity of the base fluid, while ρ_f and ρ_s are the densities of the pure fluid and the nanoparticle respectively. The constants k_f and k_s are the thermal conductivities of the base fluid and the nanoparticle respectively. Using Rosseland approximation given by $q_r = \frac{4\sigma'}{3k'}\frac{\partial T'^4}{\partial y'}$ with Taylor's series expansion and differentiation, equation (8) becomes

$$(\rho c_p)_{nf}\left[u'\frac{\partial T'}{\partial x'} + v'\frac{\partial T'}{\partial y'}\right] = K_{nf}\frac{\partial^2 T'}{\partial y'^2} + \frac{16T_\infty^3\sigma'}{3k'}\frac{\partial^2 T'}{\partial y'^2} \quad (11)$$

Introducing the following variables for transformation

$$u = \frac{u'}{\sqrt{av_f}} v = \frac{v'}{\sqrt{av_f}}, \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, x = \frac{x'}{\sqrt{\frac{v_f}{a}}} . y = \frac{y'}{\sqrt{\frac{v_f}{a}}} \quad (12)$$

Equation (12) transform equation (6),(7) and (11) to the followings

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (13)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{(1-A)\rho_f+\rho_s}\left(\frac{1}{(1-A)^{2.5}}\frac{\partial^2 u}{\partial y^2}-Mu+G_r\theta\right) \quad (14)$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{p_r}\frac{1}{(1-A)(\rho C_p)_f+A(\rho C_p)_s}\left(\frac{k_{nf}}{k_f}+R_d\right)\frac{\partial^2 \theta}{\partial y^2} \quad (15)$$

Where $M=\frac{\sigma\beta_0}{a}$ is the magnetic field parameter, $p_r=\frac{v_f}{\mu_{nf}}$is the Prandtl number, $R_d=\frac{16\sigma' T_\infty'^3}{3K^*\mu_{nf}}$ is the radiation parameter,

$G_r=\frac{g\beta_t(T_w'-T_\infty')x}{a}$ is the Grashof number

And the corresponding boundary conditions is

$$\left.\begin{aligned}&u=x, v=0, \theta=1 \ at\ y=0\\ &u\to 0, \theta\to 0\ as\ y\to\infty\end{aligned}\right\} \quad (16)$$

Using $u=\frac{\partial\psi}{\partial y}, V=-\frac{\partial\psi}{\partial x}$, $\eta=y, \psi=xf(\eta), \theta=\theta(\eta)$ (17)

$$f'''+(1-A)^{2.5}\left\{ff''-(f')^2\right\}\left[(1-A)\rho_f+\rho_s)-(Mf'+G_r\theta)\right]=0 \quad (18)$$

$$\frac{1}{P_r}\frac{1}{(1-A)(\rho C_p)_f+A(\rho C_p)_s}\left[\frac{k_{nf}}{k_f}+R_d\right]\theta''(\eta)+f(\eta)\theta'(\eta)=0 \quad (19)$$

$$\left.\begin{aligned}&f(0)=0,\ \ f'(0)=1\ at\ \ \eta=0\\ &f\to 0\ at\ \eta\to\infty.\\ &\theta(0)=1\ and\ \theta(\infty)=0,\end{aligned}\right\} \quad (20)$$

Solution by Application of HPM

The transformed non-linear equations can be written as

$$f'''+\alpha\big(\beta(ff''-(f')^2)\big)-kf'+Gr\theta=0$$

$$f\theta'+H\theta''=0$$

Where $\alpha=(1-A)^{2.5}$,$k=M$,$\beta=(1-A)\rho_f+\rho_s$,

$$H=\frac{1}{Pr}\frac{1}{(1-A)(\rho c_p)_f+A(\rho c_p)_s}\left(\frac{k_{nf}}{k_f}+R\right)$$

We construct the homotopy of the transformed equations as follows

$$(1-p)(f'''-f'''u_0)+p\big(f'''+\alpha\big(\beta(ff''-(f')^2)\big)-kf'+Gr\theta\big)=0$$

And

$$(1-p)(\theta''-\theta''t_0)+p(f\theta'+H\theta'')=0$$

We assume f and θ in the following form

$$f = f_0 + pf_1 + p^2 f_2$$
$$\theta = \theta_0 + p\theta_1 + p^2\theta_2$$

and group the terms according to the order:
For order zero, we have

$$\frac{d^3 f_0}{d\eta^3} - \frac{d^3 v_0}{d\eta^3} = 0$$
$$\frac{d^2\theta_0}{d\eta^2} - \frac{d^2 t_0}{d\eta^2} = 0$$

With boundary conditions

$$f_0(0) = 0, f_0{}'(0) = 0, f_0{}'(\infty) = 1, \theta_0(0) = 1, \theta_0(\infty) = 0$$

For order one, we have

$$\frac{d^3 f_1}{d\eta^3} - \frac{d^3 v_0}{d\eta^3} + \alpha\left(Gr\theta_0 - k\frac{df_0}{d\eta} + \beta\left(f_0\frac{d^2 f_0}{d\eta^2} - \left(\frac{df_0}{d\eta}\right)^2\right)\right) = 0$$
$$\frac{d^2\theta_1}{d\eta^2} + f_0\frac{d\theta_0}{d\eta} + (H-1)\frac{d^2\theta_0}{d\eta^2} + \frac{d^2 t_0}{d\eta^2} = 0$$

With boundary conditions

$$f_1(0) = 0, f_1{}'(0) = 0, f_1{}'(\infty) = 0, \theta_1(0) = 1, \theta_1(\infty) = 0$$

For order two, we have

$$\frac{d^3 f_2}{d\eta^3} + \alpha\left(Gr\theta - k\frac{df_1}{d\eta} + \beta\left(f_1\frac{d^2 f_1}{d\eta^2} - \left(\frac{df_1}{d\eta}\right)^2\right)\right) = 0$$
$$\frac{d^2\theta_2}{d\eta^2} + f_1\frac{d\theta_1}{d\eta} + (H-1)\frac{d^2\theta_1}{d\eta^2} = 0$$

With boundary conditions

$$f_2(0) = 0, f_2{}'(0) = 0, f_2{}'(\infty) = 0, \theta_2(0) = 1, \theta_2(\infty) = 0$$

Solving the equations with their respective boundary conditions, we have the following solutions

$$f_0 = \frac{\eta^2}{12}$$
$$f_1 = \frac{c_1\eta^2}{2} - \alpha Gr\frac{\eta^3}{6} + \alpha(Gr + k)\frac{\eta^4}{144} + \alpha\beta\frac{\eta^5}{4320}$$

$$f_2 = c_3\frac{\eta^2}{2} + g_8\frac{\eta^4}{24} + g_9\frac{\eta^5}{60} + g_{10}\frac{\eta^6}{120} + g_{11}\frac{\eta^7}{210} + g_{12}\frac{\eta^8}{336} + g_{13}\frac{\eta^9}{504} + g_{14}\frac{\eta^{10}}{720} + g_{15}\frac{\eta^{11}}{990}$$
$$\theta_0 = 1 - \frac{\eta}{6}$$
$$\theta_1 = \frac{\eta^4}{864} - \frac{\eta}{4}$$
$$\theta_2 = c_2\eta + \frac{g_1}{4}\eta^4 - \frac{g_2}{5}\eta^5 + \frac{g_3}{6}\eta^6 + \frac{g_4}{7}\eta^7 + \frac{g_5}{8}\eta^8 - \frac{g_6}{9}\eta^9 - \frac{g_7}{10}\eta^{10}$$

where

$$c_1 = 2\alpha Gr - \alpha k - \frac{\alpha\beta}{4}$$

$$c_2 = -\frac{216}{4}g_1 + \frac{1296}{5}g_2 - 1296g_3 - \frac{46656}{7}g_4 - \frac{279936}{8}g_5 + \frac{1679616}{9}g_6 + \frac{10077696}{10}g_7$$

$$c_3 = -(6g_8 + 18g_9 + \frac{1296}{20}g_{10} + \frac{7776}{30}g_{11} + \frac{46656}{42}g_{12} + \frac{279936}{56}g_{13} + \frac{1679616}{72}g_{14} + \frac{10077696}{90}g_{15})$$

$$g_1 = \frac{c_1}{4} - \frac{(H-1)}{72}$$

$$g_2 = \frac{\alpha Gr}{24}$$

$$g_3 = \frac{\alpha(Gr + k)}{576}$$

$$g_4 = \frac{\alpha\beta}{17280} - \frac{c_1}{432}$$

$$g_5 = \frac{\alpha Gr}{1296}$$

$$g_6 = \frac{\alpha(Gr + k)}{144}$$

$$g_7 = \frac{\alpha\beta}{933120}$$

$$g_8 = c_1\alpha k + \frac{\alpha Gr}{4}$$

$$g_9 = c_1\alpha\beta - \frac{\alpha^2 GrK}{2} - \frac{\alpha\beta\, {C_1}^2}{2}$$

$$g_{10} = \frac{\alpha^2 K(Gr + K)}{36} - \frac{\alpha^2 C_1 \beta\, Gr}{3}$$

$$g_{11} = \frac{\alpha^2 K\,\beta - \alpha Gr}{864} + \alpha\beta\left(\frac{c_1\alpha(Gr + k) + 12\alpha^2 Gr^2}{144}\right)$$

$$g_{12} = \alpha\beta\left(\frac{4C_1\alpha\beta}{2160} - \frac{\alpha^2 Gr(Gr + K)}{144}\right)$$

$$g_{13} = \alpha\beta\left(\frac{\alpha^2(Gr + K)^2}{5184} + \frac{\alpha^2 Gr\beta}{270}\right)$$

$$g_{14} = \alpha\beta\left(\frac{\alpha^2\beta(Gr + K)}{77760}\right)$$

$$g_{15} = \frac{\alpha^3\beta^3}{746496}$$

Results

The thermo physical properties of pure water and those of the nanoparticles as given by Hamad (2011) will now be substituted.

Table 1: Thermo physical properties of water and nanoparticles Hamad (2011)

Compound	ρ (kg/m^3)	C_p(J/kgK)	k (W/mK
Pure water	997.1	4179	0.613
Copper (Cu)	8933	385	401
Alumina (Al_2O_3)	3970	765	40
Silver (Ag)	10500	235	429
Titanium Oxide (TiO_2)	4250	686.2	8.9538

Substituting the thermo physical properties and evaluation using Matlab, generate the following table for the rate of heat transfer.

Table 2: Variation of the rate of heat transfer with different values of A, M and Gr

			$-\theta(0)$	$-\theta(0)$	$-\theta(0)$	$-\theta(0)$
A	M	Gr	Cu	Al_2O_3	Ag	TiO_2
0.2	0.5	0.2	-6.3599×10^3	-2.8533×10^3	-7.4671×10^3	-3.0511×10^3
0.3	0.5	0.2	-4.5049×10^3	-1.9933×10^3	-5.2976×10^3	-2.1350×10^3
0.4	0.5	0.2	-3.0301×10^3	-1.3219×10^3	-3.5695×10^3	-1.4183×10^3
0.5	0.5	0.2	-1.8996×10^3	-816 6862	-2.2415×10^3	-877 7803
0.6	0.5	0.2	-1.0754×10^3	-455 5434	-1.2711×10^3	-490 5157
0.1	0.6	0.2	-8.5324×10^3	-3.8888252×10^3	-1.0019×10^4	-4.0908×10^3
0.1	0.7	0.2	-8.4332×10^3	-3.7259×10^3	-9.9194×10^3	-3.9915×10^3
0.1	0.8	0.2	-8.3339×10^3	-3.6266×10^3	-9.8201×10^3	-3.8922×10^3
0.1	0.9	0.2	-8.2346×10^3	-3.5274×10^3	-9.7209×10^3	-3.7929×10^3
0.1	1.0	0.2	-8.1353×10^3	-3.4281×10^3	-9.6216×10^3	-3.6937×10^3
0.1	0.5	0.3	-8.5332×10^3	-3.8260×10^3	-1.0019×10^4	-4.0916×10^3
0.1	0.5	0.4	-8.4348×10^3	-3.7275×10^3	-9.9210×10^3	-3.9931×10^3
0.1	0.5	0.5	-8.3363×10^3	-3.6290×10^3	-9.8225×10^3	-3.8946×10^3
0.1	0.5	0.6	-8.2378×10^3	-3.5306×10^3	-9.7241×10^3	-3.7961×10^3
0.1	0.5	0.7	-8.1393×10^3	-3.4321×10^3	-9.6256×10^3	-3.6977×10^3

Discussion

Numerical evaluation of the solutions was performed with mathematical software "Matlab" and the results are presented in tabular form. This was done to illustrate effect of some governing parameters involved. The rate of heat transfer for different value of volume fraction (A), magnetic parameter(M) and Grashof number(Gr) are obtained as shown in Table 2. As can be depicted from the table, Increase in the values of A, M and Gr led to increase in the values of the heat transfer coefficient $-\theta(0)$. That is to say, increase in solid volume fraction, magnetic field and Grashof number increases the rate of heat transfer for different nanoparticles in the presence of thermal radiation.

Conclusion

In this work, the dimensionless equations of the governing equations were solved with HPM. And we notice that M, Gr and A increases the rate of heat transfer in the presence of thermal radiation irrespective of the nanoparticles.

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