DYNAMICS OF VIRAL MARKETING ON NETWORKS

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Abstract
Viral marketing is the process of marketing products and services through internet. In this paper, a network of consumers is constructed. The dissemination of information about products and services is simulated via the network. The results show that viral marketing is an effective means for promoting and increasing sales of products and services.

Keywords: *viral marketing, social networks, internet.*

1. Introduction

Viral marketing ***i***s a marketing strategy that focuses on spreading information and opinions about a product orservice from person to person, especially by using unconventional means such as the Internet or email [1].Viral marketing can be thought of as a diffusion of information about the product and its adoption over the network. Viral marketing exploits existing social networks by encouraging customers to share product information with their friendsand campaigns with their friends, through email or social Networks [2, 3].

Viral marketing utilizes social networks to proliferate the publicity of a product or brand, thereby generating more sales [4].

One way to encourage positive viral marketing is by distributing reduced or free products to target consumers who will then discuss the product with their friends and encourage those friends to buy the product [5] or by providing other forms of incentives.

According to [6], all viral marketing campaigns are based on one of three models namely, thepromotions model, the incentive-based model and the loyalty-based model. Jokes, games and competitions rank highest in the viral marketing stakes. These fall into the category of viral marketing that for argument's sake,can be called the *promotions model*. This model creates enough interest that consumers arewilling to pass along the content to their friends, without receiving any incentives for doingso. According to Wilding, the trick is that the content must in some way enhance ordemonstrate the personality of the sender or match the personality of the receiver so closelythat the sender is compelled to bring it to his or her attention. Alternatively, it must be funny.With *incentive-based models*, the sender is rewarded for his or her actions.

According to [6] this model will increasingly become the most common and probably mostcontentious one, given that the recipient is merely a component in a money-makingmechanism. The saving grace thus far has been that monetary rewards are rare. This isunsurprising, considering the vast size of digital media and the difficulty of making payments. Instead, incentive-based viral marketing models involve some kind of rewardscheme, for instance free products or Web currency. From a business point of view, a more controlled method is the *loyalty-based model* thatcombines elements of affiliate programmes and gives financial rewards, but only when therecipient engages in some way [6].

The underlying concept behind viral marketing is that people will interact with other people,spreading information about products or services. The spread of information by parties otherthan the organization can convey to potential customers what a product or service can belike. The only difference is the source of origin. Therefore, although consumers receiveincentives to spread messages, it can be considered as a form of viral marketing because it isstill the consumer who chooses the next recipients of the message or viral object [6].

From mathematical modeling point of view, [3,7] have formalized the concept of viral marketing as an analogy of infectious disease models. [7] Cross-fertilizes the ideas of the SIR and SEIAR models of infectious disease spread to the dissemination of marketing information. [3] views SIR model as an analogy of a viral marketing propagation model. In this paper, we view viral marketing as a close derivative of stochastic models of infectious disease spread on networks.The plan of this paper is as follows. Graphs and modeling is presented in section 2. Modeling description is devoted to section 3. Simulation is done in section 4. Results are presented in section 5. Discussion and conclusive remark are passed in sections 6 and 7.

2. Graphs and Modeling

Graphs used in the literature can be classified based on the properties of interest. From the dynamism point of view, graphs or networks can be classified as static or dynamic depending on whether their structures change with time. From the field of application perspective, we have social networks, information networks, technological networks, epidemic networks, to mention a few. Each of these types of networks can be narrowed to specific networks. Graph classifications based on degree distribution exist. For instance, scale-free graphs, Poisson graphs. Graphs such as unipartite, bipartite or multipartite are based on the node types. For a general knowledge of graphs and their theory, refer to [8, 9, 10, 11, 12, 13, 14, 16, 17, 19].

Real world network are large, and in most cases it is virtually impossible to describe them in detail or to give an accurate model for how they came to be. To circumvent this problem, random graphs have been considered as network models. The field of random graphs was established in late 1950s and early 1960s. For detail, see [8].

In this article, our interest is in social networks and how they affect viral advertising. A social network is a social structure made up of individuals (or organizations) called nodes which are connected by some specific types of interdependency, such as friendship, enmity, common interest, financial exchange, dislike, sexual relationship or relationship of beliefs, knowledge or prestige. For detail of social network analysis, the reader is referred to [19].

3. Model Description

We construct a graph or network model, wherein each individual is represented by a node and the edges are the links between the individuals. A Poisson distribution is used to generate degree sequence; and the graph is constructed using the mechanism of configuration model.

We simulate viral marketing on our graph based on the following procedure.

1. Specify the total population $T = N$.
2. Specify the degree distribution as a Poisson distribution with the parameter value λ.
3. Generate the graph by the mechanism of configuration model.
4. At each time step, a target node may be informed by a marketer (one of those sharing market information)) with probability β

5. At each time step, a marketer stops sharing market information with probability γ.
Repeat these steps until statistical significance is obtained.

4. Simulation

We use the following parameter values in Table 1.

Table 1: parameters for numerical simulations

parameters	Definition	Parameter value	source
β	Market contact probability per day	$0.02 - 0.1$	[3]
γ	Fixed probability for stopping to share	0.02	[3]
λ	Mean number of neighbours	$4 - 10$	Assumed

5. Result

The results of our simulation experiments are as follows.

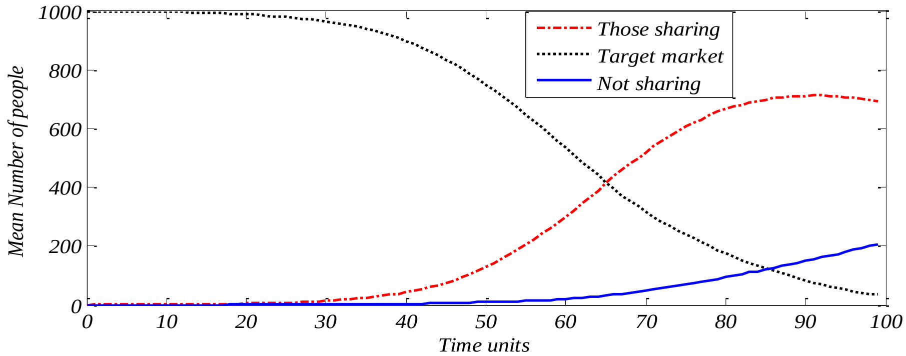

Figure 1: Graph showing the dynamics of target consumers, consumers sharing and not sharing product information, $\beta = 0.1, \gamma = 0.02, \lambda = 10$

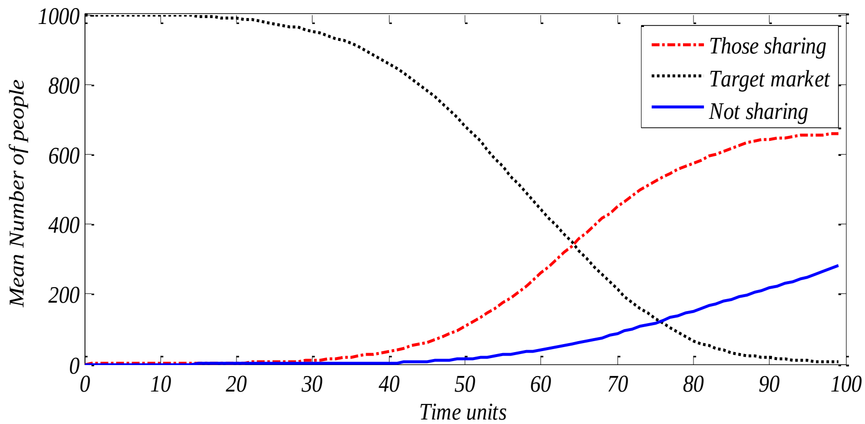

Figure 2: Graph showing the dynamics of target consumers, consumers sharing and not sharing product information, $\beta = 0.08, \gamma = 0.02, \lambda = 8$

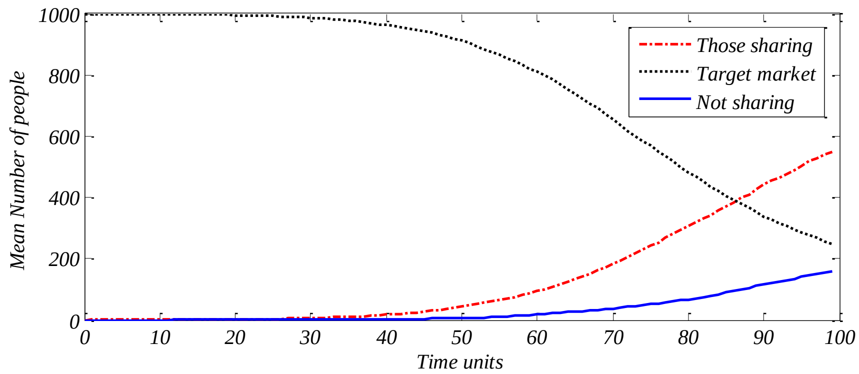

Figure 3: Graph showing the dynamics of target consumers, consumers sharing and not sharing product information, $\beta = 0.06, \gamma = 0.02, \lambda = 7$

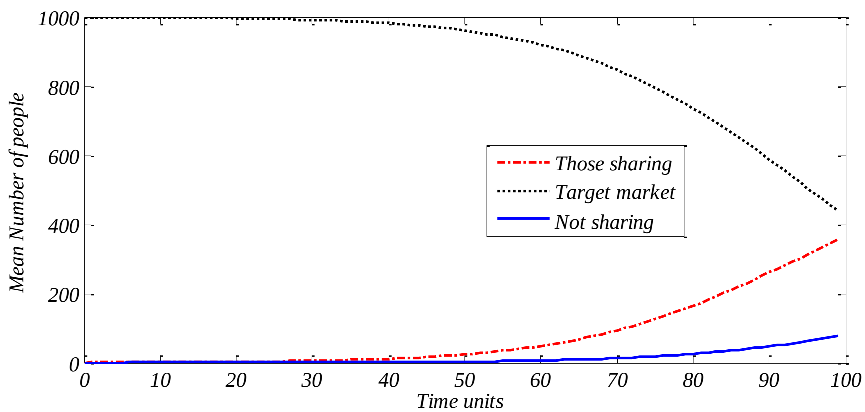

Figure 4: Graph showing the dynamics of target consumers, consumers sharing and not sharing product information, $\beta = 0.04, \gamma = 0.02, \lambda = 5$

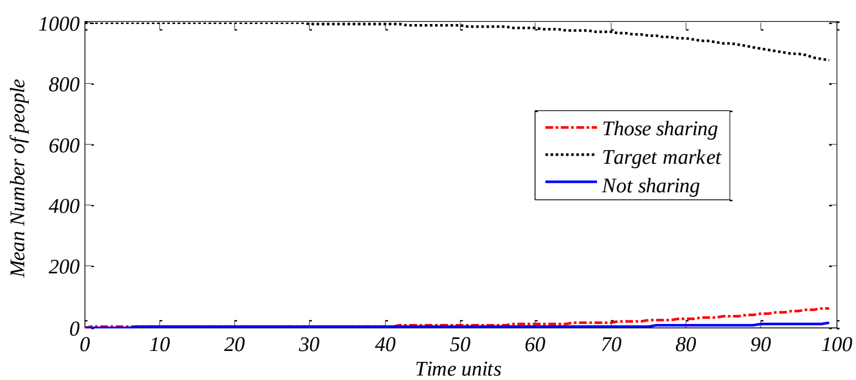

Figure 5: Graph showing the dynamics of target consumers, consumers sharing and not sharing product information, $\beta = 0.02, \gamma = 0.02, \lambda = 4$

6. Discussion

This section discusses the results of our simulation experiments. The experiments were performed for varying product/service information sharing rates. The rates range from 0.02-0.1. The results are shown in Figures 1 through 5. In our experiments, the highest sharing rate is 0.1 and the outcome is depicted in Figure 1. The lowest sharing rate is 0.02 and the result is displayed in Figure 5. The results show that the denser the consumer network and or the higher the sharing rate, the more are the product consumers aware of the information about the product.

7. Conclusion

This paper focuses on product/service information dissemination on network, otherwise known as viral marketing. Simulation to ascertain the levels of information dissemination was done for varying sharing rates. The results can be seen in Figures 1 through 5. The results show that product/service information dissemination can be fruitful via consumer networks.

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THE VOLUME OF A RECTANGULAR FRUSTUM: AN ALTERNATIVE METHOD

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Abstract
This paper presents an improvement on an early presentations on volumes of Frustums, a useful tool in a construction outfit. As a follow-up to an earlier presentation that considered Square and Conical Frustums, this paper considers the Volumes of a Rectangular Frustum as related to a Square Frustum, noting how they could depend on each other for estimation. It was noted that earlier papers discussed how the Egyptians obtained the volume of the pyramid as one-third the height multiplied by the sum of the two different areas A_1 *(from a large pyramid) and* A_2 *(a smaller pyramid), added to the square-root of the product of the two areas {i.e.* $V = \frac{1}{3}h\left[A_1 + A_2 + \sqrt{A_1 A_2}\right]$*}. It was also noted that the usual method of estimating the volume of such dissected solid figures was by first calculating the volume of the original big pyramidal container, chopping off the top part as needed, calculating the volume of the chopped-off pyramid, and then subtracting the chopped-off volume from the original big pyramid. Such cumbersome calculation having been reduced by Olagunju (2011), to obtain a proven formula for a Square Frustum as* $V_{SF} = \frac{1}{6}h(D^2 + Dd + d^2)$*, and in furtherance to this, Olagunju (2016) showed that the volume of a square frustum could be estimated using the known volume of a conical frustum (and vice versa). This has now been extended to obtain the volume of a Rectangular Frustum as* $V_{RF} = \frac{h}{3\sqrt{5}}(D^2 + Dd + d^2)$. *Where* V_{RF} *= Volume of Rectangular Frustum, D = Diagonal of the large Rectangular base, d = diagonal of the small Rectangular top, and h = the height of the Rectangular Frustum.*

Keywords: *Volume, Pyramids, Frustum, Rectangular, Square, Diagonals.*

1.0 Background
Scientific knowledge is expected to be progressive in order to bring about some improvement in application for the purpose of sustaining progressive development. This explains why man looks for new ways of doing things every new day. Thus, the essence of education, especially Mathematics, is to find a way of improving on earlier findings. Sometimes, this could be achieved by establishing the relationship between existing models. This accounts for why the established formula for the Volume of Square Frustums is considered here in order to generate a formula for the Rectangular Frustum.

2.0 Significance of the Study

This work seeks to obtain a formula for estimating the volume of a Rectangular based Frustum, and illustrate same with a view of comparing the results of the new formula with the old existing method. This will help teachers and students of Mathematics as well as construction firms to gain some time in their effort in estimating such.

3.0 Some Clarifications

3.1 Pyramids and their Classification

According to Hart (2005), as reported in Olagunju (2011, 2016), a Pyramid is a Polyhedron having one polygonal face (called 'base') and all other faces as Triangles, meeting at the Vertex (called 'Apex'). A special kind of Pyramid whose base is circular and all slant-edge lines meet at the vertex is referred to as a Cone or a Conical Pyramid. When a part of a Pyramid is chopped off, a plane parallel to the base is created and it becomes a Truncated Pyramid, usually referred to as a Frustum. Pyramids are classified by their dimensions. While a *Regular pyramid* is one with a base with regular polygon (e.g. Square-Based, Rectangular-Based), a *Right pyramid* is one whose apex is joined to the center of the base by a perpendicular line. Another type with one single cross-sectional shape having lengths scaling linearly with its height is referred to as an *Arbitrary pyramid*.

3.2 Frustums and their Classification

A Frustum is a Truncated Pyramid in which one part has been chopped-off to a given height. At the point of cut, the top platform is usually parallel to the base platform. Frustums are classified using the shape of their bases and tops which are usually the same. A frustum is said to be *Rectangular* when the Base and Top are both Rectangular, or *Square* when the Base and Topare both in the form of Squares (usually, the area of one end-face is smaller than the other), and *Conical* when the Base and Top are circular (usually, the radius of one end-face is smaller than the other).

4.0 Brief Information on Volumes of Allied Shapes

4.1 Volumes of Pyramids and Pyramidal Frustums

As noted by Olagunju (2011, 2016), Harris and Stocker (1998) showed that the Volume of a pyramid is given as one-third of the product of the base-area and the perpendicular height.

i.e. Pyramidal Volume = $\frac{1}{3}$(base-area x $\perp$ height).

Thus, the volume of the Truncated Pyramid (i.e. Pyramidal Frustum) is given as the difference between the complete pyramid and the chopped small pyramid.

i.e. $V_{PF} = V_{BP} - V_{SP}$ **where**

V_{PF} = Volume of Pyramidal Frustum,

V_{BP} = Volume of Complete Big Pyramid,

V_{SP} = Volume of the Small chopped pyramid

4.2 Volume of a Square Frustum

Consider the work of Olagunju (2016) on the Square Pyramidal Frustum, where $l = b$ (square base.) and base area = l x b,

Since $V_P = \frac{1}{3} l \text{ x } b \text{ x } h = \frac{1}{3} l^2 h = \frac{1}{3} b^2 h$

Then, we obtain the volume of a Truncated Square Pyramid thus:

If the volume of a Big Square-based Pyramid is $V_{BP} = \frac{1}{3} L^2 H$

And if the volume of the chopped-off small Pyramid is $V_{SP} = \frac{1}{3} l^2 x$

Then, $V_{SF} = V_{BP} - V_{SP} \Rightarrow V_{PF} = \frac{1}{3} L^2 H - \frac{1}{3} l^2 x = \frac{1}{3}(L^2 H - l^2 h)$ **(4.1)**

Where: l = length of Small-Square-Top (Base of chopped-off top pyramid),
x = height of Small Pyramid
L = Length of Big-Square-Base Pyramid,
H = Height of Big Pyramid [(H = x + h), h = height of Frustum],.

V_{SF} = Volume of Square Frustum

V_{BP} = Volume of Big Pyramid

V_{SP} = Volume of Small Pyramid

This leads to the fact that if the base and top diagonals of a Square Frustum are given as D and d respectively, and if the height is h, then, the Volume of the Square frustum designated as V_{SF} is obtained as $V_{SF} = \frac{h}{6}\left[D^2 + Dd + d^2\right]$ **(4.2)**

Where: d = Diagonal of the Small-Square-Top,
D = Diagonal of the Big-Square Base
L = Length of Big-Square-Base Pyramid,
h = Height of Square Frustum

V_{SF} = Volume of Square Frustum

5.0 Formula for the Volume of a Rectangular Frustum

5.1 Important Information concerning the Rectangular-Based Model

5.11 Since the original Pyramid has a Rectangular base, then, the top chopped-off Small Pyramid also has a Rectangular base.

5.12 If the ratio of the rectangular base-length to the base-breadth is 2:1 (i.e. **2L:L**), then, the top rectangular chopped-off pyramid, top-length and top-breadth will also be of ratio 2:1 (i.e. ***2l:l***).

5.13 The ratio of the height of the top chopped-off pyramid to the height of the original big pyramid equals the ratio of the diagonal of the top chopped-off pyramid to the Diagonal of the original big pyramid ***x : H = d : D***

5.14 Since the Big Rectangular Base-Length and the Small Rectangular Top-length are in the ratio L : *l* , and this affects their diagonals, then, $\mathbf{D^2 : d^2 = L : \mathit{l}}$

5.2 Developing Formula for the Volume of a Rectangular Frustum

As against method of subtracting the volume of a small chop-off rectangular pyramid from the complete big rectangular pyramid as earlier mentioned, we proceed to obtain a formula for the volume of a rectangular frustum as follows, considering Figure 5.1 below:

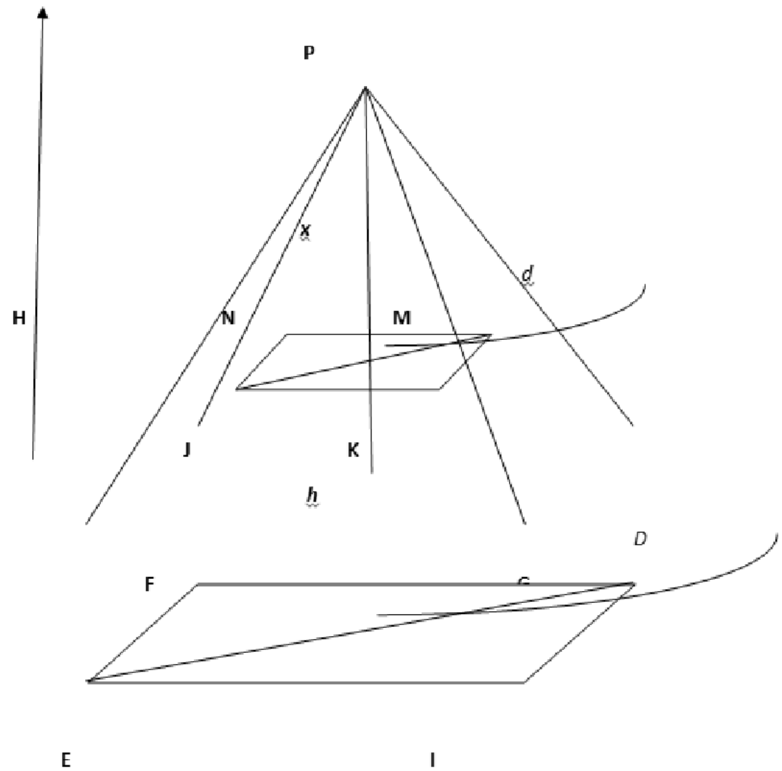

Figure 5.2

Given application of the listed dimensions below to Fig. 5.2 above,
x *= height of Small Pyramid,*
h *= height of Frustum,*
H *= Height of Large Pyramid, (height of small pyramid + height of Frustum)*
Base-Length = ***2L***
Base-breadth = ***L***
Top-Length = ***2l***
Top-breadth = ***l***
We note if the base and top diagonals of the pyramidal frustum be ***D*** and ***d*** respectively.
Since its height is ***h***, and the Volume is designated as V_{RF} **,**
Then, by the old method, the Volume of Rectangular Frustum will be the difference between the Volume of large Rectangular Pyramid and Volume of chopped Rectangular Pyramid.
We note here that $H = h + x$ (Heights of the pyramids) **(5.21)**
By Pythagoras applied to rect. JKMN and rect. EFGI above,

From the Small Top-Rectangle, $d = \sqrt{5}l$ and $d^2 = 5l^2$ **(5.22)**

From the Big Base-Rectangle, $D = \sqrt{5}L$ and $D^2 = 5L^2$ **(5.23)**

But Base-Area of the Big Rectangular Pyramid = $2L$ x $L = 2L^2$

While the Volume of the Big Rectangular Pyramid = $2L^2H = 2L^2(x+h)$

Also, Base- Area of the chopped Small Rectangular Pyramid = $2l$ x $l = 2l^2$

While the Volume of the chopped Small Rectangular Pyramid = $2l^2x$

Substituting these in the old technique whereby

$$V_{PF} = \tfrac{1}{3}L^2H - \tfrac{1}{3}l^2x = \tfrac{1}{3}(L^2H - l^2h) \qquad \textbf{(4.2)}$$

We have equation (4.2) becoming

$$V_{RF} = \tfrac{1}{3}\left[2L^2(x+h) - 2l^2x\right]$$

$$\Rightarrow \quad V_{RF} = \tfrac{2}{3}\left[L^2(x+h) - l^2x\right]$$

But equations (5.22) and (5.23) above, $d = \sqrt{5}l$ and $D = \sqrt{5}L$

Thus, $V_{RF} = \tfrac{2}{3}\left[\left(\tfrac{D}{\sqrt{5}}\right)^2(x+h) - \left(\tfrac{d}{\sqrt{5}}\right)^2 x\right]$

$$\Rightarrow \quad V_{RF} = \tfrac{2}{3}\tfrac{1}{5}\left[D^2(x+h) - d^2x\right]$$

$$\Rightarrow \quad V_{RF} = \tfrac{2}{15}\left[D^2h + D^2x - d^2x\right]$$

$$\Rightarrow \quad V_{RF} = \tfrac{2}{15}\left[D^2h + x\left(D^2 - d^2\right)\right]$$

$$\Rightarrow \quad V_{RF} = \tfrac{2}{15}\left[D^2h + x\left(D-d\right)\left(D+d\right)\right] \qquad \textbf{(5.2)}$$

Considering similar ΔPJM and ΔPEG

We note that $\dfrac{x}{d} = \dfrac{x+h}{D}$

Implying that $x(D-d) = dh$ **(5.3)**

Putting (5.3) into (5.2), we have

$$V_{RF} = \tfrac{2}{15}\left[D^2h + dh(D+d)\right]$$

Hence $V_{RF} = \tfrac{2}{15}\left[D^2h + Ddh + d^2h\right]$

$$\Rightarrow \quad V_{RF} = \tfrac{2h}{15}\left[D^2 + Dd + d^2\right] \qquad \textbf{(5.4)}$$

Equation (5.4) determines the Volume of the said Rectangular Frustum

6.0 Illustrations – of Rectangular Frustum (Old and New Methods)

For the purpose of illustration, we shall consider few examples below, calculating the volumes using the method of subtracting smaller chopped pyramids from the original big pyramids, in comparison with the use of the formula just derived. For this purpose, we shall designate the volume and height of the original big rectangular pyramid by $\boldsymbol{V_{BRP}}$ and ($\boldsymbol{x + h}$ i.e. H) respectively, volume and height of the small chopped-off rectangular pyramid by $\boldsymbol{V_{SRP}}$ and $\boldsymbol{x}$ respectively, and the volume and height of the Rectangular Frustum by $\boldsymbol{V_{RF}}$ and $\boldsymbol{h}$ respectively.

6.1 Illustration I

When x = 5, h = 5 , l = 4 and L = 8.
From equation (5.21), ***H : (x + h)***.

Solution: $V_{SP} = \frac{2}{3}xl^2 = \frac{2}{3}(5)4^2 = \frac{160}{3}$

$V_{BP} = \frac{2}{3}(h+x)L^2 = \frac{2}{3}(5+5)8^2 = \frac{1280}{3}$

Thus, $V_{RF} = V_{BP} - V_{SP} = \frac{1280}{3} - \frac{160}{3} = \frac{1120}{3}$ unit^3 (6.1.1)

Now, by the new formula, from equations (5.22) and (5.23),
We have $D^2 = 5L^2$**,** $d^2 = 5l^2$ and $Dd = 5Ll$
Also, from equation (5.4),
We have $V_{RF} = \frac{2h}{15}\left[D^2 + Dd + d^2\right]$

Now, h = 5, $d^2 = 5l^2 = 5(4)^2 = 80$**,** $D^2 = 5L^2 = 5(8)^2 = 320$ and
$Dd = 5Ll = 5(8)(4) = 160$

Hence, $V_{RF} = \frac{2h}{15}\left[D^2 + Dd + d^2\right]$ becomes

$$V_{RF} = \frac{10}{15}\left[320 + 160 + 80\right] = \frac{1120}{3} \text{ unit}^3 \quad (6.1.2)$$

It is noted that results in (6.1.1) and (6.1.2) confirm the authenticity of the formula.

6.2 Illustration II

When x = 7, h = 7 , l = 11 and L = 22.
From equation (5.21), ***H : (x + h)***.

Solution: $V_{SP} = \frac{2}{3}xl^2 = \frac{2}{3}(7)11^2 = \frac{1694}{3}$

$V_{BP} = \frac{2}{3}(h+x)L^2 = \frac{2}{3}(7+7)22^2 = \frac{13552}{3}$

Thus, $V_{RF} = V_{BP} - V_{SP} = \frac{13552}{3} - \frac{1694}{3} = \frac{11858}{3}$ unit^3 (6.2.1)

Now, by the new formula, from equations (5.22) and (5.23),
We have $D^2 = 5L^2$**,** $d^2 = 5l^2$ and $Dd = 5Ll$
Also, from equation (5.4),
We have $V_{RF} = \frac{2h}{15}\left[D^2 + Dd + d^2\right]$

Now, h = 7, $d^2 = 5l^2 = 5(11)^2 = 605$, $D^2 = 5L^2 = 5(22)^2 = 2420$ and $Dd = 5Ll = 5(22)(11) = 1210$

Hence, $V_{RF} = \frac{2h}{15}\left[D^2 + Dd + d^2\right]$ becomes

$$V_{RF} = \tfrac{14}{15}[2420 + 1210 + 605] = \frac{11858}{3} \text{ unit}^3 \qquad (6.2.2)$$

It is noted that results in (6.2.1) and (6.2.2) confirm the authenticity of the formula.

6.3 Illustration III

When x = 0.8, h = 0.8 , l = 0.3 and L = 0.6.
From equation (5.21), ***H : (x + h)***.

Solution: $V_{SP} = \frac{2}{3}xl^2 = \frac{2}{3}(0.8)(0.3)^2 = \frac{0.144}{3}$

$$V_{BP} = \tfrac{2}{3}(h + x)L^2 = \tfrac{2}{3}(0.8 + 0.8)(0.6)^2 = \frac{1.152}{3}$$

Thus, $V_{RF} = V_{BP} - V_{SP} = \frac{1.152}{3} - \frac{0.144}{3} = \frac{1.008}{3}$ unit^3 (6.3.1)

Now, by the new formula, from equations (5.22) and (5.23),
We have $D^2 = 5L^2$, $d^2 = 5l^2$ and $Dd = 5Ll$
Also, from equation (5.4),
We have $V_{RF} = \frac{2h}{15}\left[D^2 + Dd + d^2\right]$

So, h = 0.8, $d^2 = 5l^2 = 5(0.3)^2 = 0.45$, $D^2 = 5L^2 = 5(0.6)^2 = 1.80$ and $Dd = 5Ll = 5(0.6)(0.3) = 0.90$

Hence, $V_{RF} = \frac{2h}{15}\left[D^2 + Dd + d^2\right]$ becomes

$$V_{RF} = \tfrac{1.6}{15}[1.80 + 0.90 + 0.45] = \frac{1.008}{3} \text{ unit}^3 \qquad (6.3.2)$$

It is noted that results in (6.3.1) and (6.3.2) confirm the authenticity of the formula.

6.4 Illustration IV

When x = 7, h = 14 , l = 4 and L = 12.
From equation (5.21), ***H : (x + h)***.

Solution: $V_{SP} = \frac{2}{3}xl^2 = \frac{2}{3}(7)4^2 = \frac{224}{3}$

$$V_{BP} = \tfrac{2}{3}(h + x)L^2 = \tfrac{2}{3}(7 + 14)12^2 = \frac{6048}{3}$$

Thus, $V_{RF} = V_{BP} - V_{SP} = \frac{6048}{3} - \frac{224}{3} = \frac{5824}{3}$ unit^3 (6.4.1)

Now, by the new formula, from equations (5.22) and (5.23),
We have $D^2 = 5L^2$, $d^2 = 5l^2$ and $Dd = 5Ll$
Also, from equation (5.4),
We have $V_{RF} = \frac{2h}{15}\left[D^2 + Dd + d^2\right]$
Now, h = 14, $d^2 = 5l^2 = 5(4)^2 = 80$, $D^2 = 5L^2 = 5(12)^2 = 720$ and
$Dd = 5Ll = 5(12)(4) = 240$
Hence, $V_{RF} = \frac{2h}{15}\left[D^2 + Dd + d^2\right]$ becomes

$$V_{RF} = \tfrac{28}{15}\left[720 + 240 + 80\right] = \frac{5824}{3} \text{ unit}^3 \quad (6.4.2)$$

It is noted that results in (6.4.1) and (6.4.2) confirm the authenticity of the formula.

6.5 Illustration V

When x = 0.9, h = 2.7 , l = 0.6 and L = 2.4
From equation (5.21), ***H : (x + h)***.

Solution: $V_{SP} = \frac{2}{3}xl^2 = \frac{2}{3}(0.9)(0.6)^2 = \frac{0.648}{3}$

$V_{BP} = \frac{2}{3}(h + x)L^2 = \frac{2}{3}(0.9 + 2.7)(2.4)^2 = \frac{41.472}{3}$

Thus, $V_{RF} = V_{BP} - V_{SP} = \frac{41.472}{3} - \frac{0.648}{3} = \frac{40.824}{3}$ unit^3 (6.5.1)

Now, by the new formula, from equations (5.22) and (5.23),
We have $D^2 = 5L^2$, $d^2 = 5l^2$ and $Dd = 5Ll$
Also, from equation (5.4),
We have $V_{RF} = \frac{2h}{15}\left[D^2 + Dd + d^2\right]$
So, h = 2.7, $d^2 = 5l^2 = 5(0.6)^2 = 1.80$, $D^2 = 5L^2 = 5(2.4)^2 = 28.80$ and
$Dd = 5Ll = 5(2.4)(0.6) = 7.20$
Hence, $V_{RF} = \frac{2h}{15}\left[D^2 + Dd + d^2\right]$ becomes

$$V_{RF} = \tfrac{5.4}{15}\left[28.80 + 7.20 + 1.80\right] = \frac{40.824}{3} \text{ unit}^3 \quad (6.5.2)$$

It is noted that results in (6.5.1) and (6.5.2) confirm the authenticity of the formula.

6.6 Illustration VI

When x = 6.8, h = 10.2 , l = 3.0 and L = 7.5
From equation (5.21), ***H : (x + h)***.

Solution: $V_{SP} = \frac{2}{3}xl^2 = \frac{2}{3}(6.8)(3.0)^2 = \frac{122.4}{3}$

$V_{BP} = \frac{2}{3}(h+x)L^2 = \frac{2}{3}(10.2+6.8)(7.5)^2 = \frac{1912.5}{3}$

Thus, $V_{RF} = V_{BP} - V_{SP} = \frac{1912.5}{3} - \frac{122.4}{3} = \frac{1790.1}{3}$ unit^3 (6.6.1)

Now, by the new formula, from equations (5.22) and (5.23),
We have $D^2 = 5L^2$, $d^2 = 5l^2$ and $Dd = 5Ll$
Also, from equation (5.4),
We have $V_{RF} = \frac{2h}{15}\left[D^2 + Dd + d^2\right]$

So, h = 10.2, $d^2 = 5l^2 = 5(3)^2 = 45$, $D^2 = 5L^2 = 5(7.5)^2 = 281.25$ and $Dd = 5Ll = 5(7.5)(3) = 112.5$

Hence, $V_{RF} = \frac{2h}{15}\left[D^2 + Dd + d^2\right]$ becomes

$$V_{RF} = \frac{20.4}{15}\left[281.25 + 112.5 + 45\right] = \frac{1790.1}{3} \text{ unit}^3 \qquad (6.6.2)$$

It is noted that results in (6.6.1) and (6.6.2) confirm the authenticity of the formula.

Conclusion and Recommendation

Having confirmed the usefulness and effectiveness of this formula, it has therefore been proven that this new method can now be used for the calculation of the volume of a Rectangular Frustum as it removes the cumbersomeness involved in the use of the old method, which involved the process of having to calculate two different volumes and then carrying out some subtraction.

Precaution

Since every model has its own precaution(s), it should be noted that this fails when the conditions are not properly applied.

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