INVESTIGATING A STATIONARITY PROPERTY OF MULTIVARIATE TIME SERIES USING NIGERIA CONSUMER PRICE INDEX RETURN SERIES

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Abstract

The motivation behind this research was to investigate stationarity of multivariate time series through positive definiteness property of the cross-autocovariance/cross-autocorrelation functions. Three vector series which represent the Nigeria Average, Urban and Rural Consumer Price Indices were used with data collected from CBN Statistical Bulletin from 1995-2018. The covariances and autocorrelation matrices of individual vector return series constituted the components of cross-covariance and cross-autocorrelation matrices. Firstly, the positive definiteness of the sub-autocorrelation matrices were investigated, and stationarity were ascertained. This was followed by the investigation on the cross-autocorrelation functions. The results of the principal minors and determinants revealed positive definiteness of the multivariate process. This paper recommends that in multivariate time series, investigation of stationarity should incorporate the sub-autocovariance or autocorrelation matrices of individual vector processes as components of the cross-covariance or cross-autocorrelation matrix.

Keywords: *Stationarity, Positive-definiteness, cross-autocovariance and cross-autocorrelations.*

1. Introduction

In time series, stationarity simply means stability or a state of equilibrium irrespective of its change in time. This is a common assumption in many time series data. Stationarity of a time series process has the property that the mean, variance and autocorrelation structure do not change over time. If the time series is not stationary, it can be transformed to become stationary with some time series operators or techniques. These may include application of difference operators $\nabla^d = (1-B)^d$ such that if $d = 1, \nabla X_t = X_t - X_{t-1}$or $T_t = a + bt$ for a non-seasonal time series process, $\nabla_s^d = (1-B^s)^d = X_t - X_{t-s}$, if $d = 1$ for a seasonal time series, taking the logarithm or square root of the series to stabilize the variance for non-constant variance, etc. The initial process taken to visualize the behaviour of a process is the time graph plotted to display some hidden features that characterise the series. The time plot exhibits certain attributes that could not be seen by raw data inspection, and these may include trend, seasonality or both, therefore, justifying the need for adoption of an operator or statistical technique to ensure equilibrium state of the process. Indisputably, most of the original economic and financial time series often show upward trend (increase in series), hence, giving room for difference or trend analysis to attain stability.

There are many statistical procedures which verify stationarity state of a time series. These are carried out through some statistical tests/investigations of some stationarity properties. Assessing the structure of the autocovariances and autocorrelations is very prominent in investigating the stationarity of time series process, Box and Jenkins (1976), Kendall and Ord (1990), Gujarity and Porter (2009). In univariate time series, positive definiteness or semi-positive definiteness property is investigated to ascertain the stationarity of the autocovariance structure with the autocorrelation matrix at different time lags. This procedure has an extension to multivariate time series, Engle and Kroner (1995). In multivariate time series, the n-dimensional cross-autcovariance or cross-autocorrelation matrix composes of sub-matrices of individual vector processes with distributed lags. For a cross-autocorrelation matrix to be positive definite, it is a justifiable assumption that the individual sub-autocorrelation matrices meet the positive definiteness condition. That means, their determinants and principal minors have positive values. Borlerslev et al (1998) considered k=2 ARCH (1) process and symmetric positive definite matrix of cross-covariances. Kiyang and Cyrus (2005) investigated stationarity of multivariate time series for correlation-based data analysis. Carsten and Subba Rao (2015) adopted Discrete Fourier Transform as a tool to test for second order stationarity of multivariate time series. In addition to the investigation of the assumption of positive definiteness of the n-dimensional cross-autocovariance matrix, it becomes more revealing to investigate the positive definiteness of the sub-autocovariance matrices of individual vectors as the components of the cross-covariance and cross-correlation matrix. This is considered the first step of verifying stationarity property before the larger cross-covariance matrix is investigated. This implies, each univariate autocovariance structure making up the cross-autocovariance matrix is verified, and the final investigation concluded in the cross-autocovariance or cross-autocorrelation matrix. This paper focuses on both the sub-covariance/correlation matrices and cross-autocovraiance/autocorrelation matrix to ascertain stationarity of the multivariate time process.

2. Methodology

A. Cross-Covariances

Here, we consider the cross-covariances of the vector processes

$$\gamma_{it+k,jt+l} = \begin{bmatrix} \gamma_{1t+k,1t+l} & \gamma_{1t+k,2t+l} & \gamma_{1t+k,3t+l} \cdots & \gamma_{1t+k,nt+l} \\ \gamma_{2t+k,1t+l} & \gamma_{2t+k,2t+l} & \gamma_{2t+k,3t+l} \cdots & \gamma_{2t+k,nt+l} \\ \gamma_{3t+k,1t+l} & \gamma_{3t+k,2t+l} & \gamma_{3t+k,3t+l} \cdots & \gamma_{3t+k,nt+l} \\ \vdots & \vdots & \vdots \cdots & \vdots \\ \gamma_{mt+k,1t+l} & \gamma_{mt+k,1t+l} & \gamma_{mt+k,1t+l} & \gamma_{mt+k,nt+l} \end{bmatrix} \quad 1$$

where $i = 1, \dots, m; j = 1, \dots, n; k = 1, \dots, r; l = 1, \dots, s$
Equation “1” is the cross-covariance matrix.

$$\gamma_{1t+k,1t} = \gamma_{1t,1t+l} = \begin{bmatrix} \gamma_{1t,1t} & & & & \\ \gamma_{1t+1,1t} & \gamma_{1t+1,1t+1} & & & \\ \gamma_{1t+2,1t} & \gamma_{1t+2,1t+1} & \gamma_{1t+2,1t+2} & & \\ \vdots & \vdots & \vdots & & \\ \gamma_{1t+r,1t} & \gamma_{1t+r,1t+1} & \gamma_{1t+r,1t+2} & \cdots & \gamma_{1t+r,1t+s} \end{bmatrix} \quad 2$$

$$\gamma_{2t+k,2t} = \gamma_{2t,2t+l} = \begin{bmatrix} \gamma_{2t,2t} & & & & \\ \gamma_{2t+1,2t} & \gamma_{2t+1,2t+1} & & & \\ \gamma_{2t+2,2t} & \gamma_{2t+2,2t+1} & \gamma_{2t+2,2t+2} & & \\ \vdots & \vdots & \vdots & & \\ \gamma_{2t+r,2t} & \gamma_{2t+r,2t+1} & \gamma_{2t+r,2t+2} & \cdots & \gamma_{2t+r,2t+s} \end{bmatrix} \quad 3$$

$$\gamma_{3t+k,3t} = \gamma_{3t,3t+l} = \begin{bmatrix} \gamma_{3t,3t} & & & & \\ \gamma_{3t+1,3t} & \gamma_{3t+1,3t+1} & & & \\ \gamma_{3t+2,3t} & \gamma_{3t+2,3t+1} & \gamma_{3t+2,3t+2} & & \\ \vdots & \vdots & \vdots & & \\ \gamma_{3t+r,3t} & \gamma_{3t+r,3t+1} & \gamma_{3t+r,3t+2} & \cdots & \gamma_{3t+r,3t+s} \end{bmatrix} \quad 4$$

$\vdots \; \vdots$

$$\gamma_{mt+k,mt} = \gamma_{mt,mt+l} = \begin{bmatrix} \gamma_{mt,mt} & & & & \\ \gamma_{mt+1,mt} & \gamma_{mt+1,mt+1} & & & \\ \gamma_{mt+2,mt} & \gamma_{mt+2,mt+1} & \gamma_{mt+2,mt+2} & & \\ \vdots & \vdots & \vdots & & \\ \gamma_{mt+r,mt} & \gamma_{mt+r,mt+1} & \gamma_{mt+r,mt+2} & \cdots & \gamma_{mt+r,mt+s} \end{bmatrix} \quad 5$$

Equations “2”, “3”, “4” and “5” are Covariances of $X_{it+k\ and}\, X_{jt}\ or\ X_{it\ and}\, X_{jt+l}$ $(i = j\ at\ k = 0, 1, \ldots, r\ or\ l = 0,1, \ldots, s)$. For $i = j\ and\ k = l$, Covariances of $X_{it+k\ and}\, X_{jt+l}$ denoted by $\gamma_{it+k,jt+l}$ are variances which are evident in the principal diagonal of the covariance matrices. For $i = j\ and\ k \neq l$, Covariances of $X_{it+k\ and}\, X_{jt+l}$ denoted by $\gamma_{it+k,jt+l}$ are the upper and lower elements of the covariance matrices.

$$\gamma_{1t+k,2t+l} = \begin{bmatrix} \gamma_{1t,2t} & \gamma_{1t,2t+1} & \gamma_{1t,2t+2} & \cdots & \gamma_{1t,2t+s} \\ \gamma_{1t+1,2t} & \gamma_{1t+1,2t+1} & \gamma_{1t+1,2t+2} & \cdots & \gamma_{1t+1,2t+s} \\ \gamma_{1t+2,2t} & \gamma_{1t+2,2t+1} & \gamma_{1t+2,2t+2} & \cdots & \gamma_{1t+2,2t+s} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \gamma_{1t+r,2t} & \gamma_{1t+r,2t+1} & \gamma_{1t+r,2t+2} & & \gamma_{1t+r,2t+s} \end{bmatrix} \quad 6$$

$$\gamma_{1t+k,3t+l} = \begin{bmatrix} \gamma_{1t,3t} & \gamma_{1t,3t+1} & \gamma_{1t,3t+2} & \cdots & \gamma_{1t,3t+s} \\ \gamma_{1t+1,3t} & \gamma_{1t+1,3t+1} & \gamma_{1t+1,3t+2} & \cdots & \gamma_{1t+1,3t+s} \\ \gamma_{1t+2,3t} & \gamma_{1t+2,3t+1} & \gamma_{1t+2,3t+2} & \cdots & \gamma_{1t+2,3t+s} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \gamma_{1t+r,3t} & \gamma_{1t+r,3t+1} & \gamma_{1t+r,3t+2} & & \gamma_{1t+r,3t+s} \end{bmatrix} \quad 7$$

$\vdots \vdots$

$$\gamma_{1t+k,nt+l} = \begin{bmatrix} \gamma_{1t,nt} & \gamma_{1t,nt+1} & \gamma_{1t,nt+2} \cdots & \gamma_{1t,nt+s} \\ \gamma_{1t+1,nt} & \gamma_{1t+1,nt+1} & \gamma_{1t+1,nt+2} \cdots & \gamma_{1t+1,nt+s} \\ \gamma_{1t+2,nt} & \gamma_{1t+2,nt+1} & \gamma_{1t+2,nt+2} \cdots & \gamma_{1t+2,nt+s} \\ & \vdots \vdots \vdots & \cdots \vdots & \\ \gamma_{1t+r,nt} & \gamma_{1t+r,nt+1} & \gamma_{1t+r,nt+2} & \gamma_{1t+r,nt+s} \end{bmatrix} \quad 8$$

Other matrices include $\gamma_{2t+k,3t+l}, \gamma_{2t+k,nt+l}, \gamma_{3t+k,nt+l}, \cdots, \gamma_{mt+k,nt+l}$ 9

Equations "6", "7", "8" and "9" are Cross-Covariances of $X_{it+k\ and}\, X_{jt+l}(i \neq j\ at\ k = 0, 1, \dots, r\ and\ l = 0,1, \dots, s)$. The matrices "6", "7", "8" and "9" are not symmetrical and the diagonal elements are not variances. The elements of the matrices are constituted by $\gamma_{it+k,jt+l}$, $for\ i \neq j, k = l\ and\ k \neq l.$

B. **Auto-Correlations and Cross-Auto-Correlations**

Autocorrelations measure the correlations between the same variable at different lags $(X_{it+k\ and}\, X_{jt}\ or\ X_{it\ and}\, X_{jt+l}; i = j\ at\ k = 0, 1, \dots, r\ or\ l = 0,1, \dots, s)$, while Cross-Autocorrelations measure correlations between different variables at different lags $(X_{it\ and}\, X_{jt}, i \neq j\ at\ k = 0, 1, \dots, r\ and\ l = 0,1, \dots, s)$.

In a univariate time series, autocorrelation is given as

$$\rho_{s,t} = \frac{E(X_t-\mu)(X_s-\mu)}{\sigma_t\sigma_s} \quad 10$$

Where "E" is the expected value operator, X_t, X_s are two processes with standard deviations σ_t, σ_s and mean μ.

Usoro (2015) obtained vector cross-correlation as

$$\rho_{it+k,jt+l} = \frac{\gamma_{it+k,jt+l}}{\sqrt{\gamma_{it}\gamma_{jt}}} \quad 11$$

The above autocorrelation is obtained by dividing the square root of the two variances $\gamma_{it} and\ \gamma_{jt}$. Hence, equations "2", "3", "4", "5", "6", "7", "8" and "9" are divided by the square root of the product variances to become

$$\rho_{1t+k,1t} = \rho_{1t,1t+l} = \begin{bmatrix} \rho_{1t,1t} & & & \\ \rho_{1t+1,1t} & \rho_{1t+1,1t+1} & & \\ \rho_{1t+2,1t} & \rho_{1t+2,1t+1} & \rho_{1t+2,1t+2} & \\ & \vdots \vdots \vdots & & \\ \rho_{1t+r,1t} & \rho_{1t+r,1t+1} & \rho_{1t+r,1t+2} & \cdots\ \rho_{1t+r,1t+s} \end{bmatrix} \quad 12$$

$$\rho_{2t+k,2t} = \rho_{2t,2t+l} = \begin{bmatrix} \rho_{2t,2t} & & & \\ \rho_{2t+1,2t} & \rho_{2t+1,2t+1} & & \\ \rho_{2t+2,2t} & \rho_{2t+2,2t+1} & \rho_{2t+2,2t+2} & \\ & \vdots \vdots \vdots & & \\ \rho_{2t+r,2t} & \rho_{2t+r,2t+1} & \rho_{2t+r,2t+2} & \cdots\ \rho_{2t+r,2t+s} \end{bmatrix} \quad 13$$

$$\rho_{3t+k,3t} = \rho_{3t,3t+l} = \begin{bmatrix} \rho_{3t,3t} & & & & \\ \rho_{3t+1,3t} & \rho_{3t+1,3t+1} & & & \\ \rho_{3t+2,3t} & \rho_{3t+2,3t+1} & \rho_{3t+2,3t+2} & & \\ \vdots & \vdots & \vdots & & \\ \rho_{3t+r,3t} & \rho_{3t+r,3t+1} & \rho_{3t+r,3t+2} & \cdots & \rho_{3t+r,3t+s} \end{bmatrix} \quad 14$$

$\vdots \vdots$

$$\rho_{mt+k,mt} = \rho_{mt,mt+l} = \begin{bmatrix} \rho_{mt,mt} & & & & \\ \rho_{mt+1,mt} & \rho_{mt+1,mt+1} & & & \\ \rho_{mt+2,mt} & \rho_{mt+2,mt+1} & \rho_{mt+2,mt+2} & & \\ \vdots & \vdots & \vdots & & \\ \rho_{mt+r,mt} & \rho_{mt+r,mt+1} & \rho_{mt+r,mt+2} & \cdots & \rho_{mt+r,mt+s} \end{bmatrix} \quad 15$$

Equations "12", "13", "14" and "15" are Autocorrelations of $X_{it+k\ and}\ X_{jt+l}(i = j\ at\ k = 0, 1, \dots, r\ or\ l = 0,1, \dots, s)$. The principal diagonal elements have correlations of same vectors with $\rho_{it+k,jt+l(i=j,k=l)} = 1$.

$$\rho_{1t+k,2t+l} = \begin{bmatrix} \rho_{1t,2t} & \rho_{1t,2t+1} & \rho_{1t,2t+2} & \cdots & \rho_{1t,2t+s} \\ \rho_{1t+1,2t} & \rho_{1t+1,2t+1} & \rho_{1t+1,2t+2} & \cdots & \rho_{1t+1,2t+s} \\ \rho_{1t+2,2t} & \rho_{1t+2,2t+1} & \rho_{1t+2,2t+2} & \cdots & \rho_{1t+2,2t+s} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \rho_{1t+r,2t} & \rho_{1t+r,2t+1} & \rho_{1t+r,2t+2} & & \rho_{1t+r,2t+s} \end{bmatrix} \quad 16$$

$$\rho_{1t+k,3t+l} = \begin{bmatrix} \rho_{1t,3t} & \rho_{1t,3t+1} & \rho_{1t,3t+2} & \cdots & \rho_{1t,3t+s} \\ \rho_{1t+1,3t} & \rho_{1t+1,3t+1} & \rho_{1t+1,3t+2} & \cdots & \rho_{1t+1,3t+s} \\ \rho_{1t+2,3t} & \rho_{1t+2,3t+1} & \rho_{1t+2,3t+2} & \cdots & \rho_{1t+2,3t+s} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \rho_{1t+r,3t} & \rho_{1t+r,3t+1} & \rho_{1t+r,3t+2} & & \rho_{1t+r,3t+s} \end{bmatrix} \quad 17$$

$\vdots \vdots$

$$\rho_{1t+k,nt+l} = \begin{bmatrix} \rho_{1t,nt} & \rho_{1t,nt+1} & \rho_{1t,nt+2} & \cdots & \rho_{1t,nt+s} \\ \rho_{1t+1,nt} & \rho_{1t+1,nt+1} & \rho_{1t+1,nt+2} & \cdots & \rho_{1t+1,nt+s} \\ \rho_{1t+2,nt} & \rho_{1t+2,nt+1} & \rho_{1t+2,nt+2} & \cdots & \rho_{1t+2,nt+s} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \rho_{1t+r,nt} & \rho_{1t+r,nt+1} & \rho_{1t+r,nt+2} & & \rho_{1t+r,nt+s} \end{bmatrix} \quad 18$$

Other matrices include $\rho_{2t+k,3t+l}, \rho_{2t+k,nt+l}, \rho_{3t+k,nt+l}, \dots, \rho_{mt+k,nt+l}$ 19

Equations "16", "17", "18" and "19" are Cross-Autocorrelations of $X_{it+k\ and}\ X_{jt+l}(i \neq j\ at\ k = 0, 1, \dots, r\ and\ l = 0,1, \dots, s)$. The upper and lower elements of the matrices are constituted by $\gamma_{it+k,jt+l}, for\ i \neq j\ and\ k \neq l$. The principal diagonal elements are cross-autocorrelations with $\rho_{it+k,jt+l(i\neq j,k=l)} \neq 1$.

C. **Positive Definitness of Auto-Correlations and Cross-Auto-Correlation Matrices**
The positive definiteness of the autocorrelation matrix requires that all the principal minors of the autocorrelation matrix are greater than zero. Also the determinant of the autocorrelation matrix is greater than zero, Box and Jenkins (1976).

Given the Auto-correlation and cross-autocorrelation matrix of stationary processes

$$\rho_{it+k,jt+l} = \begin{bmatrix} \rho_{1t+k,1t+l} & \rho_{1t+k,2t+l} & \rho_{1t+k,3t+l} \cdots & \rho_{1t+k,nt+l} \\ \rho_{2t+k,1t+l} & \rho_{2t+k,2t+l} & \rho_{2t+k,3t+l} \cdots & \rho_{2t+k,nt+l} \\ \rho_{3t+k,1t+l} & \rho_{3t+k,2t+l} & \rho_{3t+k,3t+l} \cdots & \rho_{3t+k,nt+l} \\ \vdots & \vdots & \vdots \cdots & \vdots \\ \rho_{mt+k,1t+l} & \rho_{mt+k,1t+l} & \rho_{mt+k,1t+l} & \rho_{mt+k,nt+l} \end{bmatrix} \quad 20$$

Positive definiteness of $\rho_{it+k,jt+l}$ requires that:

(i) The minors of $\rho_{1t+k,1t+l}, \rho_{2t+k,2t+l}, \rho_{3t+k,3t+l}$ and $\rho_{mt+k,nt+l}$ are greater than zero

(ii) The determinant of $\rho_{it+k,jt+l} > 0$

3. Numerical Verification
This paper considers Average Consumer Price Index, Urban Consumer Price Index and Rural Consumer Price Index of Nigeria obtained from CBN Statistical Bulletin from 1995 to 2018. The autocorrelations and cross-autocorrelations are obtained from the return series of Average Consumer Price Index, Urban Consumer Price Index and Rural Consumer Price Index represented by X_{1t}, X_{2t} and X_{3t} respectively.

Given the Autocorrelation and Cross-Autocorrelation Matrix as:

$$\rho_{it+k,jt+l} = \begin{bmatrix} 1 & 0.258 & 0.250 & 0.603 & 0.269 & 0.280 & 0.922 & 0.213 & 0.213 \\ 0.258 & 1 & 0.257 & 0.202 & 0.602 & 0.269 & 0.204 & 0.922 & 0.213 \\ 0.259 & 0.257 & 1 & 0.153 & 0.202 & 0.602 & 0.228 & 0.204 & 0.922 \\ 0.603 & 0.202 & 0.153 & 1 & 0.258 & 0.112 & 0.419 & 0.163 & 0.143 \\ 0.269 & 0.602 & 0.202 & 0.258 & 1 & 0.258 & 0.227 & 0.419 & 0.163 \\ 0.280 & 0.269 & 0.602 & 0.112 & 0.258 & 1 & 0.259 & 0.226 & 0.419 \\ 0.922 & 0.204 & 0.228 & 0.419 & 0.227 & 0.259 & 1 & 0.175 & 0.192 \\ 0.213 & 0.922 & 0.204 & 0.163 & 0.419 & 0.226 & 0.175 & 1 & 0.174 \\ 0.213 & 0.213 & 0.922 & 0.143 & 0.163 & 0.419 & 0.192 & 0.174 & 1 \end{bmatrix} \quad 21$$

where $i = 1,2,3; j = 1,2,3; k = 0,1,2; l = 0,1,2.$

Components of the Autocorrelation and Cross-Autocorrelations
The sub-Matrices which make up the components of $\rho_{it+k,jt+l}$ (21) are provided below

$$\rho_{1t,1t+l} = \rho_{1t+k,1t} = \begin{bmatrix} 1 & 0.258 & 0.250 \\ 0.258 & 1 & 0.257 \\ 0.250 & 0.257 & 1 \end{bmatrix}, \rho_{1t+k,2t+l} = \begin{bmatrix} 0.602 & 0.269 & 0.280 \\ 0.202 & 0.602 & 0.269 \\ 0.153 & 0.202 & 0.602 \end{bmatrix}$$

$$\rho_{1t+k,3t+l} = \begin{bmatrix} 0.922 & 0.213 & 0.213 \\ 0.204 & 0.922 & 0.213 \\ 0.228 & 0.204 & 0.922 \end{bmatrix}, \rho_{2t+k,1t+l} = \begin{bmatrix} 0.602 & 0.202 & 0.153 \\ 0.269 & 0.602 & 0.202 \\ 0.280 & 0.269 & 0.602 \end{bmatrix}$$

$$\rho_{2t,2t+l} = \rho_{2t+k,2t} = \begin{bmatrix} 1 & 0.258 & 0.112 \\ 0.258 & 1 & 0.258 \\ 0.112 & 0.258 & 1 \end{bmatrix}, \rho_{2t+k,3t+l} = \begin{bmatrix} 0.419 & 0.163 & 0.143 \\ 0.227 & 0.419 & 0.163 \\ 0.259 & 0.226 & 0.419 \end{bmatrix}$$

$$\rho_{3t+k,1t+l} = \begin{bmatrix} 0.922 & 0.204 & 0.228 \\ 0.213 & 0.922 & 0.204 \\ 0.213 & 0.213 & 0.922 \end{bmatrix}, \rho_{3t+k,2t+l} = \begin{bmatrix} 0.419 & 0.227 & 0.259 \\ 0.163 & 0.419 & 0.226 \\ 0.143 & 0.163 & 0.419 \end{bmatrix}$$

$$\rho_{3t,3t+l} = \rho_{3t+k,3t} = \begin{bmatrix} 1 & 0.175 & 0.192 \\ 0.175 & 1 & 0.174 \\ 0.192 & 0.174 & 1 \end{bmatrix} \qquad 22$$

3.1 Positive Definiteness of 3x3 Component Autocorrelation Matrices

Firstly, positive definiteness of the sub-matrices is considered.
From "22" all the sub-matrices have their principal minors and determinants greater than zero. It is verified that the principal minors and determinants of $\rho_{1t,1t+l}, \rho_{1t+k,2t+l}, \rho_{1t+k,3t+l}, \rho_{2t+k,1t+l}, \rho_{2t,2t+l}, \rho_{2t+k,3t+l}, \rho_{3t+k,1t+l}, \rho_{3t+k,2t+l}$ and $\rho_{3t,3t+l} > 0$.

3.2 **Positive Definiteness of 9x9 Autocorrelation Matrix**

Here, the principal minors and determinant of the autocorrelation matrix are considered. Thus, The first row and column of the 9x9 autocorrelation matrix are $\rho_{it+k,jt+l[(i=1;\,k=0),(j=1,2,3;l=0,1,2)]}$ and $\rho_{it+k,jt+l[(i=1,2,3;\,k=0,1,2),(j=1;l=0)]}$. Since "21" is symmetric, the crossed elements of the first row and column can be written as $\rho_{it+k,jt+l[(i=1;\,k=0),(j=1,2,3;l=0,1,2)]}$ and $\rho^{I}_{it+k,jt+l[(i=1;\,k=0),(j=1,2,3;l=0,1,2)}$ with the respective minor

$\rho_{it+k,jt+l[(i=1;\,k=1,2;\,i=2,3,k=0,1,2),(j=1,l=1,2;\,j=2,3,l=0,1,2)]} =$

$$\begin{bmatrix} 1 & 0.257 & 0.202 & 0.602 & 0.269 & 0.204 & 0.922 & 0.213 \\ 0.257 & 1 & 0.153 & 0.202 & 0.602 & 0.228 & 0.204 & 0.922 \\ 0.202 & 0.153 & 1 & 0.258 & 0.112 & 0.419 & 0.163 & 0.143 \\ 0.602 & 0.202 & 0.258 & 1 & 0.258 & 0.227 & 0.419 & 0.163 \\ 0.269 & 0.602 & 0.112 & 0.258 & 1 & 0.259 & 0.226 & 0.419 \\ 0.204 & 0.228 & 0.419 & 0.227 & 0.259 & 1 & 0.175 & 0.192 \\ 0.922 & 0.204 & 0.163 & 0.419 & 0.226 & 0.175 & 1 & 0.174 \\ 0.213 & 0.922 & 0.143 & 0.163 & 0.419 & 0.192 & 0.174 & 1 \end{bmatrix} = 0.0037$$

The second row and column of the 9x9 autocorrelation matrix are $\rho_{it+k,jt+l[(i=1;\,k=1),(j=1,2,3;l=0,1,2)]}$ and $\rho_{it+k,jt+l[(i=1,2,3;\,k=0,1,2),(j=1;l=1)]}$. Since "21" is symmetric, the crossed elements of the first row and column can be written as $\rho_{it+k,jt+l[(i=1;\,k=1),(j=1,2,3;l=0,1,2)]}$ and $\rho^{I}_{it+k,jt+l[(i=1;\,k=1),(j=1,2,3;l=0,1,2)}$ with its respective minor

$\rho_{it+k,jt+l[(i=1;\,k=0,2;\,i=2,3,k=0,1,2),(j=1,l=0,2;\,j=2,3,l=0,1,2)]} =$

$$\begin{bmatrix} 1 & 0.250 & 0.602 & 0.269 & 0.280 & 0.922 & 0.213 & 0.213 \\ 0.250 & 1 & 0.153 & 0.202 & 0.602 & 0.228 & 0.204 & 0.922 \\ 0.602 & 0.153 & 1 & 0.258 & 0.112 & 0.419 & 0.163 & 0.143 \\ 0.269 & 0.202 & 0.258 & 1 & 0.258 & 0.227 & 0.419 & 0.163 \\ 0.280 & 0.602 & 0.112 & 0.258 & 1 & 0.259 & 0.226 & 0.419 \\ 0.922 & 0.228 & 0.419 & 0.227 & 0.259 & 1 & 0.175 & 0.192 \\ 0.213 & 0.204 & 0.163 & 0.419 & 0.226 & 0.175 & 1 & 0.174 \\ 0.213 & 0.922 & 0.143 & 0.163 & 0.419 & 0.192 & 0.174 & 1 \end{bmatrix} = 0.0037$$

The third row and column of the 9x9 autocorrelation matrix are $\rho_{it+k,jt+l[(i=1;\,k=2),(j=1,2,3;l=0,1,2)]}$ and $\rho_{it+k,jt+l[(i=1,2,3;\,k=0,1,2),(j=1;l=2)]}$. Since "21" is symmetric, the crossed elements of the first row and column can be written as $\rho_{it+k,jt+l[(i=1;\,k=2),(j=1,2,3;l=0,1,2)]}$ and $\rho^{I}_{it+k,jt+l[(i=1;\,k=2),(j=1,2,3;l=0,1,2)}$ with its respective minor

$\rho_{it+k,jt+l[(i=1;\,k=0,1;\,i=2,3,k=0,1,2),(j=1,l=0,1;\,j=2,3,l=0,1,2)]} =$

$$\begin{bmatrix} 1 & 0.258 & 0.602 & 0.269 & 0.280 & 0.922 & 0.213 & 0.213 \\ 0.258 & 1 & 0.202 & 0.602 & 0.269 & 0.204 & 0.922 & 0.213 \\ 0.602 & 0.202 & 1 & 0.258 & 0.112 & 0.419 & 0.163 & 0.143 \\ 0.269 & 0.602 & 0.258 & 1 & 0.258 & 0.227 & 0.419 & 0.163 \\ 0.280 & 0.269 & 0.112 & 0.258 & 1 & 0.259 & 0.226 & 0.419 \\ 0.922 & 0.204 & 0.419 & 0.227 & 0.259 & 1 & 0.175 & 0.192 \\ 0.213 & 0.922 & 0.163 & 0.419 & 0.226 & 0.175 & 1 & 0.174 \\ 0.213 & 0.213 & 0.143 & 0.163 & 0.419 & 0.192 & 0.174 & 1 \end{bmatrix} = 0.0036$$

The fourth row and column of the 9x9 autocorrelation matrix are $\rho_{it+k,jt+l[(i=2;\,k=0),(j=1,2,3;l=0,1,2)]}$ and $\rho_{it+k,jt+l[(i=1,2,3;\,k=0,1,2),(j=2;l=0)]}$.
This can be written as $\rho_{it+k,jt+l[(i=2;\,k=0),(j=1,2,3;l=0,1,2)]}$ and $\rho^{I}_{it+k,jt+l[(i=2;\,k=0),(j=1,2,3;l=0,1,2)}$ with its respective minor

$\rho_{it+k,jt+l[(i=2;\,k=1,2;\,i=1,3,k=0,1,2),(j=2,l=1,2;\,j=1,3,l=0,1,2)]} =$

$$\begin{bmatrix} 1 & 0.258 & 0.25 & 0.269 & 0.280 & 0.922 & 0.213 & 0.213 \\ 0.258 & 1 & 0.257 & 0.602 & 0.269 & 0.204 & 0.922 & 0.213 \\ 0.25 & 0.257 & 1 & 0.202 & 0.602 & 0.228 & 0.204 & 0.922 \\ 0.269 & 0.602 & 0.202 & 1 & 0.258 & 0.227 & 0.419 & 0.163 \\ 0.280 & 0.269 & 0.602 & 0.258 & 1 & 0.259 & 0.226 & 0.419 \\ 0.922 & 0.204 & 0.228 & 0.227 & 0.259 & 1 & 0.175 & 0.192 \\ 0.213 & 0.922 & 0.204 & 0.419 & 0.226 & 0.175 & 1 & 0.174 \\ 0.213 & 0.213 & 0.922 & 0.163 & 0.419 & 0.192 & 0.174 & 1 \end{bmatrix} = 0.0007$$

The fifth row and column of the 9x9 autocorrelation matrix are $\rho_{it+k,jt+l[(i=2;\,k=1),(j=1,2,3;l=0,1,2)]}$ and $\rho_{it+k,jt+l[(i=1,2,3;\,k=0,1,2),(j=2;l=1)]}$.
This can be written as

$\rho_{it+k,jt+l[(i=2;\, k=1),(j=1,2,3;l=0,1,2)]}$ and $\rho^{I}_{it+k,jt+l[(i=2;\, k=1),(j=1,2,3;l=0,1,2)}$ with its respective minor

$\rho_{it+k,jt+l[(i=2;\, k=0,2;\, i=1,3,k=0,1,2),(j=2,l=0,2;\, j=1,3,l=0,1,2)]} =$

$$\begin{bmatrix} 1 & 0.258 & 0.25 & 0.602 & 0.280 & 0.922 & 0.213 & 0.213 \\ 0.258 & 1 & 0.257 & 0.202 & 0.269 & 0.204 & 0.922 & 0.213 \\ 0.25 & 0.257 & 1 & 0.153 & 0.602 & 0.228 & 0.204 & 0.922 \\ 0.602 & 0.202 & 0.153 & 1 & 0.112 & 0.419 & 0.163 & 0.143 \\ 0.280 & 0.269 & 0.602 & 0.112 & 1 & 0.259 & 0.226 & 0.419 \\ 0.922 & 0.204 & 0.228 & 0.419 & 0.259 & 1 & 0.175 & 0.192 \\ 0.213 & 0.922 & 0.204 & 0.163 & 0.226 & 0.175 & 1 & 0.174 \\ 0.213 & 0.213 & 0.922 & 0.143 & 0.419 & 0.192 & 0.174 & 1 \end{bmatrix} = 0.0007$$

The minor of the sixth row and column of the matrix is

$$\rho_{it+k,jt+l[(i=2;\, k=0,1;\, i=1,3,k=0,1,2),(j=2,l=0,1;\, j=1,3,l=0,1,2)]} = 0.0007$$

The minor of the seventh row and column of the matrix is

$$\rho_{it+k,jt+l[(i=3;\, k=1,2;\, i=1,2,k=0,1,2),(j=3,l=1,2;\, j=1,2,l=0,1,2)]} = 0.0028$$

The minor of the eighth row and column of the matrix is

$$\rho_{it+k,jt+l[(i=3;\, k=0,2;\, i=1,2,k=0,1,2),(j=3,l=0,2;\, j=1,2,l=0,1,2)]} = 0.0028$$

The minor of the ninth row and column of the matrix is

$$\rho_{it+k,jt+l[(i=3;\, k=0,1;\, i=1,2,k=0,1,2),(j=3,l=0,1;\, j=1,2,l=0,1,2)]} = 0.0028$$

$$\left|\rho_{it+k,jt+l}\right| = \begin{vmatrix} 1 & 0.258 & 0.250 & 0.603 & 0.269 & 0.280 & 0.922 & 0.213 & 0.213 \\ 0.258 & 1 & 0.257 & 0.202 & 0.602 & 0.269 & 0.204 & 0.922 & 0.213 \\ 0.259 & 0.257 & 1 & 0.153 & 0.202 & 0.602 & 0.228 & 0.204 & 0.922 \\ 0.603 & 0.202 & 0.153 & 1 & 0.258 & 0.112 & 0.419 & 0.163 & 0.143 \\ 0.269 & 0.602 & 0.202 & 0.258 & 1 & 0.258 & 0.227 & 0.419 & 0.163 \\ 0.280 & 0.269 & 0.602 & 0.112 & 0.258 & 1 & 0.259 & 0.226 & 0.419 \\ 0.922 & 0.204 & 0.228 & 0.419 & 0.227 & 0.259 & 1 & 0.175 & 0.192 \\ 0.213 & 0.922 & 0.204 & 0.163 & 0.419 & 0.226 & 0.175 & 1 & 0.174 \\ 0.213 & 0.213 & 0.922 & 0.143 & 0.163 & 0.419 & 0.192 & 0.174 & 1 \end{vmatrix}$$

$$= 0.0003$$

The results of the computations of the principal minors and determinant of the 9x9 autocorrelation matrix have positive values (greater than zero). This reveals cross-autocovariances/cross-autocorrelations stationarity of the multivariate process.

Conclusion

Positive Definiteness of autocovariance and autocorrelations for univariate time series and cross-autocovariance and cross-autocorrelations for multivariate time series is a property investigated to ascertain stationarity of a time series. What account for positive definiteness in a stationary process are the conditions that the principal minors and determinant of the autocorrelations and cross-autocorrelations have values greater than zero. In this paper, the return series of the three vector series were obtained. These were succeeded with the computations of the cross-covariances and cross-autocorrelations as tools to investigate the stationarity of the multivariate process. The results have confirmed stationarity of the cross-covariance process of Average, Urban and Rural Nigeria Price Indices. This paper recommends that in multivariate time series, investigation of stationarity should incorporate the sub-autocovariance and autocorrelation matrices of individual vector process as well as the larger cross-covariance and cross-autocorrelation matrices.

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