A NEW ORDERING POLICY FOR THE FIXED LIFETIME INVENTORY SYSTEM

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Abstract
The fixed lifetime inventory system is one of the most critical aspect of operations management. This is because of the effect of outdating on products, when not used to meet demand during their useful lifetime in inventory. Over the years, fixed lifetime inventory models have based the decision to reorder on the quantity of products on hand, rather than the age of items on hand at the point of reorder. In this work, we propose a new fixed lifetime inventory model where the decision to reorder is based on the number of useful lifetime remaining on the products rather than just the quantity of products on hand. The model assumes lost sales.

Keywords and Phrases: *Fixed lifetime, outdates, useful lifetime, inventory, lost sales, reorder point.*

1. Introduction

The fixed lifetime inventory system consists of products with expiration date. Fixed lifetime products have deterministic shelf life i.e. if a product remains unused by the end of its useful lifetime in inventory, it is considered outdated. Outdating refers to a sudden death in value or usability of a product over time and outdated products must be discarded. However, all units which have not expired have constant utility, Omosigho (2002). Over the years, fixed lifetime inventory models have based the decision to reorder on the quantity of products on hand, rather than the age of items on hand at the point of reorder. Authors in the literature with ordering policies based on the quantity of products on hand include; Chiu (1994), Bookbinder and Cakanyildirim (1999), Mohammad et al (2007), Hariga (2010) and Siriruk (2012) all of whom considered the ordering policy (Q,r) which order Q whenever inventory on hand drops to the reorder point r. Nahmias (1978), Hollier et al (1995), Liu and Lian (1999) and Silver et al (2012) considered the ordering policy (s,S) which order up to S when inventory on hand drops to the reorder point s. Schmidt and Nahmias (1985), Perry and Posner (1998), Kranenburg and Houtum (2007) and Olsson and Tydesjo (2010) considered the ordering policy $(S-1,S)$ in which an order is placed for exactly one item each time inventory is depleted by either demand or outdating. Nahmais and Pierskalla (1973) considered the ordering policy (x,y) where y is the ordered quantity and x the sum of items (items with different age categories) on hand must be less than a certain quantity. Shen et al (2012) considered an ordering policy that maintains a minimum volume of inventory, whenever inventory drops to this level a new order is placed. As a deviation from these ordering policies, we propose a new fixed lifetime inventory model where the decision to reorder is based on the number of useful lifetime remaining on the items on hand rather than just the quantity of items on hand. The ordering policy of our model is based on the number of useful lifetime remaining on the items on hand when placing new orders and not the quantity. Our policy takes the form $(y, m-1)$ interpreted as order y when the useful lifetime remaining on the items on hand is one period. One advantage of our ordering policy over existing policies is that, the policy is not fixed. If the demand is high, the inventory manager can decide to reorder new items with two periods remaining on the items on hand instead of one period. This is very common during festive periods when sales are high. If the demand drops the inventory manager can reverse back to placing new orders with one useful period remaining on the items on hand. If new orders are placed with two periods remaining, our ordering policy will be $(y, m-2)$ order y when the useful lifetime remaining on the items on hand is two periods.

The other sections of this work are as follows; section2 presents the assumptions and notation of the model, section3 gives a description of the model, section4 derivation of costs components, section5 the difference between our model and existing models, section6 shows the convex property of our total cost function, section7 illustrating our model on two fixed lifetime products, bread and egg. Bread has a fixed lifetime of 4days and egg has a fixed lifetime of 5weeks, section8 data analysis, section9 comparing result from our model with result from existing models, section10 research observation and recommendations and section11 conclusion.

2. Assumptions and Notation of the model

2.1 Assumptions

(1) Ordering of new products is based on the remaining useful lifetime of the items on hand. We place order for new items when the useful lifetime remaining on the items on hand is one period. One period is particular to the model presented in this work.

(2) Only one order is placed at a time. There are no outstanding orders.

(3) The new order arrive instantly, whenever orders are placed.

(4) Order received is used to satisfy demand in period 1,2, . . . m .

(5) The fixed lifetime of the product is a positive integer m

(6) A complete cycle consist of two consecutive order receive. One at the beginning of the cycle and the other at the end of the cycle. The time in between is the cycle length.

(7) Shortage occur whenever the on hand inventory is not able to satisfy all the demand in a period. Only a part of the demand is satisfied while the other part is lost. That is lost sale is assumed and a shortage cost is charged against the inventory manager.

(8) Item(s) not used to meet demand by the end of period m , outdates and are discarded. An outdate cost is charged against the inventory manager.

(9) The issuing policy is FIFO, that is oldest units must be used to meet demand before the new ones are used.

(10) All units of the new order are of the same age and arrive into the inventory at age zero.

(11) Demand in each period are not known but assumed to be independent and identically distributed random variables $d_1, d_2, \ . \ . \ .$ with known distribution f. The demand $t = \sum_{i=1}^{m} d_i$ has density f^* which is the n -fold convolution of f with itself.

2.2 Notation

m = lifetime of the product. m is a positive integer

$d_i = demand\ in\ period\ i$

$t = \sum_{i=1}^{m} d_i$, $total\ demand\ with\ distribution\ f^*(t)$

x represent the quantity of products with one useful period remaining in them. That is $x = y - \sum_{i=1}^{m-1} d_i$

y = new products ordered/entering into inventory with age zero

$T_i = period\ i\ (periods\ can\ be\ in\ hours,\ days,\ weeks\ or\ years\ depending\ on\ the\ product)$

θ = outdate cost per unit
v = shortage cost per unit
h = holding cost per unit
k = fixed ordering cost per unit.

3. Model description

The model involves a single fixed lifetime product in a single location. New orders arrives with one useful lifetime remaining on the items on hand. The total cost function for the model consist of the following cost components:

(1) Ordering cost: cost of placing new orders.
(2) Holding cost: For every amount of products held in inventory there is a holding cost per unit.
(3) Shortage cost: for every demand that cannot be satisfied from the stock on hand, a shortage cost is charged against the inventory manager.
(4) Outdate cost: for every product not used to satisfy/meet demand at the end of its useful lifetime in inventory, an outdate cost is charged against the inventory manager.

Table 1 shows the model for a product with m useful lifetime.

Table 1: Orders and Age distribution for a product with m lifetime

$Order$	T_1	T_2	T_3	$\cdots$	T_{m-1}	T_m	T_{m+1}	T_{m+2}
1	$(y-d_1)^+$ $age\,1$	$(y-\sum_{i=1}^{2} d_i)^+$ $age2$	$(y-\sum_{i=1}^{3} d_i)^+$ $age3$	$\ldots$	$(y-\sum_{i=1}^{m-1} d_i)^+$ $age\,m-1$	$x-d_m$ $age\ m$		

2	$(y-d_1)^+$ $age\,1$	$(y-\sum_{i=1}^{2} d_i)^+$ $age\,2$	....	$(y-\sum_{i=1}^{m-1} d_i)^+$ $age\,m-1$
3				y $age\,0$

In Table1, T_i is period i. The first order y arrives at the start of period1 at age zero. At the end of period1 it reduces by the demand in period1 and becomes $(y-d_1)^+$ age1. The amount of items brought into period2 is $(y-d_1)^+$ and this reduces (by demand) in period2 and becomes $(y-d_1-d_2)^+$ age2. This continues until the reordering period $m-1$ when the first order reduces to $y-\sum_{i=1}^{m-1} d_i$ and the second order arrives. Finally, at the end of the m^{th} period items from the first order not used to meet demand outdates leaving only items from the second order. The process continues with the second, third, orders. Next, we consider the outlook of two fixed lifetime products with useful life of four and five periods respectively.

4 Derivation of Costs Components

4.1 Shortage Cost

Shortage, unsatisfied demand, run out, or stock out occurs when total demand exceeds on hand inventory. The units on hand are wholly/completely used to satisfy only a part of the demand, while the unsatisfied demand is lost. This is because the model assumes lost sales of excess demand.

The expected unsatisfied demand per order for our model is

$$Expected\ \ (unsatisfied\ demand) = \int_{x+y}^{\infty} (t-(x+y)) f^*(t)dt \tag{1}$$

With v as the shortage cost per unit, our shortage cost is

$$shortage\ \cos t = v \int_{x+y}^{\infty} (t-(x+y)) f^*(t)dt \tag{2}$$

4.2 Outdate Cost

Outdate or expiration of products occurs when the total demand is less than the on hand inventory. Outdate is caused by products over staying their useful lifetime in inventory because of low demand. The demand is satisfied by a part of the on hand inventory while the other part not used to meet demand will outdate. Since new orders arrives with one useful lifetime left in the items from the previous order, only items from the previous order can outdate, if not used to meet demand in their last useful period.

From Table 1, to determine the outdates from order 1 our interest will be on the m^{th} period which is the last useful period for order1 and the second useful period for order 2. This imply that only items from order 1 not used to meet demand will outdate at this point, since the issuing policy is first in first out. The total amount of items on hand from order 1 at the end of the m^{th} period is $x - d_m$.

To obtain the outdate quantity, we consider the following cases.

Case1: if $d_m = x$. The amount of m periods old items from the order1 is equal to the demand in the m^{th} period. Since the issuing policy is FIFO, there will be no outdating as the demand is completely met by the m periods old items in inventory.

Case 2: if $d_m > x$. The amount of m periods old items from the order1 is less than, the demand in the m^{th} period. Part of the demand is satisfied by all of the m periods old items while the other part is satisfied by items from the new order and there will be no outdating.

Case3: if $d_m < x$. The amount of the m periods old items from the order1 is more than the demand in the m^{th} period. Since the issuing policy is oldest units first, part of the m periods old items will be used to meet the demand, while the other part will outdate. Thus the expected outdate quantity per order will be $x - t$, that is

$$Expected \quad (outdate) \quad = \int_0^x (x-t) f(t) dt \qquad (3)$$

With an outdate cost of θ per unit, our outdate cost is

$$outdate \cos t = \theta \int_0^y (y-t) f^*(t) dt \qquad (4)$$

4.3 Holding Cost

With a holding cost of $h > 0$ per unit held in inventory our holding cost is

$$Holding \cos t = h \int_0^{x+y} (x+y-t) f^*(t) dt \qquad (5)$$

4.4 Ordering Cost

There is a fixed ordering cost k per unit ordered, so that our ordering cost is ky.

Therefore our total cost function is

$$C(x,y) = \min_{y \geq 0} \{ky + h \int_0^{x+y} (x+y-t) f^*(t) dt + v \int_{x+y}^{\infty} (t-(x+y)) f^*(t) dt + \theta \int_0^y (y-t) f^*(t) dt\} \qquad (6)$$

where

$f^*(t) =$ *distribution of total demand*.

Now, an inventory policy answers two questions; what quantity to order and when to order the quantity. The answer to the second question is already known with our model (that is when the useful lifetime remaining on the items from the previous order is one period). To answer the first question, we minimize equation (6) and solve for y.

5.2 Outdate Alarm System (OAS)

In March 2016, NAFDAC destroyed expired malt and glucose belonging to Guinness Nigeria PLC worth millions of naira (PM news 9th march, 2016). The two products are normally stored in large structures called silo. Motivated by this lost, we are working on an outdate alarm system (OAS) which can be installed in these structures. The alarm goes off when the number of useful lifetime remaining on the products drops to the required reorder period eg when the remaining useful period is one or when the remaining useful period is two. This will check the outdating of products kept in silos or other similar structures as they are not visible to the eyes.

5.3: Outdate code

During the course of this research, the following were our observations.

Firstly, many inventory managers are involved in repackaging of outdated products in order to maximize profit. Repackaging is the act of exchanging the original pack of a product with the right expiration date, with a new pack carrying a false or wrong expiration date. This is very common with inventory managers in the food and pharmaceuticals sectors.

In response to this, we recommend that the expiration date be written on the product and not on the park.

Secondly, some products (especially pharmaceuticals) carry verification code on them. The code is sent to the manufacturing company and an instant message sent back saying the product is authentic and safe for consumption. It however does not tell you weather the product has expired or when it will expire. We recommend that an expiration code be added to the verification code. The expiration code can be sent via sms to the producing company and an instant message sent back to the consumer with either one of the following messages (i) expired on the 2nd April, 2019, for an expired product or (ii) Not expired, will expire on 2nd April 2020, for a product that has not expired. The expiration code should be on the product and not on the pack. The introduction of the expiration code and the verification code (already being practice) will make the products safer for Nigerians. Together, the codes will tell you that the product is authentic and when it will expire.

Thirdly, the verification code and the expiration code can be combined into one code called the verification and expiration code or simply (VEC). This will go a long way in reducing the problem of repackaging.

6. Conclusion

We have used the remaining useful life of a product as a decision variable for developing inventory policy for the fixed lifetime inventory system. The model is easy to implement and can be extended to product with m useful lifetime. The problem of rebranding can be eliminated with the introduction of the expiration code on the product.

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