THE FLEXIBILITY OF THE USEFUL LIFETIME BASED FIXED LIFE INVENTORY MODEL

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Abstract
The ordering policies for a fixed lifetime inventory system is a policy statement which determine when to order and how much to order. Over the years the decision to order was based on the quantity of items on hand at the time of placing the order. Recently, ordering policies based on the number of useful lifetime remaining on the items on hand, were developed. In this work, we examine the flexibility of the ordering polices based on the number of useful lifetime remaining on the items on hand and give some advantages it hold over the quantity based policies.

Keywords: *Quantity, ordering, policy, flexibility, period*.

Introduction

The ordering policies of a fixed lifetime inventory system tell the inventory manager when to order and how much to order. Policies, based on quantity on hand, orders new items when the quantity of items on hand drops to the reorder level. The items on hand are reduced by demand or outdating. Some ordering policies based on quantity on hand include; (Q, r) [Chiu (1995), Nahmias (1978), Hariga (2010)] which order Q when the inventory drops to r , (s, S) [Liu and Lian (1999), Silver et al (2012)] which order up to S when inventory drops to s and $(S, S-1)$ [Olsson and Tydesjo (2010)] which order one anytime an item is used to meet demand. The condition for applying these policies is that the quantity of items on hand must drop to the reorder point. They do not consider the age of items in inventory while waiting for inventory to drop to the reorder point, thereby increasing the chances of outdating. On the other hand the useful lifetime based policies do not consider the quantity of items, but the number of useful lifetime remaining on the items on hand. The age of the items is tracked from the time they arrive into inventory to the time they are used to meet demand or outdate. Tracking these items with their age help to minimize outdate. Example of useful lifetime based model include; $(y, m-1)$ [Izevbizua and Omosigho (2017)] which order y when the useful lifetime remaining on the items on hand is one period, $(y, m-2)$ which order y when the useful lifetime remaining is 2 periods and $(y, x, m-1)$ which order y when the useful lifetime remaining on the items x on hand is one period.

While the reorder point of the inventory system is fixed for quantity based model, the number of useful lifetime remaining before placing a new order can be reviewed from time to time depending on demand for useful lifetime model. When demand is high, we increase the number of periods and when demand is low, we reduce the number of useful periods left before placing a new order. This flexibility gives the useful lifetime based model an advantage.

Features of Useful Lifetime Model

(i) Placement of new order depends on the number of useful life remaining on the items on hand.
(ii) The quantity ordered is never fixed
(iii) The number of useful periods remaining on items on hand before placing a new order can be changed depending on prevailing demand.

Features of Quantity Model

(i) Placement of new order depends on the quantity of items on hand.
(ii) The quantity ordered is fixed for most of the quantity based models
(iii) The quantity of items on hand before placing a new order is fixed.

Description of the Useful Lifetime Based Model

We present the model's outlook for two ordering periods (when one useful period is remaining and the case when two useful periods are remaining) for products with four, five and m useful lifetime.

Case1: Useful life of product is 4periods. One period left before ordering

Period	1	2	3	4	5	6	7
	y_1	$y_1 - d_1$	$y_1 - \sum_{i=1}^{2} d_i$	$y_1 - \sum_{i=1}^{3} d_i$	outdating		
			y_2	$y_2 - d_1$			

Case2: Useful life of product is 5periods. Two periods left before ordering.

Period	1	2	3	4	5	6	7
	y_1	$y_1 - d_1$	$y_1 - \sum_{i=1}^{2} d_i$	$y_1 - \sum_{i=1}^{3} d_i$	$y_1 - \sum_{i=1}^{4} d_i$	outdating	
			y_2	$y_2 - d_1$	$y_2 - \sum_{i=1}^{2} d_i$		

Case3: Useful lifetime of product is m . One period left before ordering.

Period	1	2	3	. . .	$m-1$	m	$m+1$
	y_1	$y_1 - d_1$	$y_1 - \sum_{i=1}^{2} d_i$		$y_1 - \sum_{i=1}^{m-2} d_i$		outdating
					y_2		

Case4: Useful lifetime of product is m . Two periods left before ordering.

Period	1	2	3	. . .	$m-2$	$m-1$	m	$m+1$
	y_1	$y_1 - d_1$	$y_1 - \sum_{i=1}^{2} d_i$		$y_1 - \sum_{i=1}^{m-3} d_i$			outdating
					y_2			

For case1, y_1 arrives in period 1. At the start of period 2 the order has reduced to $y_1 - d_1$ *where* d_1 is the demand in period 1. The first order reduces to $y_1 - \sum_{i=1}^{2} d_i$ in period 3 and a new order arrives. items from y_1 not used to meet demand in period 4 outdates in period 5, since the useful period of the product is 4periods.

For case4, y_1 arrives in period 1 and reduces to $y_1 - d_1$ at the start of period 2. In period $m-2$, the items from y_1 reduces to $y_1 - \sum_{i=1}^{m-3} d_i$ and a new order arrives, since we order with two useful life left on the items on hand. Items not used to meet demand in period m outdate in period $m+1$.

Next, we derive the one period cost function per order for the useful lifetime based model and give numerical examples of the model.

Notation of the model

m = lifetime of product
d_i = demand in period i
$f(d)$ = demand distribution
y_i = order i, $i = 1,2,3,4,5...$
x = quantity on hand at the last useful period
θ =outdate cost per unit
α = shortage cost per unit
h = holding cost
β = ordering cost

Derivation of Cost Function

On hand inventory refers to the amount of items in inventory at any given time. Since items are reduced by demand during their useful periods, we that the on hand per order is given as

Holding cost = $h\,(y - \sum_{i=1}^{p} d_i)$, $p \geq 2$ (1).

To see this, if y arrives in period 1,

At the start of period 2 it reduces to $y - d_1$

At the start of period 3 it reduces to $y - d_1 - d_2$

At the start of period 4 it reduces to $y - d_1 - d_2 - d_3$

At the start of period m it reduces to $y - d_1 - d_2 - .,..- d_{m-1}$

The issuing policy is FIFO that is oldest units must be use to meet demand before the items arriving. Items from the old order outdate before items from the new order. If d_m^1 represent all the demand during the useful life of an order then the outdate quantity will be $y - d_m^1$. If $d_m^1 < y$

then the outdates will be $\int_0^y (y-d_m^1)\,f(d_m^1)dd_m^1$. With a cost of θ per unit, our outdate cost will be

$$\text{Outdate cost} = \theta\int_0^y (y-d_m^1)\,f(d_m^1)dd_m^1 \qquad (2)$$

Similarly, if d_m^1 is greater than y, our shortage quantity will be $\int_y^\infty (d_m^1-y)\,f(d_m^1)dd_m^1$. With a shortage cost of λ per unit our shortage cost is

$$\text{Shortage cost} = \lambda\int_y^\infty (d_m^1-y)\,f(d_m^1)dd_m^1 \qquad (3)$$

The ordering cost per unit is β and our ordering cost is $\beta\, y$.
Therefore our one order total cost function for the model is

$$f(y)=\beta y+h(y-\sum_{p=1}^{m}d_i)+\theta\int_0^y (y-d_m^1)\,f(d_m^1)dd_m^1+\lambda\int_y^\infty (d_m^1-y)\,f(d_m^1)dd_m^1 \qquad (4)$$

Equation (4) can minimized by differentiating and equating to zero, to obtain a minimum value for y when required.

Numerical Examples

The ordering policy of the model was implemented on two fixed lifetime products with useful life 4 and 6 periods. The results are shown in the Table 5 and Table 6.

Table 5: Ordering policy for a product with four useful life and placing order with one useful life remaining on items on hand

Day	Ordered Quantity	On hand inventory	Daily demand	Outdates	Shortages
1	100	100	20	-	-
2		80	25	-	-
3	80	55+80	30	-	-
4		25+80	40	-	-
5	55	65+55	52	-	-
6		13+55	36	-	-
7	100	32+100	30	-	-
8		2+100	43	-	-
9	90	59+90	50	-	-
10		9+90	45	-	-
11	80	54+80	40	-	-
12		14+80	54	-	-
13	80	30+80	53	-	-
14		57	30	-	-
15	100	27+100	64	-	-

16		63	65	-	2
17	100	100	54	-	-
18		46	36	-	-
19	90	10+90	56	-	-
20		44	40	-	-

The Table shows the quantity of items ordered and the daily demand. For day 1 the quantity ordered was 100 and the daily demand was 20. For day 2, on hand inventory drops to 80 and the daily demand was 25. For day 3, items from the first order have 1 useful life left, so a new order of 80 arrives making inventory 135 (55 from old order and 80 from the new order) and daily demand is 30. Since the issuing policy is FIFO, items from the old order must be used to meet demand before items from the new order. Also, observe that after 20days and 10 orders, there are no outdates and only two shortages from order 8.

Table 6: Ordering policy for a product with six useful life and placing order with two useful life remaining on items on hand

Day	**Ordered quantity**	**Inventory on hand**	**Daily demand**	**Outdates**	**Shortages**
1	150	150	40	-	-
2		110	25	-	-
3		85	10	-	-
4	80	75+80	15	-	-
5		60+80	30	-	-
6		30+80	28	2	-
7	40	80+40	45	-	-
8		35+40	40	-	-
9		35	10	-	-
10	60	25+60	16	-	-
11		9+60	32	-	-
12		37	15	-	-
13	80	22+80	28	-	-
14		72	15	-	-
15		57	20	-	-
16	90	37+90	30	-	-
17		7+90	30	-	-
18		57	17	-	-
19	80	40+80	48	-	-
20		72	25	-	-

Table 6 shows the ordering policy for a product with six useful periods and placing new order with two useful periods remaining. Again, after 20 days and 7 orders there are two outdates and no shortages.

The number of periods before placing order can be changed from time to time depending on the prevailing demand. When the demand is high the number can be increase and when demand is low the number can be reduced. This is the flexibility associated with the useful lifetime based models.

Discussion

Placing new order on the basis of the useful lifetime remaining on the items on hand, help to reduce the quantity outdating in the fixed lifetime inventory system. As shown in Table 6, there were only two outdates from 7 orders. Inventory managers are able control when to order or not to order. When demand is high the number of useful period(s) remaining on the items on hand before placing order is increased and decreased when the demand is low.

Conclusion

The biggest problem associated with the fixed lifetime inventory is outdating of products. Beside the economic loss suffered by the inventory manager, outdated products (especially food and drugs) can negatively affect the health of the citizens. The useful lifetime based model was designed to minimize the amount of items outdating from a fixed lifetime inventory, using the number of useful life left in an order as a bases for placing new orders. The inventory manager can either increase or decrease the required number of useful left before placing a new order following the prevailing demand.

References

Chiu, H.N. (1995). An Approximation to the Continuous review Inventory Model with Perishable Items and Lead Times. European Journal to Operational Research. Vol. 87. pp. 93-108.

Hariga, M. (2010). A single-item continuous review inventory problem with space restriction. International journal of production economics. Vol 128, pp 153-158.

Liu, L. and Lian, Z. (1999). (s,S) Continuous Review Models for Products with Fixed Lifetime. Operations Research, Vol. 47(1). pp. 150-158.

Olsson, F. and Tydesjo, P. (2010). Inventory Problems with Perishable items: Fixed Lifetimes and Backlogging. European Journal of Operational Research. Vol. 202. pp .131-137.

Silver, E.A, Bischak, D.P and Kok, T (2012). Determining the reorder point and the order-up-to level to satisfy two constraints in a periodic review system under negative binomial demand.

Nahmias, S (1978). The fixed-charge perishable inventory problem. Journal of Operations Research. Vol 26, (3) pp. 464-481.

Izevbizua, O and Omosigho, S.E. (2017). A review of the fixed lifetime inventory system. Journal of Mathematical Association of Nigeria. Vol 44 (2), pp. 188-198.