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Modified 3-step Implicit Linear Multistep Method for Time Series Forecasting

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Abstract
In this paper, a time series smoothing technique derived from the modification of the 3-step implicit linear multistep numerical method was developed. The modified time series smoothing technique proposed in this paper was called 3-step mAMT and is applied to a dataset with a high fluctuation. The 3-step mAMT is compared with the simple moving average and the simple exponential smoothing ($\alpha = 0.8$) and the 3-step mAMT performed better than the two other techniques. The measures of best fit used are the MAE, MSE, RMSE, and MAPE. The mAMT Order-4 produced a better result when compared to moving average and the simple exponential smoothing having an R^2 greater than the other techniques.
Keywords: Time series, linear Multistep, Adams-Moulton, Implicit, 3-step, Smoothing technique, Forecasting.

Introduction

Due to the presence of random variations in real time series data and the effects they pose to forecast results, it is sometimes important to reduce these variations by way of smoothing the data using adequate time series smoothing techniques. These smoothing techniques are essential as they are deployed to reduce white nose from the time series data and highlight some useful details contained in the data. Fluctuations presence in the time series data can be carefully removed, so that appropriate forecast can be adequately made.

There are numerous time series smoothing techniques available to data scientists but a few are widely used. Because of its simplicity, exponential smoothing is widely used and produces wonderful results as they assign more weights to the most recent observations during the computation process. Exponential smoothing is the old smoothed value plus a systemic adjustment of errors that occurred in the new smoothed values (Ostertagova et al., 2012). This technique is best applied to time series data that do not have trends or seasonality (Paul, 2011; Ravinder, 2013; Kumari et al., 2014; Silitonga et al., 2020). It is also the smoothing technique that is most adequate in analyzing univariate time series data (Attanayake et al., 2020). The accuracy of the exponential smoothing technique relies heavily on the smoothing constant. There are diverse types of the exponential smoothing techniques (Pronchakovet al., 2019) but the most common is the simple exponential smoothing (SES).

The simple exponential smoothing equation can be written as:

$$Y_{t+1} = \alpha f_t + (1 + \alpha) Y_t \tag{1}$$

where f_t is the actual observation, Y_t is the smoothened value of the f_t at time t, Y_{t+1} is the smoothened value of f_t at time $t + 1$, and α is the smoothing constant.

The beautiful time series smoothing technique developed by Nwokike et al., (2021), is another newly introduced data smoothing technique that produces some very good results. The technique was derived from the modification of the Adams-Bashforth oder-4 numerical technique and used for time series data smoothing in a way similar to what we have developed in this research work. The model can be seen below:

$$Y_t = \frac{1}{24}(55f_t - 59f_{t-1} + 37f_{t-2} - 9f_{t-3}) \tag{2}$$

The moving average smoothing technique is another widely used time series smoothing technique (Pierrefeu, 2019), and has similarity to the exponential smoothing technique in removing white noise in time series data. This technique is easy to implement and for this reason is seen as common and useful if the series has no identifiable tendency of trend or seasonality (Ostertagova 2016). There are many types of the moving average smoothing technique (Ostertagova 2016; Pierrefeu, 2019).

The simple moving average equation can be written as:

$$Y_{t+1} = \frac{1}{n}\sum_{j=1}^{n-1} f_{t-n+1+j} \tag{3}$$

Where;
$t = n, n + 1, \dots, N$
Y_t =New forecast
f_t =Original time series observation
$n =$ Number of observation.

The Holt-Trend exponential smoothing and ARIMA forecasting technique was applied by Khamis et al. (2020) to forecast gold bullion coin prices in Malaysia. Time series smoothing techniques have also been studied in the works of Kolkova (2018), Pronchakov et al. (2019), Raudys et al. (2018), Duttadeka et al. (2014), Alajbeg et al. (2017), Indriana et al. (2010), Mustapa et al. (2019), and Hung (2016).

In this research, focused is on developing a new time series smoothing technique derived from the modification of a linear multistep method called the Oder-4 Adams-Moulton method, which is suitable for all kind of data sets

The linear multistep numerical method is a numerical method for providing solution to ordinary and partial differential equations where the initial value problems are first given. The linear multistep methods are popular for solving first order initial value problem (IVP) but are also adequate for solving higher orders by first reducing them to first order IVPs (Jator, 2008). The generic description of a linear multistep method is described by Alen Alexanderian (2018) as: $x_n = a + nh,\ n = 0, \dots, N$ with $h = (b-a)/N$. The general form of a multistep method is

$$y_{n+1} = \sum_{j=0}^{p} \alpha_j y_{n-j} + h \sum_{j=0}^{p} \beta_j f(x_{n-j}, y_{n-j}),\ \ n \geq p \qquad (4)$$

Where $\{\alpha_i\}_{i=0}^{p}$ and $\{\beta_i\}_{i=-1}^{p}$ are constant coefficients, and$p \geq 0$. If $\alpha_p \neq 0$ or $\beta_p \neq 0$ either, then the method is called a $p+1$ step method. The initial values,$y_0, y_1, y_2, \dots, y_p$ must be obtained by other means (e.g., an appropriate implicit method). Note that if $\beta_{-1} = 0$ then the method is explicit, and if $\beta_{-1} \neq 0$, the method is implicit. Also, here we use the convention that $y(x_n)$ is the exact value of y at x_n and y_n is the numerically computed approximation to $y(x_n)$.

The 3-step Adams-Moulton method considered in this research is a special case of the linear multistep method called the implicit schemes. The 3-step Adams-Moulton method which we have stated are implicit, have minimal error constants, involves less steps and computation, and have a better stability region than the explicit linear multistep methods (Dattani, 2008).

The method leading to the derivation of the 3-step Adams-Moulton method was first developed by John Cauchy Adams who applied the implicit equation to derive the formulae. In 1926, Forest Ray Moulton in his research, discovered that the method can be applied simultaneously with the Adams-Bashforth method to produce a predictor-corrector numerical method (Ernst et al., 1993). This method is based on the idea of replacing with a polynomial the integrand that interpolates the function $f(x, y)$ at the points $x_n,\ y_n$.

Motivation

Science is emerging with unending ideas. It is interesting to see the advent of new techniques in problem solving especially in the field of mathematics and engineering. Scientific problems come in different shades and require careful study and recommendation for lasting solutions. The motivation for this work is the many uses of logarithms Laplace transformation, Fourier transforms and other mathematical techniques to solve problems in other diverse fields of study. This research is geared towards proposing a better time series smoothing technique, developed from the modification of an implicit linear multistep method also called the Order-4 Adams-Moulton method.

Derivation of the Modified3-step Adams-Moulton Smoothing Technique

The time series smoothing technique proposed in this paper is a modification of the3-step implicit linear multistep numerical method also referred to as the 3-step Adams-Moulton technique. The modified 3-step Adams-Moulton technique which we shall also call: *3-step mAMT* is derived as follows:
The linear multistep method is based on the Stone-Weierstrass Theorem.

Theorem: The Stone-Weierstrass Theorem - Let $f(t) : \mathbb{R} \to \mathbb{C}$ be continuous on $t \in [a, b]$. For all $\epsilon > 0, \exists$ a polynomial $\varphi(t) \ni ||f(t) - \varphi(t)|| < \epsilon$.
This is to say that; any continuous function can be approximated to an arbitrary accuracy by a polynomial; generally, the more demanding the accuracy of the approximation, the higher the order needed of such a polynomial (Dattani, 2008).
With the Stone-Weierstrass theorem stated, let;

$$y' = f(x, y), \quad y(x_0) = y_0 \tag{5}$$

Integrate both sides to have:

$$\int_{x_t}^{x_{t+1}} y'(x)dx = y(x_{t+1}) - y(x_t) = \int_{x_t}^{x_{t+1}} f\big(x, y(x)\big)\, dx \tag{6}$$

Integrate $f\big(x, y(x)\big)$ analytically would require no numerical methods to determine the solution to problem. According to the Stone-Weierstrass Theorem, if analytic integration is unlikely, then, we approximate it with arbitrary accuracy by a polynomial$\varphi(x)$. Since all polynomials can be integrated analytically, then, we have a fair approximation of the solution to the ODE:

$$y(x_{t+1}) - y(x_t) \approx \int_{x_t}^{x_{t+1}} \varphi_{k-1}(x)\, dx \tag{7}$$

To ensure that the approximation is reasonable, let $\varphi_{k-1}(x)$ be a polynomial such that $k = 4$, such that the 3-step mAMT is achieved by interpolating the polynomial $(\varphi_3(x))$ using the Newton-Gregory backward:

$$\varphi_3(x) = f_{t+1} + p\nabla f_{t+1} + \frac{p(p+1)}{2!}\nabla^2 f_{t+1} + \frac{p(p+1)(p+2)}{3!}\nabla^3 f_{t+1} \tag{8}$$

Where $p = \frac{(x-x_{n+1})}{h}$, we integrate over x from $x_n \ to \ x_{n+1}$. This is the similar to integrating over p from $-1 \ to \ 0$ (Kreyszig 2011).

$$y_{t+1} - y_t = \int_{-1}^{0} \left(f_{t+1} + p\nabla f_{t+1} + \frac{p(p+1)}{2!}\nabla^2 f_{t+1} + \frac{p(p+1)(p+2)}{3!}\nabla^3 f_{t+1} \right) h\, dp \qquad (9)$$

$$y_{t+1} - y_t = \left[\left(p f_{t+1} + \frac{p^2}{2}\nabla f_{t+1} + \left(\frac{2p^3 + 3p^2}{12}\right)\nabla^2 f_{t+1} + \left(\frac{p^4 + 4p^3 + 4p^2}{24}\right)\nabla^3 f_{t+1} \right) h \right]_{-1}^{0} \qquad (10)$$

$$y_{n+1} - y_n = h\left(f_{n+1} - \frac{1}{2}\nabla f_{n+1} - \frac{1}{12}\nabla^2 f_{n+1} - \frac{1}{24}\nabla^3 f_{n+1} \right) \qquad (11)$$

We expand the following to have:

$$\nabla f_{n+1} = f_{n+1} - f_n \qquad (a)$$

$$\nabla^2 f_{n+1} = f_{n+1} - 2f_n + f_{n-1} \qquad (b)$$

$$\nabla^3 f_{n+1} = f_{n+1} - 3f_n + 3f_{n-1} - f_{n-2} \qquad (c)$$

Substituting for Eq. (a, b, and c) in Eq. (11)

$$y_{n+1} - y_n = h\left(f_{n+1} - \frac{1}{2}f_{n+1} + \frac{1}{2}f_n - \frac{1}{12}f_{n+1} + \frac{1}{6}f_n - \frac{1}{12}f_{n-1} - \frac{1}{24}f_{n+1} + \frac{1}{8}f_n + \frac{1}{8}f_{n-1} + \frac{1}{24}f_{n-2} \right) \qquad (12)$$

$$y_{n+1} - y_n = h\left(\frac{24 - 12 - 2 - 1}{24}\right)f_{n+1} + \left(\frac{24 + 4 + 3}{24}\right)f_n + \left(\frac{-2 - 3}{24}\right)f_{n-1} + \left(\frac{1}{24}\right)f_{n-2} \qquad (13)$$

$$y_{n+1} - y_n = \frac{h}{24}(9f_{n+1} + 19f_n - 5f_{n-1} + f_{n-2}) \qquad (14)$$

Note: The implicit LMM involves $f_{n+1} = f(x_{n+1}, \ y_{n+1})$ at the right, so that it defines y_{n+1} implicitly (Kreyszig, 2011). We move also to say, let $y_t = 0, h = 1,$ and $y_{t+1} = Y_t$. Let time intervals between observations be $t, t+1, t+2, t+3, .., t-N$. Then, time between two observations can be stated as $h = (t+1) - (t) = 1$. Where t is weekly, monthly or yearly data. Then;

$$Y_t = \frac{1}{24}(9f_{t+1} + 19f_t - 5f_{t-1} + f_{t-2}) \qquad (15)$$

By means of simple moving average technique, we find the smoothened value (forecast) f_{t+1} by taking the average of the three previous observations. Thus;

$$f_{t+1} = \left(\frac{f_t + f_{t-1} + f_{t-2}}{3}\right) \tag{16}$$

Substituting Eq. (16) in (15) with the order-3 moving average now imbedded in the scheme, we have:

$$Y_t = \frac{1}{24}\left(9\left(\frac{f_t + f_{t-1} + f_{t-2}}{3}\right) + 19f_t - 5f_{t-1} + f_{t-2}\right) \tag{17}$$

$$Y_t = \frac{1}{24}(3(f_t + f_{t-1} + f_{t-2}) + 19f_t - 5f_{t-1} + f_{t-2}) \tag{18}$$

$$Y_t = \frac{1}{24}(22f_t - 2f_{t-1} + 4f_{t-2}) \tag{19}$$

By dividing the right hand side by 2, and making $t = t + 2$, Eq. (19) becomes:

$$Y_{t+2} = \frac{1}{12}(11f_{t+2} - f_{t+1} + 2f_t) \tag{20}$$

Y_t is the smoothened value at time t.
$f_t\ \forall\ t = t,\ t-1,\ t-2$, are successive time series observations.
The constant coefficients are the coefficient of data adjustment and the constant, 12, in the denominator is the averaging constant.
The Eq. (20) above is therefore the new smoothing technique, which we called the 3-Step mAMT.

Procedure for updating the parameters of the Scheme

Given a set of data $f_t\ \forall\ t = 1, 2, 3, 4, 5, \ldots\ldots N$
The first smoothened value Y_{t+2} takes values such that;
At $t = 1$, (20) becomes:

$$Y_3 = \frac{1}{12}(11f_3 - f_2 + 2f_1) \tag{21}$$

For the second smoothened value;
At $t = 2$, (20) becomes:

$$Y_4 = \frac{1}{12}(11f_4 - f_3 + 2f_2) \tag{22}$$

The procedure is continued in this manner.

Procedure for Obtaining the Smoothing

This technique removes any missing smoothened value (forecast) or be subjected to carrying over actual data to the forecast column for the first few vacant forecasts just like is in the case of other smoothing techniques (moving average techniques, exponential smoothing techniques and so on). We shall use the algorithm below to solve this problem.

There shall be a forward and backward application of the 3-step mAMT to obtain smoothened values of the dataset and then an average of both (forward and backward) taken to obtain the final smoothened values.

Illustration: let $x_1, x_2, x_3, x_4, x_5, \dots, x_{12}$ be time series dataset.
Using the 3-step mAMT, we demonstrate the procedure below:
The forward smoothing is denoted by Y_{t+2}^{f} while the backward smoothing by Y_{t+2}^{b}

$$forward:\ at\ t = 1, \qquad Y_3^f = \frac{1}{12}(11x_3 - x_2 + 2x_1) \tag{23}$$

$$forward:\ at\ t = 2, \qquad Y_4^f = \frac{1}{12}(11x_4 - x_3 + 2x_2) \tag{24}$$

$$backward:\ at\ t = 1, \qquad Y_{10}^b = \frac{1}{12}(11x_{10} - x_{11} + 2x_{12}) \tag{25}$$

$$backward:\ at\ t = 2, \qquad Y_9^b = \frac{1}{12}(11x_9 - x_{10} + 2x_{11}) \tag{26}$$

Table 1
Method Demonstration using variables.

Serial Number	*Observation*	*Forward mAMT 3-step*	*Backward mAMT 3-step*	*Average = Smoothened Value*
1.	x_1		Y_{12}^b	Y_{12}^b
2.	x_2		Y_{11}^b	Y_{11}^b
3.	x_3	Y_3^f	Y_{10}^b	$(Y_3^f + Y_{10}^b)/2$
4	x_4	Y_4^f	Y_9^b	$(Y_4^f + Y_9^b)/2$
5.	x_5	Y_5^f	Y_8^b	$(Y_5^f + Y_8^b)/2$
6.	x_6	Y_6^f	Y_7^b	$(Y_6^f + Y_7^b)/2$
7.	x_7	Y_7^f	Y_6^b	$(Y_7^f + Y_6^b)/2$
8.	x_8	Y_8^f	Y_5^b	$(Y_8^f + Y_5^b)/2$
9.	x_9	Y_9^f	Y_4^b	$(Y_9^f + Y_4^b)/2$

10.	x_{10}	Y_{10}^{f}	Y_{3}^{b}	$(Y_{10}^{f} + Y_{3}^{b})/2$
11.	x_{11}	Y_{11}^{f}		Y_{11}^{f}
12.	x_{12}	Y_{12}^{f}		Y_{12}^{f}

Efficiency Comparisons

Comparison of mAMT Order-4 to Simple Moving Average (SMA) and Simple Exponential Smoothing (SES)

In this paper, we use the dataset of Obite et al. (2021) to show the efficiency of the proposed 3-step mAMT smoothing technique and compare same with the simple moving average (SMA) order-3 and simple exponential smoothing techniques (SES), with the SES having a smoothing constant of $\alpha = 0.8$.

Measuring Model Performance

In practice, it is mostly recommended to test the efficiency of smoothing techniques with real datasets so as to determine its performance characteristics by way of comparison with the other actual data. This paper utilizes the SMA and SES on the same datasets and a comparison of the performances of the various models have been determined using some common indicators. There are various determinants and measures for smoothing technique best fit, but some of the common indicators are the Mean absolute error (MAE), Mean squared error (MSE), Root mean squared error (RMSE) or Mean absolute percentage error (MAPE) (Ostertagova 2012 and Oyewale et al. 2013):

$$MAE = \frac{1}{n}\sum_{t=1}^{n}|e_t| \tag{27}$$

$$MSE = \frac{1}{n}\sum_{t=1}^{n} e_t^2 \tag{28}$$

$$RMSE = \sqrt{MSE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n} e_t^2} \tag{29}$$

$$MAPE = \frac{1}{n}\sum \frac{|e_t|}{f_t}.100\% \tag{30}$$

Results

Table 2
The 3-step mAMT, SMA, and SES smoothened values.

Serial	*Actual*	*Forward mAMT 3-step*	*Backward mAMT 3-step*	*Average = Smoothened Value*	*SMA Order-3*	*SES* $\alpha = 0.8$
1.	2.25		2.249	**2.249**	***2.25**	***2.25**
2.	2.32		2.338	**2.338**	***2.32**	**2.306**
3.	2.28	2.272	2.288	**2.28**	**2.283**	**2.285**
4.	2.41	2.406	2.41	**2.408**	**2.337**	**2.385**
5.	2.39	2.37	2.391	**2.38**	**2.36**	**2.389**
6.	2.4	2.403	2.397	**2.4**	**2.4**	**2.398**
7.	2.4	2.398	2.392	**2.395**	**2.397**	**2.4**
8.	2.38	2.382	2.38	**2.381**	**2.393**	**2.384**
9.	2.34	2.347	2.335	**2.341**	**2.373**	**2.349**
10.	2.36	2.365	2.355	**2.36**	**2.36**	**2.358**
⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮
168.	2.07	2.052	2.068	**2.06**	**2.033**	**2.066**
169.	2.04	2.043	1.992	**2.017**	**2.06**	**2.045**
170.	2.04	2.045	2.006	**2.025**	**2.05**	**2.041**
171.	1.75	1.774	1.745	**1.76**	**1.943**	**1.808**
172.	1.69	1.743	1.683	**1.713**	**1.827**	**1.714**
173.	1.69	1.7	1.687	**1.693**	**1.71**	**1.695**
174.	1.65	1.653	1.632	**1.643**	**1.677**	**1.699**
175.	1.65	1.657	1.653	**1.655**	**1.663**	**1.652**
176.	1.54	1.549		**1.549**	**1.613**	**1.562**
177.	1.61	1.623		**1.623**	**1.6**	**1.6**

Application of the Method

The proposed time series technique is adaptive and adequate for time series datasets with very high fluctuation. The application of the 3-step mAMT can be used to provide smoothing for time series data without the application of the backward and averaging technique that was introduced in table 1 and 2. Without applying the backward and averaging procedures, the 3-step mAMT can be compared with moving average and exponential smoothing techniques and great results will still be posted as seen in column 3 of table 2. This will only result to missing smoothing values for the first 2 entries just as is in the case of other smoothing techniques. But, with the approach applied in this paper, the problems of carrying over actual data and placing them as startup smoothing values as seen in column 6 and 7 (denoted with "*") has been eliminated.

Table 3
Performance measurement of 3-step mAMT, SMA, and SES smoothing techniques.

Serial	*3-step mAMT*	*SMA Order-3*	*SES* ($\alpha = 0.8$)
R^2	0.9962156	0.8898824	0.9904286
MAE	0.0111554	0.0583616	0.0164302
MSE	0.0001997	0.0058098	0.0005050
RMSE	0.0000998	0.0029049	0.0002525
MAPE	0.5352254	2.7996229	0.7935339

Discussion and Conclusion

Discussion

The objective of this paper is to provide a very compatible time series smoothing technique for time series data of low, moderate and high fluctuations (white noise). The paper applied the Stone-Weierstrass Theorem to derive the original Order-4 Adams-Moulton method and modifying the method to be able to fit in time series data. The technique developed is highly flexible and efficient method of analyzing time series data and understanding the degree of noise presence in the data. The paper also utilized a method of forward and backward application of the derived smoothing technique on the data and then taking the average in such a way that no missing smoothened value would be recorded. The paper also compared the proposed technique to the moving average of order-3 smoothing technique. A demonstration of the forward and backward application of the technique is demonstrated on Table 1.

Conclusion

In this paper, a new time series smoothing technique derived from the modification of the Adams-Moulton Oder-4 numerical method was proposed. The new time series smoothing technique proposed in this paper was called mAMT Order-4. The mAMT Order-4 was adjusted to perform a backward smoothing procedure starting from the last observation to the top as demonstrated in (23) and (24) and Tables 1 and 2. In the calculation, the original mAMT Order-4 was called the forward smoothing as seen in (21) and (22). The mAMT Order-4 was compared with the simple moving average (SMA) and the simple exponential smoothing (SES) techniques and results of the smoothing is shown in Table 2. Statistical performance measurements between the techniques is shown in Tables 3. The proposed mAMT Order-4 is very adequate and performed measurably better than the others as shown in Table 3.

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