A PRODUCTION INVENTORY MODEL WITH LINEAR LEVEL PRODUCTION RATE, LINEAR LEVEL DEPENDENT DEMAND AND CONSTANT HOLDING COST

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ABSTRACT
In this paper, we develop a production inventory model in which both production rate and demand rate are linearly dependent on inventory. The model considers small amount of decay and production starts with a buffer stock reaching a desired level of inventory. After reaching the desired level, the inventory begins to deplete due to deterioration and demand. The objective of this paper is to find out the optimal inventory cost and optimal cycle length. The cost function has been shown to be convex and a numerical example is given to illustrate the model. Later, a sensitivity analysis is carried out to see the effect of parameters changes.

Keywords: *Production, Inventory, Deterioration items, Backlog, Holding Cost*.

INTRODUCTION

Inventory handling is an important part of manufacturing, retail and distribution. In practice, it has been observed that in many situations, production is influenced by on-hand inventory, in that, the production rate may play an opposite role if the on-hand inventory level increases or decreases. In this case, the amount of inventory has a motivational effect on the manufacturing. Lower level of inventory always triggers the producers to produce more and vice versa. It is also known that production is influenced by demand in that, production goes up or down with demand rate. This situation generally arises in the case of inventories of highly demandable products. If the demand rate increases, consumption by the customer will be more and to meet the requirement of the customers, the manufacturers have to increase their production. Similarly, consumption by the customers is less as the demand of the particular commodity goes down and, in that case, to avoid unnecessary inventory, the manufacturers will have to cut down their production.

Many papers have discussed production inventory model under different conditions. By considering the production rate as a variable, Mark (1982), developed a production lot size inventory model with uniform demand rate over a fixed time horizon. Mark argued that a finite planning horizon is preferable to an infinite one for a replenishment policy of production inventory models. He proposed a corrected infinite planning inventory model for the first replenishment cycle. Hollier and Mark (1983), discussed inventory replenishment policies for deteriorating items with a declining demand. They considered constant partial backlogging rate during the shortage period. Urban (1995), discussed a model where demand is partially dependent on instantaneous stock level. Urban also considered a constant partial backlogging during the shortage period. Balkhi and Benkherouf (1996), developed a production lot size inventory model with arbitrary production and demand rate which depends on production cycle time and inventory cycle time. Bhunia and Maiti (1997), developed two inventory systems. In the first system, the production rate was dependent on the inventory level, while in the second system the production rate was dependent upon the demand. However, in both cases the demand rate at any moment of time is a linear function of time for the scheduling period. Both models were formulated and solved without shortages. Su and Lin (2001), combined the two models of Bhunia and Maiti (1997) creating a model where production rate is dependent on both inventory level and demand. They assumed an exponentially decreasing demand and that shortages were allowed and completely backlogged. Chund and Wee (2008), developed an integrated two stage production inventory deterioration model for the buyer and the supplier on the basis of stock dependent selling rate considering imperfect items and just in time multiple deliveries. Other inventory models include the work of Cheng and Wang, (2009), who presented an inventory model for deteriorating items with trapezoidal type of demand rate, where the demand rate is a piecewise linear function. Zhou and Min (2009), discussed an inventory model with stock dependent demand and linear holding cost. .Mishra *et al*., (2013), presented an inventory model for deteriorating items with time dependent demand rate and time varying holding cost under partial backlogging. Islam *et al*., (2015), developed a production inventory model for different classes of demands with constant production rate. Ukil *et al*., (2015), considered a production inventory model with power demand and constant production rate where the products have finite shelf life. Shirajul Islam and Sharifuddin (2016), formulated an inventory model with constant production rate, linear level dependent demand with buffer stock. Sharif *et al*., (2017), developed a production inventory model with time dependent linear demand and constant holding cost. They assumed the production rate to be constant. .

In this paper we develop a production inventory model that considers both production rate and demand rate to be functions of the on-hand inventory where the production starts with a buffer stock. The difference between this paper and that of Shirjul Islam and Sharifuddin, (2016) is the fact that in Shirjul Islam and Sharifuddin (2016), the demand rate is linear level dependent and the production rate is constant whereas in this paper both production rate and demand rate are functions of the inventory .The convexity of the cost function is established and a numerical example is given to illustrate the model developed. Later, a sensitivity analysis is carried out to see the effect of the parameter changes.

Assumptions

(i) The production rate $\lambda + \alpha I(t)$ is linearly dependent on the on-hand inventory, where λ and α are constants.

(ii) The rate of decay μ is small and constant. Since decay is small it is assumed that there is no deterioration cost as in Shirajul Islam and Sharifuddin (2016).

(iii) The demand rate $a + bI(t)$ before production is linearly level dependent, where a and b are constants and satisfying the condition that $\lambda + \alpha I(t) > a + bI(t)$.

(iv) The demand rate $c+fI(t)$ after production is linearly dependent, where c and f are constants.

(v) Production starts with a buffer stock.

(vi) Inventory level is highest at the end of production and after this, the inventory depletes due to demand and deterioration.

(vii) Shortages are not allowed.

Notations:

$I(t)$ = Inventory level at any instant t

I_{1h} = The total undecayed inventory for the period from 0 to t_1

I_{2h} = The total undecayed inventory for the period from t_1 to T_1

D_{1h} = The total deteriorated inventory for the period from 0 to t_1

D_{2h} = The total deteriorated inventory for the period from t_1 to T_1

Q and Q_1 are the inventory levels at time $t = 0$ and t_1 respectively. Here, Q is the buffer stock.

dt = Very small portion of instant t

K_0 = Set up cost

h = Holding cost per unit

$TC(T_1)$ = Total average inventory cost in a unit time

t_1 = Time when inventory gets to the maximum level

T_1 = The time cycle

Q^* = Optimal order quantity

t_1^* = Optimal time for maximum inventory

$TC(T_1)^*$ = Optimal average inventory cost per unit time

Model Formulation

Here it is considered that at time $t=0$, the production starts with a buffer stock where the production rate $\lambda+\alpha I(t)$ is linearly dependent on inventory. The inventory changes (increases) at the rate of $\lambda+\alpha I(t)-a-bI(t)-\mu I(t)$ between $t=0$ to t_1. The market demand is $a+bI(t)$ and $\mu I(t)$ is the decay of $I(t)$ inventory at an instant t. We get the following equations by using the above facts.

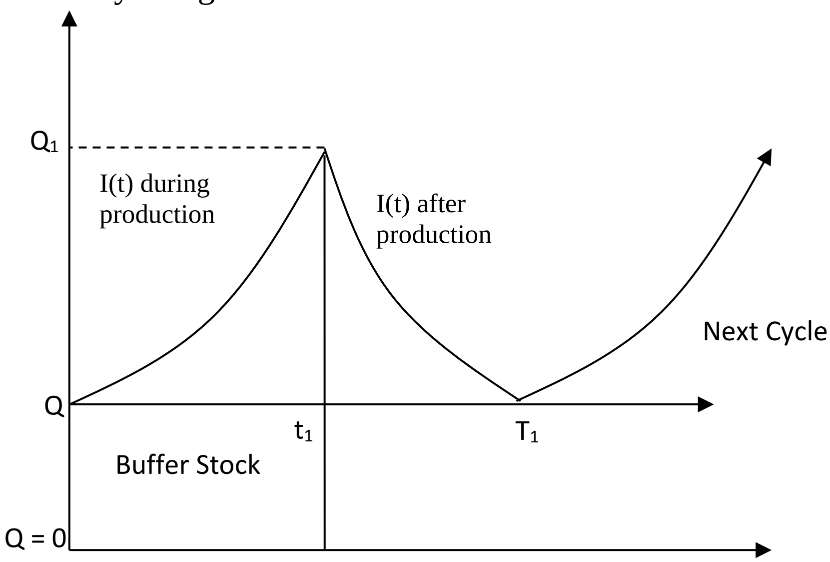

Figure 1: Inventory situation before and after production

$$I(t+dt)=I(t)+\{\lambda+\alpha I(t)-a-bI(t)-\mu I(t)\}dt$$

$$\Rightarrow I(t+dt)-I(t)=\{\lambda+\alpha I(t)-a-bI(t)-\mu I(t)\}dt$$

$$\Rightarrow \lim_{dt\to 0}\frac{I(t+dt)-I(t)}{dt}=\lambda+\alpha I(t)-a-bI(t)-\mu I(t) \text{ or}$$

$$\therefore I(t)=\frac{\lambda-a}{(\mu+b-\alpha)}+Ke^{-(\mu+b-\alpha)t} \tag{1}$$

Which is the general solution to the differential equation (1). Applying the following initial/matching condition

$I(t)=Q$ at $t=0$ we get

$$\therefore K=Q-\frac{\lambda-a}{\mu+b-\alpha} \tag{2}$$

$$I(t)=\frac{\lambda-a}{\mu+b-\alpha}+\left(Q-\frac{\lambda-a}{\mu+b-\alpha}\right)e^{-(\mu+b-\alpha)t} \tag{3}$$

From other boundary/matching conditions i.e. at $t=t_1$, $I(t)=Q_1$ taking up to first degree of μ to obtain the following equation.

$$Q_1=\frac{\lambda-a}{\mu+b-\alpha}+\left(Q-\frac{\lambda-a}{\mu+b-\alpha}\right)e^{-(\mu+b-\alpha)t_1} \tag{4}$$

$$=Q+(\lambda-a-Q\mu-Qb+Q\alpha)t_1 \tag{5}$$

Using equation (3) and considering up to the second degree of μ we obtain the total undecayed inventory in the period $t=0$ to t_1 as follows

$$I_{1h}=\int_0^{t_1}I(t)\,dt=\int_0^{t_1}\left[\frac{\lambda-a}{\mu+b-\alpha}+\left(Q-\frac{\lambda-a}{\mu+b-\alpha}\right)e^{-(\mu+b-\alpha)t}\right]dt$$

$$=\left[\left(\frac{\lambda-a}{\mu+b-\alpha}\right)t+\left\{Q-\frac{\lambda-a}{\mu+b-\alpha}\right\}\frac{e^{-(\mu+b-\alpha)t}}{-(\mu+b-\alpha)}\right]_0^{t_1}$$

$$=\left(\frac{\lambda-a}{\mu+b-\alpha}\right)t_1-\left\{Q-\frac{\lambda-a}{\mu+b-\alpha}\right\}\left\{\frac{1}{(\mu+b-\alpha)}\right\}\left\{-(\mu+b-\alpha)t_1+\tfrac{1}{2}(\mu+b-\alpha)^2t_1^{\,2}\right\}$$

$$\Rightarrow I_{1h}=Qt_1-\tfrac{1}{2}Q(\mu+b-\alpha)t_1^{\,2}+\tfrac{1}{2}(\lambda-a)t_1^{\,2} \tag{6}$$

Now we calculate total deteriorating items in the same time period *0* to t_1 as follows:

$$D_{1h}=Q\mu t_1-\tfrac{1}{2}\mu(\mu+b-\alpha)t_1^{\,2}+\tfrac{1}{2}\mu(\lambda-a)t_1^{\,2} \tag{7}$$

Next during the time period $t=t_1$ to T_1 the inventory changes (decreases) at the rate of $c+fI(t)+\mu I(t)$ as there is no production after time t_1 . The demand during production is considered to be different from the demand after production. The inventory reduces due to demand and deterioration using the same argument as before, we get

$I(t+dt)=I(t)+\{-c-fI(t)-\mu I(t)\}dt$ or

$I(t+dt)-I(t)=\{-c-fI(t)-\mu I(t)\}dt$ or

$\lim\limits_{dt\to 0}\dfrac{I(t+dt)-I(t)}{dt}=\{-c-fI(t)-\mu I(t)\}$ or

$$\therefore I(t)=\frac{-c}{\mu+f}+Re^{-(\mu+f)t} \tag{8}$$

Which is the general solution to the differential equation (8). Applying the boundary/matching condition where

$I(t)=Q$ at $t=T_1$ we obtain

$$R=\left(Q+\frac{c}{\mu+f}\right)e^{(\mu+f)T_1}$$

$$\therefore I(t)=\frac{-c}{\mu+f}+\left(Q+\frac{c}{\mu+f}\right)e^{(\mu+f)(T_1-t)} \tag{9}$$

Now putting boundary/matching conditions $I(t)=Q_1$ at $t=t_1$ taking up to the first degree of μ we get the following equations.

$$Q_1=\frac{-c}{\mu+f}+\left(Q+\frac{c}{\mu+f}\right)e^{(\mu+f)(T_1-t_1)}$$

$$Q_1=\frac{-c}{\mu+f}+\left\{Q+\frac{c}{\mu+f}\right\}\{1+(\mu+f)(T_1-t_1)\}$$

$$\therefore Q_1=Q+\left(Q(\mu+f)+c\right)(T_1-t_1) \tag{10}$$

Now using equation (9) and considering up to the first degree of μ we get the total undecayed inventory for the period $t=t_1$ to T_1 as

$$I_{2h}=\int_{t_1}^{T_1}I(t)\,dt=\int_{t_1}^{T_1}\left[\frac{-c}{\mu+f}+\left\{Q+\frac{c}{\mu+f}\right\}e^{(\mu+f)(T_1-t)}\right]dt$$

$$=\left[\left(\frac{-c}{\mu+f}\right)t+\left\{Q+\frac{c}{\mu+f}\right\}\left\{\frac{e^{(\mu+f)(T_1-t)}}{-(\mu+f)}\right\}\right]\Bigg|_{t_1}^{T_1}$$

$$I_{2h}=\frac{-c(T_1-t_1)}{\mu+f}-\left\{Q+\frac{c}{\mu+f}\right\}\left\{\frac{1}{\mu+f}\right\}\left\{1-1-(\mu+f)(T_1-t_1)\right\}$$

$$=\frac{-c(T_1-t_1)}{\mu+f}+Q(T_1-t_1)+\frac{c(T_1-t_1)}{\mu+f}$$

$$=Q(T_1-t_1) \tag{11}$$

Now considering the decay of the items, we calculate the total deteriorating items during the period $t=t_1$ to $t=T_1$as below.

$$D_{2h}=\int_{t_1}^{T_1}\mu I(t)\,dt=\int_{t_1}^{T_1}\mu\left[\frac{-c}{\mu+f}+\left\{Q+\frac{c}{\mu+f}\right\}e^{(\mu+f)(T_1-t)}\right]dt$$

$$D_{2h}=Q\mu(T_1-t_1) \tag{12}$$

Now because of continuity at $t_1$we equate equations (5) and (10) to obtain.

$$Q+(\lambda-a-Q\mu-Qb+Q\alpha)t_1=Q+\left(Q(\mu+f)+c\right)(T_1-t_1)$$

$$\therefore t_1=\frac{(Q\mu+Qf+c)T_1}{\lambda-a+c+Q(-b+\alpha+f)} \tag{13}$$

$$\text{Let}\,V=\frac{(Q\mu+Qf+c)}{\lambda-a+c+Q(-b+\alpha+f)} \tag{14}$$

$$\Rightarrow t_1=VT_1 \tag{15}$$

The total cost function in the cycle is given as follows

$$TC(T_1)=\frac{K_0+h(I_{1h}+D_{1h}+I_{2h}+D_{2h})}{T_1} \tag{16}$$

By substituting the equation (6), (7), (11) and (12) in (16) we get the total average inventory Cost per unit time as follows:

$$TC(T_1)=\frac{1}{T_1}\left\{K_0+h\left\{\begin{matrix}Qt_1-\frac{Q(\mu+b-\alpha)t_1^{\,2}}{2}+\frac{(\lambda-a)t_1^{\,2}}{2}+Q\mu t_1-\frac{1}{2}(\mu+b-\alpha)\mu t_1^{\,2}+\left(\frac{\lambda-a}{2}\right)\mu t_1^{\,2}+\\ Q(T_1-t_1)+Q\mu(T_1-t_1)\end{matrix}\right\}\right\}$$

$$\therefore TC(T_1)=\frac{K_0}{T_1}+\frac{hQ(1+\mu)t_1}{T_1}-\frac{hQ(\mu+b-\alpha)(1+\mu)t_1^{\,2}}{2T_1}+\frac{h(\lambda-a)(1+\mu)t_1^{\,2}}{2T_1}+\frac{hQ(T_1-t_1)(1+\mu)}{T_1}$$

Now substitute $t_1=VT_1$ to obtain

$$\therefore TC(T_1)=\frac{K_0}{T_1}+\frac{hQ(1+\mu)VT_1}{T_1}-\frac{hQ(\mu+b-\alpha)(1+\mu)V^2T_1^2}{2T_1}+\frac{h(\lambda-a)(1+\mu)V^2T_1^2}{2T_1}+\frac{hQ(T_1-t_1)(1+\mu)}{T_1}$$

$$=\frac{K_0}{T_1}+hQ(1+\mu)V-\frac{hQ(\mu+b-\alpha)(1+\mu)V^2T_1}{2}+\frac{h(\lambda-a)(1+\mu)V^2T_1}{2}$$
$$+hQ(1+\mu)-hQ(1+\mu)V \qquad (17)$$

The main objective is to find the value of T_1 which gives the minimum variable cost per unit time. The necessary and sufficient condition to minimize $TC(T_1)$ are respectively:

$$\frac{dTC(T_1)}{dT_1}=0 \quad and \quad \frac{d^2TC(T_1)}{dT_1^2}>0$$

Therefore, to satisfy the necessary condition we have to differentiate equation (17)with respect to T_1 as follows:

$$\frac{dTC(T_1)}{dT_1}=-\frac{K_0}{T_1^2}-\frac{hQ(\mu+b-\alpha)(1+\mu)V^2}{2}+\frac{h(\lambda-a)(1+\mu)V^2}{2} \qquad (18)$$

Equating this value to zero, we obtain

$$\frac{K_0}{T_1^2} = -\frac{hQ(\mu+b-\alpha)(1+\mu)V^2}{2} + \frac{h(\lambda-a)(1+\mu)V^2}{2} \quad (19)$$

Lemma: The value of $T_1 = \sqrt{\frac{2K_0\{\lambda-a+c+Q(-b+\alpha+f)\}^2}{h\left[\left(-Q(\mu+b-\alpha)+(\lambda-a)\right)(1+\mu)\{Q(\mu+f)+c\}^2\right]}}$

Proof: From equation (19) and equation (14) we get

$$\frac{K_0}{T_1^2} = -\frac{hQ(\mu+b-\alpha)(1+\mu)}{2}\left\{\frac{Q(\mu+f)+c}{\lambda-a+c+Q(-b+\alpha+f)}\right\}^2 + \frac{h(\lambda-a)(1+\mu)}{2}\left\{\frac{Q(\mu+f)+c}{\lambda-a+c+Q(-b+\alpha+f)}\right\}^2$$

$$T_1 = \sqrt{\frac{2K_0\{\lambda-a+c+Q(-b+\alpha+f)\}^2}{h\left[\left(-Q(\mu+b-\alpha)+(\lambda-a)\right)(1+\mu)\{Q(\mu+f)+c\}^2\right]}} \quad (20)$$

Theorem1: The value of $t_1 = \sqrt{\frac{(2K_0)}{\left\{h\left[\left(-Q(\mu+b-\alpha)+(\lambda-a)\right)(1+\mu)\right]\right\}}}$

Proof: From the lemma, with the help of equations (14) and (15), we get

$$t_1 = \frac{Q(\mu+f)+c}{\lambda-a+c+Q(-b+\alpha+f)}\sqrt{\frac{2K_0\{\lambda-a+c+Q(-b+\alpha+f)\}^2}{h\left[\left(-Q(\mu+b-\alpha)+(\lambda-a)\right)(1+\mu)\{Q(\mu+f)+c\}^2\right]}}$$

$$= \sqrt{\frac{(2K_0)}{\left\{h\left[\left(-Q(\mu+b-\alpha)+(\lambda-a)\right)(1+\mu)\right]\right\}}} \quad (21)$$

Theorem2: The cost function $TC(T_1)$ is convex.

Proof: From equation (18) we note that

$\frac{dTC(T_1)}{dT_1} = -\frac{K_0}{T_1^2} - \frac{hQ(\mu+b-\alpha)(1+\mu)V^2}{2} + \frac{h(\lambda-a)(1+\mu)V^2}{2}$ so that

$$\frac{d^2TC(T_1)}{dT_1^2} = \frac{2K_0}{T_1^3} > 0$$

Therefore, the *function* $TC(T_1)$ is convex. Hence, there is an optimal solution in T_1.

Numerical Example

To illustrate the model, an example is considered based on the following value of parameters taken from Shiarjul Islam and Sharifuddin (2016). $K_0 = 100$, $Q = 10$, $h = 4$, $\lambda = 50$, $b = 0.8$, $f = 0.6$, $a = 5$, $\alpha = 2$, $c = 3$ and $\mu = 0.01$ per unit time. Substituting the above values of parameter into equations (5), (16) (18) and (20) respectively gives $Q_1^* = 63.09013$, $t_1^* = 0.933042756$, $T_1^* = 6.767123$ (2471days) and $TC(T_1)^* = 69.96373$.

Sensitivity Analysis

We study the effect of changes of parameter K_O, Q, h, λ, b, f, a, α, c and μ on the opitimal length of ordering cycle t_1^*, optimal cycle time T_1^*, optima production quantity Q_1^* and the total average inventory cost per unit time $TC(T_1)^*$. The sensitivity is performed by changing each of the parameter by 50%, 25%, 10%, 5%, -5%, -10%, -25%-50% taking each parameter at a time and keeping the remaining parameters unchanged.

Table 1: Sensitivity Analysis on the numerical example 1 to see the changes in the values of Q_1^*, t_1^*, T_1^*, and $TC(T_1)^*$

Parameter	% Change in Parameter	Q_1^*	t_1^*	T_1^*	$TC(T_1)^*$
K_0	50%	75.01929	1.142694064	8.287671 (3026 days)	76.60803
	25%	69.34488	1.042968037	7.564384 (27 62 days)	73.45326
	10%	65.66941	0.978372769	7.09589 (2591 days)	71.4067
	5%	64.40127	0.956085513	6.934247 (2532 days)	70.69381
	0%	**63.09013**	**0.933042756**	**6.767123 (2471 days)**	**69.96373**
	-5%	61.73601	0.9092445	6.594521 (2408 days)	69.21516
	-10%	60.3604	0.885068493	6.419178 (2344 days)	68.44662

	-25%	55.97563	0.808007472	5.860274 (2140 days)	66.00294
	-50%	47.54998	0.659929431	4.786301 (1748 days)	61.30472
Parameter	% Change in Parameter	Q_1^*	t_1^*	T_1^*	$TC(T_1)^*$
Q	50%	70.79014	0.887671233	5.479452 (2001 days)	97.10676
	25%	66.96368	0.909623045	6.035616 (2204 days)	83.64884
	10%	64.6315	0.923248475	6.446575 (2354 days)	75.46744
	5%	63.86874	0.928232692	6.60274 (2411 days)	72.72075
	0%	**63.09013**	**0.933042756**	**6.767123 (2471 days)**	**69.96373**
	-5%	62.30993	0.937926057	6.942466 (2535 days)	67.19602
	-10%	61.51946	0.942729484	7.128767 (2603 days)	64.41774
	-25%	59.18008	0.958369529	7.780822 (2841 days)	56.01037
	-50%	55.22634	0.985796683	9.287671 (3291 days)	41.735667
h	50%	53.35336	0.761921959	5.526027 (2018 days)	96.80803
	25%	57.4802	0.834449929	6.052055 (2210days)	83.55326
	10%	60.61833	0.889601494	6.452055 (2356 days)	75.44671
	5%	61.80049	0.91037775	6.60274 (2411 days)	72.71381
	0%	**63.09013**	**0.933042756**	**6.767123 (2471 days)**	**69.96373**

	-5%	64.46575	0.957218763	6.942466 (2535 days)	67.19516
	-10%	65.94883	0.9828352	7.131507 (2604 days)	64.40662
	-25%	71.30083	1.077343296	7.813699 (2853 days)	55.90294
	-50%	85.07848	1.31948112	9.569863 (3494 days)	41.10472
λ	50%	73.68005	0.777534247	7.775342 (2839 days)	66.12454
	25%	68.63001	0.844812844	7.287671 (2661 days)	67.85094
	10%	65.36175	0.894373915	6.978082 (2548 days)	69.06382
	5%	64.24317	0.913184682	6.873973 2510 days)	69.5038
	0%	**63.09013**	**0.933042756**	**6.767123 (2471 days)**	**69.96373**
	-5%	61.90151	0.954071837	6.657534 (2431 days)	70.44504
	-10%	60.6972	0.976824613	6.547945 (2391 days)	70.94928
	-25%	56.8854	1.055977468	6.208219 (2267 days)	72.61618
	-50%	49.74638	1.245967257	5.613699 (2050 days)	76.03351

Parameter	% Change in Parameter	Q_1^*	t_1^*	T_1^*	$TC(T_1)^*$
b	50%	61.18093	0.967503314	6.591781 (2407 days)	70.74473
	25%	62.14043	0.949734589	6.679452 (2439 days)	70.347
	10%	62.70708	0.939520128	6.731507(2458 days)	70.11535

	5%	62.288805	0.936071667	6.747945 (2464 days)	70.03927
	0%	**63.09013**	**0.933042756**	**6.767123 (2471 days)**	**69.96373**
	-5%	63.27037	0.929674864	6.783562 (2477 days)	69.88875
	-10%	63.45024	0.926347305	6.8000 (2483 days)	69.8143
	-25%	64.00931	0.916966156	6.852055 (2502 days)	69.59415
	-50%	64.9292	0.901808219	6.936986 (2533 days)	69.23751
F	50%	63.08904	0.933023625	5.320548 (1943 days)	78.00089
	25%	63.07387	0.932756976	5.939726 (2169 days)	74.07161
	10%	63.08373	0.93293019	6.405479 (2339 days)	71.62909
	5%	63.08927	0.933027542	6.580822 (2403 days)	70.80018
	0%	**63.09013**	**0.933042756**	**6.767123 (2471 days)**	**69.96373**
	-5%	63.07772	0.932824587	6.964384 (2543 days)	69.11965
	-10%	63.08378	0.932931172	7.178082 (2621 days)	68.26782
	-25%	63.08491	0.932951046	7.917808 (2891 days)	65.66479
	-50%	63.08613	0.932972385	9.635616 (3510 days)	61.16114
A	50%	61.90151	0.954071837	6.657534 (2431 days)	70.44504
	25%	62.49767	0.943354313	6.712329 (2451 days)	70.20162
	10%	62.85358	0.937120151	6.745205 (2463 days)	70.05824
	5%	62.97193	0.935073702	6.756164 (2467 days)	70.01088

	0%	**63.09013**	**0.933042756**	**6.767123 (2471 days)**	**69.96373**
	-5%	63.2082	0.931027139	6.778082 (2475 days)	69.9168
	-10%	63.32613	0.929026676	6.789041 (2479 days)	69.87009
	-25%	63.65755	0.9227438	6.819178 (2490 days)	69.7311
	-50%	64.24317	0.913184682	6.873973 (2510 days)	69.5038

Parameter	% Change in Parameter	Q_1^*	t_1^*	T_1^*	$TC(T_1)^*$
C	50%	63.07387	0.932756976	5.939726 (2169 days)	74.07161
	25%	63.09335	0.933099379	6.323288 (2309 days)	72.04075
	10%	63.08927	0.933027542	6.580822 (2403 days)	70.80018
	5%	63.07992	0.932863252	6.671233 (2436 days)	70.3829
	0%	**63.09013**	**0.933042756**	**6.767123 (2471 days)**	**69.96373**
	-5%	63.07554	0.93278622	6.863014 (2506 days)	69.54265
	-10%	63.07772	0.932824587	6.964384 (2543 days)	69.11965
	-25%	63.08481	0.932949142	7.290411 (2662 days)	67.83897
	-50%	63.08491	0.932951046	7.917808 (2891 days)	65.66479
α	50%	65.5611	0.860465032	7.186301 (2624 days)	68.23855
	25%	65.36175	0.894373915	6.978082 (2548 days)	69.06382
	10%	64.00931	0.916966156	6.852055 (2502 days)	69.59415
	5%	63.54004	0.924698426	6.808219 (2486 days)	69.77728
	0%	**63.09013**	**0.933042756**	**6.767123 (2471 days)**	**69.96373**
	-5%	62.61645	0.941260274	6.723288 (2455 days)	70.15361

	-10%	62.14043	0.949734589	6.679452 (2439 days)	70.347
	-25%	60.6972	0.976824613	6.547945 (2391 days)	70.94928
	-50%	53.19136	1.027534247	6.323288 (2309 days)	72.03337
μ	50%	62.92466	0.930952677	6.715068 (2452 days)	70.38656
	25%	62.99716	0.931818182	6.739726 (2461 days)	70.1751
	10%	63.05308	0.932555417	6.756164 (2467 days)	70.04827
	5%	63.07163	0.932799502	6.761644 (2469 days)	70.006
	0%	**63.09013**	**0.933042756**	**6.767123 (2471 days)**	**69.96373**
	-5%	63.10859	0.933285181	6.772603 (2473 days)	69.92147
	-10%	63.10553	0.93334944	6.775342 (2474 days)	69.87922
	-25%	63.16054	0.933869863	6.791781 (2480 days)	69.75247
	-50%	63.22993	0.93467829	6.816438 (2489 days)	69.54131

Discussion of Results

From the results obtained in Table 1, it can be deduced that

i. With increase in the value of the parameter K_0 (set up cost), the values of Q_1^*,t_1^*, T_1^* and $TC(T_1)^*$ all increase. This is clearly expected since excess stocking is encouraged as a result of high set up cost. The total average inventory cost per unit time $TC(T_1)^*$ is therefore expected to increase due to increase in stocking cost. The values of Q_1^*, t_1^* andT_1^*all increases due to high set up cost as well as stockholding.

ii. With increase in the value of the parameter Q (Buffer Stock), the values of Q_1^* and $TC(T_1)^*$ increase while the values of t_1^* and T_1^* decrease. This is expected because if Q increases the total average inventory cost per unit time $TC(T_1)^*$ andQ_1^*, increase due to increase in the value of the holding cost for buffer stock. The inventory produced takes a shorter time to finish and this forces a reduction of the optimal cycle timeT_1^*as well as optimal time for maximum inventoryt_1^*.

iii. With the increase in the value of the parameter h (holding cost) the values of Q_1^*, t_1^* andT_1^* decrease while the value of $TC(T_1)^*$ increases. This is so because increase in the holding cost of the items will increase the stocking cost and so increases the total average cost per unit time $TC(T_1)$.* To reduces the stocking holding cost, the model will now lower the values ofQ_1^* thereby reducing both $t_1^*$and the cycle time T_1^*.

iv. With increase in the value of the parameter λ (production rate), there is decrease in the values of t_1^* and $TC(T_1)^*$ but increase in the values of T_1^* and Q_1^* . The value t_1^* decreases due to an increase in the production rate as seen in equation (21). The value of T_1^* increases since much has been produced and so takes longer time to finish. Similarly, based on equation (5), the value of Q_1* increases, and the model adjusts other parameters which probably forces a reduction in the value of $TC(T_1)^*$.

v. With increase in the value of the parameter b (stock dependent part of the demand during production), the values of t_1^* and $TC(T_1)^*$ increase, while the values of Q_1^* and T_1^* decrease. This is expected in the case of t_1* and T_1* since if the stock dependent demand rate is higher, the inventory will finish earlier and so T_1^* will decrease. On the other hand increasing the stock- dependent demand will effectively increase the total demand and so t_1* increases. The model will now adjust other parameters which probably forces a decrease in Q_1* and an increase in the value of $TC(T_1)^{*.}$

vi. With increase in the value of the parameter f (stock dependent part of the demand after production), the values of Q_1^* and t_1^* are unstable, while the T_1^* decreases and $TC(T_1)^*$ increases. This is so because if the stock dependent part of the demand rate increases, the demand will increase also. Due to high demand, the stock will finish earlier and this will lower the value of T_1^*. Increase in the value of the total average inventory cost per unit time $TC(T_1)^*$ is probably due to instability of t_1^* and Q_1^*.

vii. With increase in the value of the parameter **a**(constant part of the demand during production) the values of t_1^* and $TC(T_1)^*$ increase while the values of Q_1^* and T_1^* decrease. This is so because since a is higher, demand rate is high, stock will take less time to finish and so Q_1^* and T_1^* reduce. Increasing the demand will also in turn increase the optimal time for maximum inventory t_1^* and total average inventory cost per unit time $TC(T_1)^*$.

viii. With increase in the value of the parameter **c**(constant part of the demand after production), the values of Q_1^* and t_1^* are unstable while the value of T_1^* decreases and that of $TC(T_1)^*$ increases. This is expected because increase in the value of parameter c will result to higher demand. Due to higher demand, stock will finish earlier and this will lower the value of T_1^*. Increase in $TC(T_1)^*$ is probably from the instability of Q_1^* and t_1^*.

ix. With increase in the value of the parameter α (stock dependent part of the production rate), the values of Q_1^* and T_1^* increase while the values of t_1^* and $TC(T_1)^*$ decrease. This is so because the value Q_1* increases since much has been produced. Similarly T_1* increases since much has been produced and takes a longer time to finish. The model adjusts other parameters which probably forces a reduction in the values of t_1* and $TC(T_1)$*.

x. With increase in the value of the parameter μ (deterioration rate), the values of Q_1^*, t_1^* and T_1^* decrease while the value of $TC(T_1)^*$ increases. The value of the total average inventory cost per unit time $TC(T_1)^*$ increases which is expected because the deterioration is high. The model now forces a decrease in the values of t_1^* and T_1* thereby resulting to a decrease in the value of Q_1^*.

Conclusion

In this paper, we have developed a production inventory model in which the demand rate and the production rate are linearly dependent on inventory. Production starts with buffer stock and after reaching its desired level, the inventory depletes due to demand and deterioration. The cost function has been shown to be convex and a numerical example is given to show the application of the model. A sensitivity analysis is then carried out on the example to determine the effect of parameter changes.

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