Oredr-4 Predictor-Corrector Time Series Smoothing Technique

By

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Abstract

This work proposes a predictor-corrector time series smoothened values technique developed from the modification of two beautiful linear multistep numerical techniques. The study modified the 3-step explicit linear multistep technique also known as the order-4 Adams-Bashforth technique and used same as an initial value predictor. For the corrector model, the study modified the 3-step implicit linear multistep technique also called the order-4 Adams-Moulton technique and designed a technique of fitting in the predicted values into the corrector model. The study also went further to develop an algorithm that performs a forward and backward smoothened values procedure before averaging these results to estimate the final smoothened value. The technique involves rigor and care but beautiful to perform and has a high performance evaluation.

Keywords: Time series, Adams-Bashforth, Adam-Moulton, Oder-4, smoothing, Time Series.

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Introduction

The predictor-corrector technique is a term associated to numerical analysis, and deals with providing numerical approximated solutions to ordinary and partial differential equations. Numerical analysis continues this long tradition of practical mathematical calculations. The numerical analysis is so much like the Babylonian approximation of square root of 2 and modern numerical analysis does not seek exact answers, because exact answers are often practically impossible to determine. Over the years, the application of numerical analysis have continued to advance from the field of engineering, physical sciences and parts of life sciences and arts. The predictor-corrector technique belong to a class of algorithm that integrate ordinary differential equations to find an unknown function that satisfies a given differential equation. The technique typically uses an explicit technique for the predictor step and an implicit technique for the corrector step (Butcher et al., 2003, and Press et al., 2007).

Time series forecasting as is the purpose of this research, can be said to be the process by which future predictions are made based on past and present data. In this work we shall apply the predictor-corrector technique using two different numerical analysis technique called the Adams-Bashforth order-4 technique and the Adams-Moulton order-4 technique. We shall use the Adams-Bashforth order-4 for prediction of the new smoothened values and then use the Adams-Moulton to make smoothened corrections to the values while also considering previous corresponding actual data.

Over the recent years some beautiful research work have been conducted in the field of time series forecasting. Guo et al. (2018), produced a model in the exponential smoothing class for new commodity demand forecasting. A study on the limitations of forecasting cryptocurrency prices was conducted by Pronchakov (2019) using a comparison technique involving several forecasting techniques. Raiyn et al. (2012) did a research on short-term travel forecast using the moving average technique. Raudys et al. (2018), used the moving averages to smooth stock price series and forecast the direction of stock market trend. A new adaptive moving average technical indicator (VAMA) was introduced by Pierrefeu (2019). Further studies in time series smoothing have been done by Raynolds et al. (2001), Handanhal (2013), Kumari et al. (2014), Ostertagová (2016), Guha et al. (2016), Majid et al. (2018), Sidqi et al. (2019), Jamil et al. (2020), etc.

Nwokike et al., (2021), proposed a time series smoothing technique which was derived from the modification of order-4 explicit linear multistep numerical technique. The proposed model was precisely modified from Adams-Bashforth order-4 explicit numerical technique originally designed to provide numerical solutions to partial and ordinary differential equations. The paper called the proposed technique; “Modified Adams-Bashforth Oder-4 (mABT Order-4)” and the model can be seen below:

$$Y_t = \frac{1}{24}(55f_t - 59f_{t-1} + 37f_{t-2} - 9f_{t-3}) \tag{1}$$

Where;

Y_t is the smoothened values at time t.

$f_t \;\forall\; t = t,\ t-1,\ t-2,\ t-3$ is the time series observations.

The constants are the coefficient of data adjustment and 24 is the averaging constant.

Methodology

This study shall modify the 3-step explicit linear multistep numerical technique and call the new model a predictor model. We shall use the predictor model as a generator of smoothed values which shall be fitted into a corrector model derived from the modification of the 3-step implicit linear multistep numerical technique.

The Modified Predictor Model

The predictor model for this new time series smoothing technique is a modification of the Adams-Bashforth Order-4 numerical technique also known as the 3-step explicit linear multistep numerical technique. Below is a derivation of the 3-step explicit linear multistep numerical technique followed by our modification.
The linear multistep technique is a special category of multistep techniques and the 3-step explicit linear multistep technique is based on the Stone-Weierstrass Theorem.

Theorem: Stone-Weierstrass Theorem - Let $f(t) : \mathbb{R} \rightarrow \mathbb{C}$ be continuous on $t \in [a, b]$. For all $\epsilon > 0, \exists$ a polynomial $\varphi(t) \ni ||f(t) - \varphi(t)|| < \epsilon$.
This is to say that; any continuous function can be approximated to an arbitrary accuracy by a polynomial; generally, the more demanding the accuracy of the approximation, the higher the order needed of such a polynomial.
Having the Stone-Weierstrass Theorem in mind, let;

$$y' = f(x, y), \quad y(x_0) = y_0 \tag{2}$$

Let us integrate both sides to have:

$$\int_{x_t}^{x_{t+1}} y'(x) dx = y(x_{t+1}) - y(x_t) = \int_{x_t}^{x_{t+1}} f\big(x, y(x)\big)\, dx \tag{3}$$

If integral $f\big(x, y(x)\big)$ can be integrated analytically, then we might need not to apply numerical techniques to determine the solution to the ODE. But, if the integration of $f\big(x, y(x)\big)$ cannot be done analytically, then according to Stone-Weierstrass Theorem, the solution can be approximated to an arbitrary accuracy by a polynomial $\varphi(x)$. Since all polynomials can be integrated analytically, then, we shall an adequate approximation of the solution to the ODE:

$$y(x_{t+1}) - y(x_t) \approx \int_{x_t}^{x_{t+1}} \varphi_{k-1}(x)\, dx \tag{4}$$

To have a reasonable approximation of the solution, let $\varphi_{k-1}(x)$ be a polynomial such that $k = 4$, then by using the Newton-Gregory backward the 3-step implicit linear multistep technique is gotten as seen below:

$$\varphi_3(x) = f_t + p\nabla f_t + \frac{p(p+1)}{2!}\nabla^2 f_t + \frac{p(p+1)(p+2)}{3!}\nabla^3 f_t \quad (5)$$

$$y_{n+1} - y_n = \int_0^1 \left(f_n + p\nabla f_n + \frac{p(p+1)}{2}\nabla^2 f_n + \frac{p(p+1)(p+2)}{6}\nabla^3 f_n \right) h\, ds \quad (6)$$

$$y_{n+1} - y_n = \left[\left(pf_n + \frac{p^2}{2}\nabla f_n + \frac{2p^3 + 3p^2}{12}\nabla^2 f_n + \frac{p^4 + 4p^3 + 4p^2}{24}\nabla^3 f_n \right) h \right]_{\mathbf{0}}^{\mathbf{1}} \quad (7)$$

$$y_{n+1} - y_n = h\left(f_n + \frac{1}{2}\nabla f_n + \frac{5}{12}\nabla^2 f_n + \frac{3}{8}\nabla^3 f_n \right) \quad (8)$$

Let us consider the expansion of $\frac{1}{2}\nabla f_n$, $\frac{5}{12}\nabla^2 f_n$, $\frac{3}{8}\nabla^3 f_n$ by Newton-Gregory backward difference formula. Thus, we have:

$$\frac{1}{2}\nabla f_n = \frac{1}{2} f_n - \frac{1}{2} f_{n-1} \quad a$$

$$\frac{5}{12}\nabla^2 f_n = \frac{5}{12} f_n - \frac{5}{6} f_{n-1} + \frac{5}{12} f_{n-2} \quad b$$

$$\frac{3}{8}\nabla^3 f_n = \frac{3}{8} f_n - \frac{9}{8} f_{n-1} + \frac{9}{8} f_{n-2} - \frac{3}{8} f_{n-3} \quad c$$

Let us substitute for Eq. (a, b and c) in Eq. (8):

$$y_{n+1} - y_n = h\left(\frac{1}{2} f_n - \frac{1}{2} f_{n-1} + \frac{5}{12} f_n - \frac{5}{6} f_{n-1} + \frac{5}{12} f_{n-2} + \frac{3}{8} f_n - \frac{9}{8} f_{n-1} + \frac{9}{8} f_{n-2} - \frac{3}{8} f_{n-3} \right) \quad (9)$$

$$y_{n+1} - y_n = \frac{h}{24}(55 f_n - 59 f_{n-1} + 37 f_{n-2} - 9 f_{n-3}) \quad (10)$$

Eq. (10) is called explicit because the right-hand-side does not have f_{n+1}, (Kreyszig 2011).

Since we are considering a time series data, we say let $n = t$.

$$y_{t+1} - y_t = \frac{h}{24}(55 f_t - 59 f_{t-1} + 37 f_{t-2} - 9 f_{t-3}) \quad (11)$$

Let $n = t$ in Eq. (11) since we are considering using this model for time series data smoothing. Let the value of the step size h be the number of trials between successive observations.

Let time intervals between successive entries be denoted as $t, t+1, t+2, t+3, t+4, .., t-N$, we can therefore derive h by saying; let $h = (t+1) - (t) = 1$. Where t represents data entries.

We assume that y_t is the initial smoothened values and we equate it to zero ($y_t = 0$) for all smoothened values, and $y_{t+1} = \dot{Y}_{t+1}$, be the new smoothened values. Hence, the predictor model is given as:

$$\dot{Y}_{t+1} = \frac{1}{24}(55f_t - 59f_{t-1} + 37f_{t-2} - 9f_{t-3}) \quad (12)$$

Let $t = t + 3$, such that we now have:

$$\dot{Y}_{t+4} = \frac{1}{24}(55f_{t+3} - 59f_{t+2} + 37f_{t+1} - 9f_t) \quad (13)$$

Where;
Y_{t+4} is the smoothened values at time t.
$f_t \; \forall \; t = t, \; t+1, \; t+2, \; t+3$, are successive time series observations.
The constants are the coefficient of data adjustment and 24 is the averaging constant.

Updating the parameters of the Predictor Model

Let $\dot{Y}_{t+4}$ be time series smoothened values such that $f_t \; \forall \; t = t, \; t+1, \; t+2, \; t+3, \ldots . t - N$ are time series observations, where $t = 1, 2, 3, 4, 5, \ldots, N$.
The first predicted primitive value (smoothened values), is estimated at $t = 1$, such that Eq. (13) becomes:

$$\dot{Y}_5 = \frac{1}{24}(55f_4 - 59f_3 + 37f_2 - 9f_1) \quad (14)$$

The second predicted primitive value (smoothened values) is estimated at $t = 2$, such that Eq. (13) becomes:

$$\dot{Y}_6 = \frac{1}{24}(55f_5 - 59f_4 + 37f_3 - 9f_2) \quad (15)$$

The model is updated in this manner.

The Modified Corrector Model

The corrector model is derived from a modification of the Adams-Moulton Order-4 numerical technique also known as the 3-step implicit linear multistep technique. Below is a derivation of the 3-step implicit linear multistep numerical technique followed by a modification to have a time series smoothing scheme.

Just like we have done above for the explicit predictor model, the 3-step implicit linear multistep corrector technique is also based on the Stone-Weierstrass Theorem.

$$y(x_{t+1}) - y(x_t) \approx \int_{x_t}^{x_{t+1}} \varphi_{k-1}(x)\, dx \qquad (16)$$

We use the cubic polynomial $\varphi_3(x)$, such that by Newton-Gregory backward scheme we have:

$$\varphi_3(x) = f_{t+1} + p\nabla f_{t+1} + \frac{p(p+1)}{2!}\nabla^2 f_{t+1} + \frac{p(p+1)(p+2)}{3!}\nabla^3 f_{t+1} \qquad (17)$$

Let $p = \frac{(x-x_{n+1})}{h}$, we can now integrate over x from $x_n\ to\ x_{n+1}$. This is the same as integrating over p from $-1\ to\ 0$ (Kreyszig 2011).

$$y_{t+1} - y_t = \int_{-1}^{0} \left(f_{t+1} + p\nabla f_{t+1} + \frac{p(p+1)}{2!}\nabla^2 f_{t+1} + \frac{p(p+1)(p+2)}{3!}\nabla^3 f_{t+1} \right) h\, dp \qquad (18)$$

$$y_{t+1} - y_t = \left[\left(pf_{t+1} + \frac{p^2}{2}\nabla f_{t+1} + \left(\frac{2p^3 + 3p^2}{12} \right)\nabla^2 f_{t+1} + \left(\frac{p^4 + 4p^3 + 4p^2}{24} \right)\nabla^3 f_{t+1} \right) h \right]_{-1}^{0} \qquad (19)$$

$$y_{n+1} - y_n = h\left(f_{n+1} - \frac{1}{2}\nabla f_{n+1} - \frac{1}{12}\nabla^2 f_{n+1} - \frac{1}{24}\nabla^3 f_{n+1} \right) \qquad (20)$$

Where,

$$\nabla f_{n+1} = f_{n+1} - f_n \qquad (a)$$

$$\nabla^2 f_{n+1} = f_{n+1} - 2f_n + f_{n-1} \qquad (b)$$

$$\nabla^3 f_{n+1} = f_{n+1} - 3f_n + 3f_{n-1} - f_{n-2} \qquad (c)$$

Substituting for Eq. (a, b, and c) in Eq. (20), we have:

$$y_{n+1} - y_n = h\left(f_{n+1} - \frac{1}{2}f_{n+1} + \frac{1}{2}f_n - \frac{1}{12}f_{n+1} + \frac{1}{6}f_n - \frac{1}{12}f_{n-1} - \frac{1}{24}f_{n+1} + \frac{1}{8}f_n + \frac{1}{8}f_{n-1} + \frac{1}{24}f_{n-2} \right) \qquad (21)$$

$$y_{n+1} - y_n = h\left(\frac{24 - 12 - 2 - 1}{24} \right) f_{n+1} + \left(\frac{24 + 4 + 3}{24} \right) f_n + \left(\frac{-2 - 3}{24} \right) f_{n-1} + \left(\frac{1}{24} \right) f_{n-2} \qquad (22)$$

$$y_{n+1} - y_n = \frac{h}{24}(9f_{n+1} + 19f_n - 5f_{n-1} + f_{n-2}) \quad (23)$$

Note: The 3-step implicit linear multistep technique involves $f_{n+1} = f(x_{n+1},\ y_{n+1})$ at the right, so that it defines y_{n+1} implicitly (Kreyszig 2011). And, as shown previously, let $y_t = 0$, $h = 1$, and $y_{t+1} = \ddot{Y}_{t+1}$.

$$\ddot{Y}_{t+1} = \frac{1}{24}(9f_{t+1} + 19f_t - 5f_{t-1} + f_{t-2}) \quad (24)$$

Let $t = t + 2$, such that we now have:

$$\ddot{Y}_{t+3} = \frac{1}{24}(9f_{t+3} + 19f_{t+2} - 5f_{t+1} + f_t) \quad (25)$$

This is the corrector and smoothened values model for the time smoothing technique.
Where;
$\ddot{Y}_{t+3}$ is the smoothened values at time t.
$f_t\ \forall\ t = t,\ t+1,\ t+2,\ t+3$, are successive time series observations.
The constants are the coefficient of data adjustment and 24 is the averaging constant.

The Predictor-Corrector Smoothened values Technique

Given the predictor model in Eq. (14) and the corrector model in Eq. (25), let the predicted values, $\dot{Y}_{t+4}\ \forall\ t = 1,\ 2, 3, \ldots N$ be assumed to be the observations $f_{t+4}\ \forall\ t = 1,\ 2, 3, \ldots N$ in Eq. (25), such that;

$$\ddot{Y}_{t+3} = \frac{1}{24}\left(9\dot{Y}_{t+3} + 19\dot{Y}_{t+2} - 5\dot{Y}_{t+1} + \dot{Y}_t\right) \quad (26)$$

Updating the parameters of the Predictor-Corrector Smoothened values Technique

Let $\ddot{Y}_{t+3}\ \forall\ t = 1,\ 2, 3, \ldots N$ be time series smoothened value such that $\dot{Y}_{t+4}\ \forall\ t = 1,\ 2, 3, \ldots N$ be predicted values (primitive smoothened values) estimated using the actual data.
The first smoothened value (corrected value), is estimated at $t = 1$, such that Eq. (26) becomes:

$$\ddot{Y}_4 = \frac{1}{24}\left(9\dot{Y}_4 + 19\dot{Y}_3 - 5\dot{Y}_2 + \dot{Y}_1\right) \quad (27)$$

The second smoothened value (corrected value) is estimated at $t = 2$, such that Eq. (26) becomes:

$$\ddot{Y}_5 = \frac{1}{24}\left(9\dot{Y}_5 + 19\dot{Y}_4 - 5\dot{Y}_3 + \dot{Y}_2\right) \qquad (28)$$

The model is updated in this manner until final value is achieved.

Procedure for Obtaining the Smoothened Values

There are three algorithmic steps to obtaining the smoothened values.

1. For the forward smoothened values (from top to down), we apply Eq. (26), placing the first smoothened value at the 4th point $(n+3), n = 1,2,3,..N$. Continue same for 5th, 6th and so on.
2. For the backward smoothened values (from down to top), we apply Eq. (26), placing the smoothened value at the 4th before last point $(N-3)$, and continue the upward smoothing procedure until last smoothened value is achieved.
3. The average of the forward and backward smoothened values are taken to achieve the final smoothened value.

Illustration: let $x_1, x_2, x_3, x_4, x_5, \dots, x_{20}$ be time series observations.
We can demonstrate the procedure for the algorithm for the proposed technique using a table as seen below.
The forward smoothing is denoted by $\ddot{Y}_t^f$ while the backward smoothing by $\ddot{Y}_t^b$

$$forward:\ at\ t = 1, \qquad \ddot{Y}_4^f = \frac{1}{24}\left(9\dot{Y}_4 + 19\dot{Y}_3 - 5\dot{Y}_2 + \dot{Y}_1\right)$$

$$forward:\ at\ t = 2, \qquad \ddot{Y}_5^f = \frac{1}{24}\left(9\dot{Y}_5 + 19\dot{Y}_4 - 5\dot{Y}_3 + \dot{Y}_2\right)$$

From bottom to top, $t = 20$ is regarded as $t = 1$

$$backward:\ at\ t = 1, \qquad \ddot{Y}_4^b = \frac{1}{24}\left(9\dot{Y}_{17} + 19\dot{Y}_{18} - 5\dot{Y}_{19} + \dot{Y}_{20}\right)$$

$$backward:\ at\ t = 2, \qquad \ddot{Y}_5^b = \frac{1}{24}\left(9\dot{Y}_{16} + 19\dot{Y}_{17} - 5\dot{Y}_{18} + \dot{Y}_{19}\right)$$

Table 1
The Order-4 Predictor-Corrector Technique demonstration

Serial Number	*Observation*	*Predicted Values*	*Forward Corrected Values*	*Backward Corrected Values*	*Average (Smoothened Value)*
1.	x_1	x_1		$\ddot{Y}_{20}^b$	$\ddot{Y}_{20}^b$

2.	x_2	x_2		$\ddot{Y}_{19}^b$	$\ddot{Y}_{19}^b$
3.	x_3	x_3		$\ddot{Y}_{18}^b$	$\ddot{Y}_{18}^b$
4	x_4	x_4	$\ddot{Y}_4^f$	$\ddot{Y}_{17}^b$	$(\ddot{Y}_4^f + \ddot{Y}_{17}^b)/2$
5.	x_5	$\dot{Y}_5$	$\ddot{Y}_5^f$	$\ddot{Y}_{16}^b$	$(\ddot{Y}_5^f + \ddot{Y}_{16}^b)/2$
6.	x_6	$\dot{Y}_6$	$\ddot{Y}_6^f$	$\ddot{Y}_{15}^b$	$(\ddot{Y}_6^f + \ddot{Y}_{15}^b)/2$
7.	x_7	$\dot{Y}_7$	$\ddot{Y}_7^f$	$\ddot{Y}_{14}^b$	$(\ddot{Y}_7^f + \ddot{Y}_{14}^b)/2$
8.	x_8	$\dot{Y}_8$	$\ddot{Y}_8^f$	$\ddot{Y}_{13}^b$	$(\ddot{Y}_8^f + \ddot{Y}_{13}^b)/2$
9.	x_9	$\dot{Y}_9$	$\ddot{Y}_9^f$	$\ddot{Y}_{12}^b$	$(\ddot{Y}_9^f + \ddot{Y}_{12}^b)/2$
10.	x_{10}	$\dot{Y}_{10}$	$\ddot{Y}_{10}^f$	$\ddot{Y}_{11}^b$	$(\ddot{Y}_{10}^f + \ddot{Y}_{11}^b)/2$
11.	x_{11}	$\dot{Y}_{11}$	$\ddot{Y}_{11}^f$	$\ddot{Y}_{10}^b$	$(\ddot{Y}_{11}^f + \ddot{Y}_{10}^b)/2$
12.	x_{12}	$\dot{Y}_{12}$	$\ddot{Y}_{12}^f$	$\ddot{Y}_9^b$	$(\ddot{Y}_{12}^f + \ddot{Y}_9^b)/2$
13.	x_{13}	$\dot{Y}_{13}$	$\ddot{Y}_{13}^f$	$\ddot{Y}_8^b$	$(\ddot{Y}_{13}^f + \ddot{Y}_8^b)/2$
14.	x_{14}	$\dot{Y}_{14}$	$\ddot{Y}_{14}^f$	$\ddot{Y}_7^b$	$(\ddot{Y}_{14}^f + \ddot{Y}_7^b)/2$
15.	x_{15}	$\dot{Y}_{15}$	$\ddot{Y}_{15}^f$	$\ddot{Y}_6^b$	$(\ddot{Y}_{15}^f + \ddot{Y}_6^b)/2$
16.	x_{16}	$\dot{Y}_{16}$	$\ddot{Y}_{16}^f$	$\ddot{Y}_5^b$	$(\ddot{Y}_{16}^f + \ddot{Y}_5^b)/2$
17.	x_{17}	$\dot{Y}_{17}$	$\ddot{Y}_{17}^f$	$\ddot{Y}_4^b$	$(\ddot{Y}_{17}^f + \ddot{Y}_4^b)/2$
18.	x_{18}	$\dot{Y}_{18}$	$\ddot{Y}_{18}^f$		$\ddot{Y}_{18}^f$
19.	x_{19}	$\dot{Y}_{19}$	$\ddot{Y}_{19}^f$		$\ddot{Y}_{19}^f$
20.	x_{20}	$\dot{Y}_{20}$	$\ddot{Y}_{20}^f$		$\ddot{Y}_{20}^f$

Model Performance and Application

Model Performance Measurement

The proposed Order-4 Predictor-Corrector model was subjected a model performance test using some common indicators such as Root mean squared error (RMSE), Mean absolute error (MAE), and Mean absolute percentage error (MAPE) (Nwokike et al., 2021). This was done to determine the performance characteristics of the Order-4 Predictor-Corrector Technique.

Application of the Technique

The proposed order-4 predictor-corrector time series smoothing technique is a great time series data smoothing tool that stands a chance amongst other time series smoothing techniques such as the simple moving average, weighted moving average, exponential smoothing techniques and others. The technique contains two smoothening techniques with first been applied as a predictor while the second acts as a corrector of the values produced by the first (predictor). Although, the model can be considered to be rigorous as it requires absolute care and time, but, with a simple computer code, it becomes absolutely easy and efficient as shown in table 2 and 3. The technique is quick adequate for removing noise (fluctuation) from large quantitative data.

Observation

1. The technique is adequate for most quantitative time series data
2. The technique requires a little rigor as it is required to compute the predictor ($\dot{Y}_t$) before computing the corrector ($\ddot{Y}_t$).
3. $\ddot{Y}_t$ cannot be calculated without first finding $\dot{Y}_t$.

Results

Table 2
The Order-4 Predictor-Corrector smoothened values.

Serial Number	*Observation*	*Predicted Values*	*Forward Corrected Values*	*Backward Corrected Values*	*Average (Smoothened Value)*
1	9.67	9.67		9.698438	**9.698438**
2	9.74	9.74		9.783125	**9.783125**
3	9.59	9.59		9.064826	**9.064826**
4	9.08	8.6225	9.199271	8.360677	**8.779974**
5	8.39	8.0375	8.248125	7.621962	**7.935043**

6	7.78	7.605833	7.818438	8.130972	**7.974705**
7	7.84	8.370417	7.844983	8.206441	**8.025712**
8	8.01	7.930833	8.35099	7.526927	**7.938958**
9	7.78	7.307083	7.591806	7.291389	**7.441597**
10	7.5	7.470417	7.282691	7.950938	**7.616814**
11	7.52	7.78625	7.642066	6.788646	**7.215356**
12	7.01	6.222917	7.245833	6.184566	**6.7152**
13	6.64	6.764583	6.152326	8.298594	**7.22546**
14	7.32	8.43875	7.547813	6.543576	**7.045694**
15	7.19	6.09	7.814427	7.756788	**7.785608**
16	7.54	8.39875	6.494566	7.468299	**6.981432**
17	7.67	7.380833	8.499688	8.616128	**8.557908**
18	8.12	8.680833	7.602483	6.979115	**7.290799**
19	7.76	6.81875	8.241632	8.874809	**8.55822**
20	8.27	9.5175	7.466267	8.179566	**7.822917**
21	8.21	7.4025	9.251753	7.471788	**8.361771**
22	7.92	7.806667	7.089115	8.48342	**7.786267**
23	8.27	9.037917	8.423872	9.368715	**8.896293**
24	9.06	9.563333	9.423316	9.614514	**9.518915**
25	9.57	9.438333	9.552726	8.916458	**9.234592**
⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
116	11.76	11.78833	11.7313	11.72002	**11.72566**
117	11.73	11.6725	11.75917	11.65427	**11.70672**
118	11.67	11.6275	11.63399	11.59731	**11.61565**
119	11.59	11.54542	11.59405	11.45634	**11.52519**
120	11.5	11.45458	11.49955	11.58297	**11.54126**
121	11.51	11.59792	11.49661	11.45	**11.47331**
122	11.48	11.39583	11.54981	11.48139	**11.5156**
123	11.48	11.51875	11.40227	11.4862	**11.44424**
124	11.49	11.49167	11.5375	11.55354	**11.54552**
125	11.54	11.59292	11.52	11.60179	**11.56089**
126	11.6	11.62292	11.62217	11.66273	**11.64245**

127	11.67	11.70917	11.65604	11.75455	**11.7053**
128	11.76	11.81708	11.76276	11.90464	**11.8337**
129	11.86	11.91042	11.86648	11.7971	**11.83179**
130	11.85	11.75417	11.86288	11.80354	**11.83321**
131	11.82	11.83042	11.75283	11.83389	**11.79336**
132	11.78	11.75958	11.82307	11.57984	**11.70146**
133	11.65	11.5175	11.65382	11.6096	**11.63171**
134	11.61	11.695	11.54667	11.76793	**11.6573**
135	11.66	11.7225	11.74498	11.5178	**11.63139**
136	11.57	11.38042	11.59141	11.34792	**11.46966**
137	11.42	11.35	11.31085	11.38814	**11.3495**
138	11.37	11.44667	11.39543		**11.39543**
139	11.43	11.50958	11.48764		**11.48764**
140	11.38	11.22667	11.40995		**11.40995**

Table 3
Performance measurement of the Order-4 Predictor-Corrector smoothing technique.

R^2	*MAE*	MSE	*RMSE*	*MAPE*
0.814867	0.314567	0.341938	0.170969	3.508546

Conclusion

The combination of two numerical techniques originally for ODE and PDE problems in this paper can be regarded as a great achievement in the field of time series smoothing. The modifications done to get this models adaptable to dataset is commendable. It can be seen that the order-4 predictor-corrector smoothing technique proposed in this paper requires absolute care and involves some rigor, but, the results are equally amazing and comparable to those of other techniques. This paper has not taken the pleasure of comparing results of the proposed technique to those of already time series smoothing technique, but, as we have seen in the literatures, there have been other techniques in existence where numerical techniques have been modified to be applicable in time series smoothing and comparison with other existing techniques were done.

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