# Modified 2-Step Implicit Linear Multistep Technique for Time Series Smoothing

**By**

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***Abstract***

*This paper presents a new time series smoothing technique developed from the modification of the 2-step implicit linear multistep numerical technique. The proposed time series smoothing technique was called 2-step mAMT in this paper. To test the efficiency of this new technique, it was applied to a real dataset (Nigeria external reserve, 1981 to 2015) with high noise (fluctuation). A comparison of the proposed 2-step mAMT was done with the simple moving average and simple exponential smoothing ($\alpha = 0.8$). As shown in table 3, the proposed technique produced an $R^2$ greater than the other techniques. The common indicators (measure of best fit) used are the MAE, MSE, RMSE, and MAPE as also displayed in table 3.*

***Keywords: Time series, Linear Multistep, Adams-Moulton, Implicit, 2-step, Forecasting, Smoothing technique.***

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## Introduction

Smoothing is not a new terminology in the field of time series, especially forecasting. Time series data collected naturally tend to have some presence of randomness (probability) which are considered as random variations as a result of uncertainty involved in the process that generate these data. There are already ways of dealing with these fluctuations found in time series so as to make these data more suitable for analysis and application for forecasting purposes. Time series smoothing is an essential technique data analysts use to reduce the noisy effects of such random variations. The moving average is predominantly used to reduce white noise caused by these fluctuations because of their ease to apply. The technique does well to smoothen time series data and make visible some important information hidden in the series.

Time series smoothing is a technique used to reduce or remove the effects of random variations from time series observations collected over time. Time series observations or data are historical information collected over a period of time and useful for making forecasts. They are simply a set of observations taken at successive intervals or over successive periods of time, Paul (2011). Time series data arise in many different field of life including sales, weather and climate, finance, economy, census, marketing, and production.

Castillo et al. (2016), and Silitonga et al. (2020), described forecasting as the art and science of predicting future events using data collected over a period of time and projecting it into the future with some form of mathematical model. In business, time series forecasting is broadly described as a technique for estimating many future aspects. Forecasting techniques can be divided into two distinguished categories namely: qualitative and quantitative. The smoothing technique proposed in this paper can only be applied on troubled quantitative time series data.

Smoothing troubled random data have come a long way in economic, financial, weather and engineering sciences. A notable smoothing technique – exponential smoothing was proposed in the late 1950s by Holt and Winters. Other notable smoothing technique is the moving average techniques.

$$Y_t = \frac{1}{k}\sum_{j=0}^{k-1} f_{t-k+1+j} \tag{1}$$

Where,
$t = k, k+1, \dots, n$
$Y_t$ = New forecast
$f_t$ = Original time series observation
$k$ = Number of observation.

One of the most popular smoothing technique is the exponential smoothing largely because of its flexibility, great results, and computational ease. The technique uses simple averaging procedure to allocate more weight to the newest observations, Muhamed et al. (2019).

Exponential smoothing models benefit from the simple and intuitive way in which their parameters are updated and their forecasts are generated, Guo et al. (2018). These models continuously enhance predictions by taking the average value of past value and refinement of the data in a decreasing way, Silitonga et al. (2020). Exponential and few other smoothing models have the capability of smoothing out random variation in the time series data to reveal underlying trends and some irregularities.

The exponential smoothing is given as:

$$Y_t = Y_{t-1} + \alpha(f_{t-1} - Y_{t-1}) \tag{2}$$

Where,
$Y_t$ = new forecast
$Y_{t-1}$ = previous forecast
$\alpha$ = smoothing constant
$f_{t-1}$ = previous actual observation

The simple exponential smoothing technique is given below as:

$$Y_{t+1} = \alpha f_t + \alpha(1-\alpha)f_{t-1} + \alpha(1-\alpha)^2 f_{t-2} + \cdots + \alpha(1-\alpha)^{t-1} f_1 + \alpha(1-\alpha)^t f_0 \quad (3)$$

Where $\alpha$ is the smoothing factor and $0 < \alpha \leq 1$.

The importance of forecasting was shown in the study done by Siregar et al. (2018) who used the exponential smoothing, weight moving average, and moving average to forecast sow talc production.

The weighted moving average technique is similar to the simple moving average computational procedure, but utilizes a coefficient of weighing and is used if there is a trend in the series, Siregar et al. (2018). It is a technique for finding average by giving the larger weight on the latest data than the weight on the previous data, Salman et al. (2014).

The general weighted moving average technique is given below as:

$$WMA_t(k) = \frac{w_t f_t + w_{t-1} f_{t-1} + w_{t-2} f_{t-2} + \cdots + w_{t-k} f_{t-k}}{w_t + w_{t-1} + w_{t-2} + \cdots + w_{t-k}} = \frac{\sum_{j=0}^{k} w_{t-j} f_{t-j}}{\sum_{j=0}^{k} w_{t-j}} \quad (4)$$

where $w_{t-j}$ is the weight of observation $f_{t-j}$ in the computation of the weighted moving average.

Some notable research have been conducted either to criticize, used or enhance existing forecast techniques. In the study by Pronchakov (2019), limitations of forecasting cryptocurrency prices using the exponential moving average, simple moving average, and weighted moving average at the financial markets was analyzed. Silitonga et al. (2020), applied the double exponential smoothing to forecast the number of students admitted in a study program at a university. The study of short-term travel prediction using the travel data based on cellular mobile service data analysis was done by Raiyn et al. (2012) using the moving average technique.

Guo et al. (2018), introduced an exponential smoothing technique that businesses can use to forecast demand of a new commodity. Parameters of the model can be updated using exponential smoothing and the model uses the Gompertz distribution to determine trend. An aggregation operator that uses properties of heavy ordered weighted averaging (OWA) and the moving averages as its main characteristics was presented by Castro et al. (2018). The techniques was called the heavy ordered weighted moving average (HOWMA) operator.

Linear multistep techniques approximate numerical values of the solution by referring to more than one previous value, Dattani (2008). The numerical techniques achieve greater accuracy than techniques that use the same number of function evaluations, since they utilize more information about the known portion of the solution.

Alen Alexanderian (2018), described the generic linear multistep technique as: $x_n = a + nh$, $n = 0, \ldots, N$ with $h = (b-a)/N$. Where the general form of a multistep technique is written as:

$$y_{n+1} = \sum_{j=0}^{p} \alpha_j y_{n-j} + h \sum_{j=0}^{p} \beta_j f(x_{n-j}, y_{n-j}), \quad n \geq p \quad (5)$$

Given $\{\alpha_i\}_{i=0}^{p}$ and $\{\beta_i\}_{i=-1}^{p}$ are constant coefficients, and $p \geq 0$. If $\alpha_p \neq 0$ or $\beta_p \neq 0$, then, the technique is called a $p+1$ step technique. The initial values, $y_0, y_1, y_2, \ldots, y_p$ can only be gotten by a different technique (like: an appropriate implicit technique). The technique is implicit if $\beta_{-1} \neq 0$, and explicit if $\beta_{-1} = 0$. We can agree that $y(x_n)$ is the exact value of $y$ at $x_n$, and $y_n$ is the numerical approximation to $y(x_n)$ when computed.

*Motivation*

This research has drawn motivation from the many recent research done by different scholars in the past in the area of time series smoothing (see; Paul, 2011; Ravinder, 2013; Kumari et al., 2014; and Attanayake et al., 2020). Some have focused more on the relevance of the smoothing constants especially in the weighted moving average and exponential smoothing techniques and others. This paper was designed to provide a smoothing technique with a fixed smoothing constant perfect for quantitative time series data of both mild and high fluctuation. It is important to mention that the derivation of this technique came from the idea used in providing numerical solutions for some differential equations, but, we have recognized that function $f_t$ can be a constant function and not necessarily an exponential, polynomial, trigonometric, logarithmic nor hyperbolic function.

*Objective and Organization*

The paper is poised to develop a time series smoothing technique capable of competing favorably with existing time series smoothing techniques. We have extended the 2-step Adams-Moulton technique to fit in a time series data. In this paper, the 2-step Adams-Moulton numerical technique was derived using the Stone-Weierstrass Theorem. The paper also considered the importance of developing a flexible and efficient time series smoothing technique that provides with less errors. The problem of missing smoothened values resulting to carrying over few actual values to fill in this gaps as is in the case of moving average and exponential techniques considered in this study was eliminated by the forward, backward, and average procedure we applied in this study. A comparison of the proposed technique with simple moving average and exponential smoothing techniques was also done and the model efficiency tests was done and results displayed.

## Methodology

This study achieved the proposed forecast technique by modifying the 2-step implicit linear multistep technique also called the 2-step Adams-Moulton technique, and then went ahead to show parametric adjustments to the new technique to achieve its purpose.

The linear multistep technique is a special category of multistep techniques. Here, the solution to the given ODE at a specific location is expressed as a linear combination of the numerical solution's values and the function's values at previous points. The 2-step Adams-Moulton technique is one of the most widely utilized linear multistep techniques used for nonstiff problems. The 2-step Adams-Moulton technique is rooted in the Stone-Weierstrass Theorem.

**Theorem:** Stone-Weierstrass Theorem - Let $f(t) : \mathbb{R} \rightarrow \mathbb{C}$ be continuous on $t \in [a, b]$. For all $\epsilon > 0, \exists$ a polynomial $\varphi(t) \ni ||f(t) - \varphi(t)|| < \epsilon$.
In other words, any continuous function can be approximated to an arbitrary accuracy by a polynomial; generally, the more demanding the accuracy of the approximation, the higher the order needed of such a polynomial.
With the Stone-Weierstrass Theorem in mind, let

$$y' = f(x, y), \quad y(x_0) = y_0 \tag{6}$$

We integrate both sides to have:

$$\int_{x_t}^{x_{t+1}} y'(x)dx = y(x_{t+1}) - y(x_t) = \int_{x_t}^{x_{t+1}} f\big(x, y(x)\big)\, dx \tag{7}$$

If we could integrate $f\big(x, y(x)\big)$ analytically, we (likely) would not need to resort to numerical techniques to determine the solution to the ODE. If we cannot integrate $f\big(x, y(x)\big)$ analytically, according to the Stone-Weierstrass Theorem above, we can approximate it with arbitrary accuracy by a polynomial $\varphi(x)$, and since all polynomials can be integrated analytically, we have an obtainable, fair approximation of the solution to the ODE:

$$y(x_{t+1}) - y(x_t) \approx \int_{x_t}^{x_{t+1}} \varphi_{k-1}(x)\, dx \tag{8}$$

Now, to ensure that our approximation is reasonable, we say, let $\varphi_{k-1}(x)$ be a polynomial such that $k = 3$, then the 2-step Adams-Moulton technique is achieved by interpolating the polynomial using the Newton-Gregory backward:

$$\varphi_4(x) = f_{t+1} + s\nabla f_{t+1} + \frac{s(s+1)}{2!}\nabla^2 f_{t+1} \tag{9}$$

We define $s = \frac{(x - x_{n+1})}{h}$ and points on the curve of the solution of Eq. (8) be $(x_0, y_0), (x_1, y_1), (x_2, y_2), .. (x_n, y_n)\ (x_{n+1}, y_{n+1})$.
We move to integrate over $x$ from $x_t\ to\ x_{t+1}$, this is same as integrating over $s$ from $0\ to\ 1$.

$$y_{t+1} - y_t = \int_{-1}^{0} \left( f_{t+1} + s\nabla f_{t+1} + \frac{s(s+1)}{2!}\nabla^2 f_{t+1} \right) h\, ds \tag{10}$$

$$y_{t+1} - y_t = \left[ \left( s f_{t+1} + \frac{s^2}{2}\nabla f_{t+1} + \left( \frac{2s^3 + 3s^2}{12} \right) \nabla^2 f_{t+1} \right) h \right]_{-1}^{0} \tag{11}$$

$$y_{t+1} - y_t = h\left( f_{t+1} - \frac{1}{2}\nabla f_{t+1} - \frac{1}{12}\nabla^2 f_{t+1} \right) \tag{12}$$

Newton-Gregory backward difference of $\nabla f_{t+1}$, $\nabla^2 f_{t+1}$ gives:

$$\nabla f_{n+1} = f_{n+1} - f_n \qquad (a)$$
$$\nabla^2 f_{n+1} = f_{n+1} - 2f_n + f_{n-1} \qquad (b)$$

We substitute for Eq. ($a$ $and$ $b$) in (12), we have:

$$y_{t+1} - y_t = h\left(f_{n+1} - \frac{1}{2}f_{t+1} + \frac{1}{2}f_t - \frac{1}{12}f_{t+1} + \frac{1}{6}f_t - \frac{1}{12}f_{t-1}\right) \qquad (13)$$

We now have:

$$y_{t+1} - y_t = h\left(\frac{12-6-1}{12}\right)f_{n+1} + \left(\frac{3+1}{6}\right)f_t - \frac{1}{12}f_{t-1} \qquad (14)$$

$$y_{t+1} - y_t = \frac{h}{12}(5f_{t+1} + 8f_t - f_{t-1}) \qquad (15)$$

*Modifying the 2-step Adams-Moulton technique to a time series forecast technique*

The number of trials between two successive elements of the series can be described as the step size $h$. Let time between successive observations be denoted as $t, t+1, t+2, t+3, \ldots N$. Then, time between two elements of the series is given as $h = (t+1) - (t) = 1$ or $h = (t+2) - (t+1) = 1$. Where $t$ is weekly, monthly or yearly data. Then, we have:

$$y_{t+1} - y_t = \frac{1}{12}(5f_{t+1} + 8f_t - f_{t-1}) \qquad (16)$$

Let $y_t$ be the ungiven initial forecast, therefore, $y_t = 0$ at all times. Let $y_{t+1} = Y_t$ for all forecast.

Then, we have:

$$Y_t = \frac{1}{12}(5f_{t+1} + 8f_t - f_{t-1}) \qquad (17)$$

Let us expand $f_{t+1}$ by taking it two steps backwards and then diving the sum to get an average. This move is similar to the simple moving average technique.

$$f_{t+1} = \left(\frac{f_t + f_{t-1}}{2}\right) \qquad (c)$$

Where $f_t$ and $f_{t-1}$ are previous observations.

We can now substitute for Eq. (c) in (17) to get have:

$$Y_t = \frac{1}{12}\left(5\left(\frac{f_t + f_{t-1}}{2}\right) + 8f_t - f_{t-1}\right) \qquad (18)$$

$$Y_t = \frac{1}{12}\left(\frac{5f_t}{2} + \frac{5f_{t-1}}{2} + 8f_t - f_{t-1}\right) \qquad (19)$$

$$Y_t = \frac{1}{12}\left(\frac{21f_t}{2} + \frac{3f_{t-1}}{2}\right) \qquad (20)$$

$$Y_t = \frac{1}{24}(21f_t + 3f_{t-1}) \qquad (21)$$

If in Eq. (21), we say, let $t = t + 1$, then, we have:

$$Y_{t+1} = \frac{1}{24}(21f_{t+1} + 3f_t) \tag{22}$$

Eq. (22) is therefore the new 2-step mAMT smoothing technique.
Where;
$Y_{t+1}$ is the smoothened value at time $t$.
$f_t \; \forall \; t = t, \; t+1$, are constant functions representing time series observations.
The coefficients are coefficients that adjusts the data values (entries), and the number 12, in the denominator, is the averaging constant.

*Updating the 2-step mAMT Parameters*

Given the time series $f_t$ such that $t = t, \; t+1, \; t+2, \; t+3, \ldots . t - N$, and $Y_{t+1}$ as smoothened values, such that $t = 1, 2, 3, 4, 5, \ldots, N$.
The first forward forecast, is estimated at $t = 1$, such that Eq. (22) becomes:

$$Y_{t+1} = \frac{1}{24}(21f_2 + 3f_1) \tag{23}$$

Continue until the last forecast is achieved.

*Forward and Backward Forecast Iteration Procedure*

Considering the dataset $f_t \; \forall \; t = 1, 2, 3, 4, 5, \ldots \ldots N$
We compute the first smoothened value $Y_{t+1}$, such that:
At $t = 1$, Eq. (22) becomes:

$$Y_2 = \frac{1}{24}(21f_2 + 3f_1) \tag{24}$$

The second smoothened value:
At $t = 2$, Eq. (22) becomes:

$$Y_3 = \frac{1}{24}(21f_3 + 3f_2) \tag{25}$$

The procedure is continued in this manner.

*Procedure for Obtaining the Smoothing*

In this paper, the proposed 2-step mAMT smoothing has been extended to have a backward smoothing procedure which shall be used to compliment deficiencies (if any) in smoothened values produced by the forward smoothing procedure. Additionally, if the forward and backward smoothened values are added and the average taken, it becomes certain that the final smoothened value column will have no missing values. This is important as it deals appropriately with the unfavorably custom of replacing missing smoothened (forecast) values with actual values. This problem is predominant in other smoothing techniques like the exponential smoothing and moving average techniques moving average techniques, exponential smoothing techniques and so on. We shall use the algorithm below to solve this problem:

**Variable Illustration:** Given the 12 point time series data $x_1, x_2, x_3, x_4, x_5, \dots, x_{12}$.

We shall demonstrate the usage of the 2-step mAMT, showing the forward and backward techniques.

Let the forward and backward smoothing be represented by $Y_t^f$ and $Y_t^b$ respectively.

Considering Eq. (19);

$$forward:\ at\ t = 1, \qquad Y_2^f = \frac{1}{24}(21f_2 + 3f_1) \tag{26}$$

$$forward:\ at\ t = 2, \qquad Y_2^f = \frac{1}{24}(21f_3 + 3f_2) \tag{27}$$

$$backward:\ at\ t = 1, \qquad Y_{11}^b = \frac{1}{24}(21f_{11} + 3f_{12}) \tag{28}$$

$$backward:\ at\ t = 2, \qquad Y_{10}^b = \frac{1}{24}(21f_{10} + 3f_{11}) \tag{29}$$

**Table 1**

Technique demonstration using variables.

| *Serial Number* | *Observation* | *Forward mAMT 2-step* | *Backward mAMT 2-step* | *Average = Smoothened Value* |
|---|---|---|---|---|
| 1. | $x_1$ | | $Y_{12}^b$ | $Y_{12}^b$ |
| 2. | $x_2$ | $Y_2^f$ | $Y_{11}^b$ | $(Y_2^f + Y_{11}^b)/2$ |
| 3. | $x_3$ | $Y_3^f$ | $Y_{10}^b$ | $(Y_3^f + Y_{10}^b)/2$ |
| 4 | $x_4$ | $Y_4^f$ | $Y_9^b$ | $(Y_4^f + Y_9^b)/2$ |
| 5. | $x_5$ | $Y_5^f$ | $Y_8^b$ | $(Y_5^f + Y_8^b)/2$ |
| 6. | $x_6$ | $Y_6^f$ | $Y_7^b$ | $(Y_6^f + Y_7^b)/2$ |
| 7. | $x_7$ | $Y_7^f$ | $Y_6^b$ | $(Y_7^f + Y_6^b)/2$ |
| 8. | $x_8$ | $Y_8^f$ | $Y_5^b$ | $(Y_8^f + Y_5^b)/2$ |
| 9. | $x_9$ | $Y_9^f$ | $Y_4^b$ | $(Y_9^f + Y_4^b)/2$ |
| 10. | $x_{10}$ | $Y_{10}^f$ | $Y_3^b$ | $(Y_{10}^f + Y_3^b)/2$ |
| 11. | $x_{11}$ | $Y_{11}^f$ | $Y_2^b$ | $(Y_{11}^f + Y_2^b)/2$ |
| 12. | $x_{12}$ | $Y_{12}^f$ | | $Y_{12}^f$ |

## Model Performance and Comparisons

*Comparison of 2-step mAMT with SMA and SES*

This paper has tested the efficiency of the proposed 2-step mAMT smoothing technique using dataset from Nigeria external reserve, 1981 to 2015 (Onwukwe et al., 2014). The results produced were also compared to results of simple exponential smoothing techniques (SES) and simple moving average (SMA) (order-2). We have chosen a smoothing constant of $\alpha = 0.8$ for the SES.

*Model Performance Measurement*

The performance of a smoothing techniques is preferably tested using real time datasets. By comparing the results to the actual data, the researcher is enabled to determine the performance characteristics of the model. Not only the performance of the proposed 2-step mAMT smoothing technique compared to the actual data but we have in this paper extended result comparison to also the SMA and SES using same dataset so as to draw a better conclusion. Performances of the various models have been done using some common indicators like the Mean absolute error (MAE), Mean squared error (MSE), Root mean squared error (RMSE), and Mean absolute percentage error (MAPE) (Ostertagova, 2016 and Oyewale et al., 2013). The level of model accuracy compared to the actual data is important in evaluating the performances of smoothing techniques in time series data analysis (Nwokike et al., 2021).

*Application of the Technique*

The proposed 2-step mAMT smoothing technique has shown to be adaptive with dataset of high fluctuation as seen in the results produced in this study. The new technique is adequate for time series smoothing and can be applied directly without the averaging component as shown in table 1 and 2. That is to say that the forward procedure can only be applied and taken as the smoothened value without carrying out the backward and averaging procedure. This proposed technique is unique because it has solved the problem of making first few actual data startup smoothened values (denoted with "*") as is the case of SMA and SES (see column 6 and 7 of table 2).

## Results

**Table 2**

The 2-step mAMT, SMA, and SES smoothened values.

| *Serial* | *Actual* | *Forward mAMT 2-step* | *Backward mAMT 2-step* | *Average = Smoothened Value* | *SMA Order-2* | *SES $\alpha = 0.8$* |
|---|---|---|---|---|---|---|
| 1. | 9.67 | | 9.67875 | **9.67875** | ***9.67** | ***9.67** |
| 2. | 9.74 | 9.73125 | 9.72125 | **9.72625** | **9.705** | **9.726** |
| 3. | 9.59 | 9.60875 | 9.52625 | **9.5675** | **9.665** | **9.6172** |
| 4. | 9.08 | 9.14375 | 8.99375 | **9.06875** | **9.335** | **9.18744** |
| 5. | 8.39 | 8.47625 | 8.31375 | **8.395** | **8.735** | **8.549488** |
| 6. | 7.78 | 7.85625 | 7.7875 | **7.821875** | **8.085** | **7.933898** |
| 7. | 7.84 | 7.8325 | 7.86125 | **7.846875** | **7.81** | **7.85878** |
| 8. | 8.01 | 7.98875 | 7.98125 | **7.985** | **7.925** | **7.979756** |
| 9. | 7.78 | 7.80875 | 7.745 | **7.776875** | **7.895** | **7.819951** |
| 10. | 7.5 | 7.535 | 7.5025 | **7.51875** | **7.64** | **7.56399** |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| 131. | 11.82 | 11.82375 | 11.815 | **11.81938** | **11.835** | **11.82543** |
| 132. | 11.78 | 11.785 | 11.76375 | **11.77438** | **11.8** | **11.78909** |
| 133. | 11.65 | 11.66625 | 11.645 | **11.65563** | **11.715** | **11.67782** |
| 134. | 11.61 | 11.615 | 11.61625 | **11.61563** | **11.63** | **11.62356** |
| 135. | 11.66 | 11.65375 | 11.64875 | **11.65125** | **11.635** | **11.65271** |
| 136. | 11.57 | 11.58125 | 11.55125 | **11.56625** | **11.615** | **11.58654** |
| 137. | 11.42 | 11.43875 | 11.41375 | **11.42625** | **11.495** | **11.45331** |
| 138. | 11.37 | 11.37625 | 11.3775 | **11.37688** | **11.395** | **11.38666** |
| 139. | 11.43 | 11.4225 | 11.42375 | **11.42313** | **11.4** | **11.42133** |
| 140. | 11.38 | 11.38625 | | **11.38625** | **11.405** | **11.38827** |

**Table 3**

Performance measurement of 2-step mAMT, SMA, and SES smoothing techniques.

| *Serial* | *2-step mAMT* | *SMA Order-2* | *SES ($\alpha = 0.8$)* |
|---|---|---|---|
| $R^2$ | 0.999324 | 0.983236 | 0.997462 |
| MAE | 0.018268 | 0.102964 | 0.042926 |
| MSE | 0.001248 | 0.030962 | 0.004688 |
| RMSE | 0.000624 | 0.015481 | 0.002344 |
| MAPE | 0.203742 | 1.132707 | 0.469588 |

## Conclusion

The technicality in the usage of the proposed technique can be said to be one of the challenges, but, then, this adds to the beauty and uniqueness of the technique. As seen in all three techniques considered in this paper, it is important to state that the proposed 2-step mAMT smoothing technique competed favorably against the simple moving average and the simple exponential smoothing techniques. The results of the mean absolute percentage error shows how closely the new technique tracks the actual data. For this reason, we recommend this technique for quantitative time series forecasting.

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