RELATING WITH THE PHRASE "A CHAIN IS AS STRONG AS ITS WEAKEST LINK" USING FUZZY LOGIC

By

Onyekonwu, A. C.

Mathematics Programme, National Mathematical Centre, Abuja. onye919@gmail.com

Abstract

Given the growing influence of fuzzy logic which gave rise to new computing process that allows for computation using linguistic variables, this paper delved into relating with the phrase "a chain is as strong as its weakest link." Using the concept of Fuzzy logic to draw and infer basic conclusion for use in our today's world.

Keywords: *Fuzzy logic, Linguistic variables, Computation.*

Introduction

The knowledge of fuzzy logic, which is rooted in the idea of a relative graded membership of a set, called fuzzy set Zadeh (1965), based on human cognitive ability Kahraman et al. (2016) can relate with information as perceived by humans. The information, considered as vague, uncertain, imprecise, partially true, partially false or without clear distinctions and so cannot be straight-way categorized like we have in classical logic. Fuzzy logic uses human perceptions, to perform computations with the help of logical connectives i.e., set theoretic operators and falls in line with the age long human ability to resolve similar problems with outstanding results. The logic is useful in resolving multi-criteria decision-making process, Yager (2015) thereby coming out with satisfactory decisions, where criteria are in the domain of linguistic variables (Zadeh,1975).

The logic plays a great role in the development of intelligent systems for decision making, identification, pattern recognition and control. For over two decades, this logic which is the basis for our new computing process, allowing for the computation of linguistic variables have found usefulness in the medical science, social sciences, engineering, psychology, as in fraud detection systems, policy sciences, assessment of credit worthiness systems and economics (Singh et al., 2013).

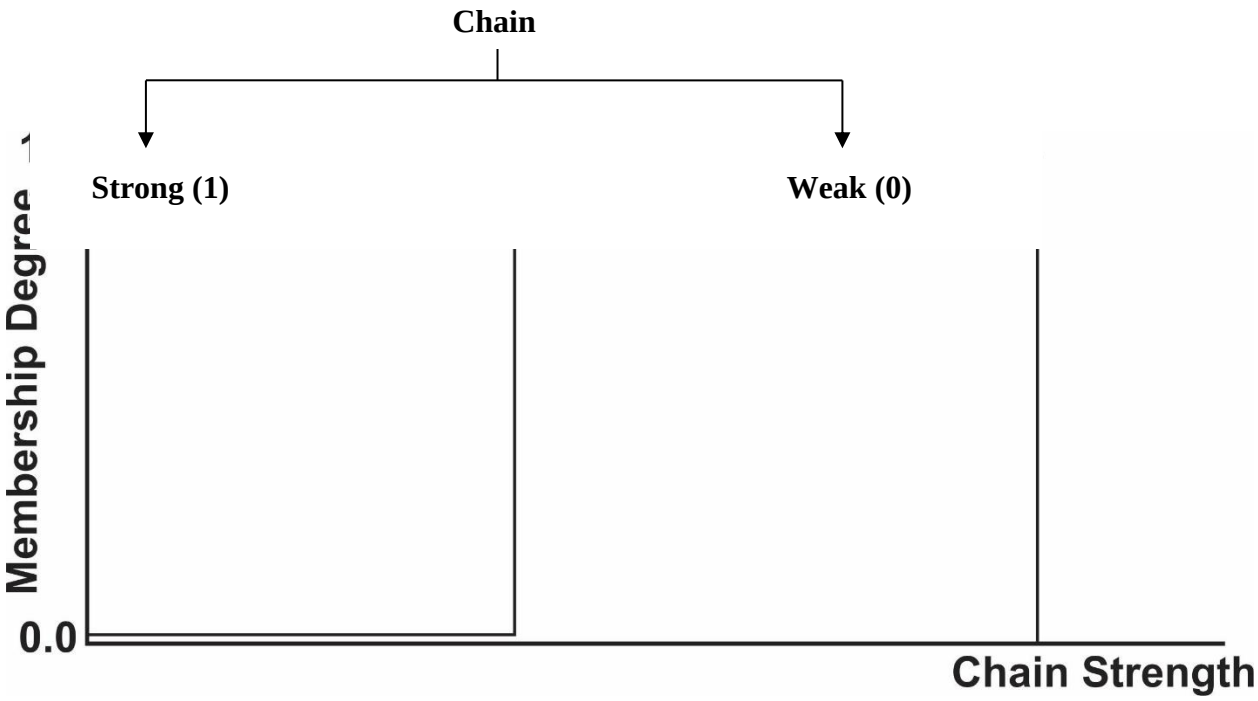
In this Work, we proceed to relate with the phrase "a chain is as strong as its weakest link" in order to drive its significance in fields of human endeavor, where interdependent entities work together to achieve common objectives, (Arshinder et al., 2007).

1.0 Fundamentals of Relating a Chain and its Weakest Link

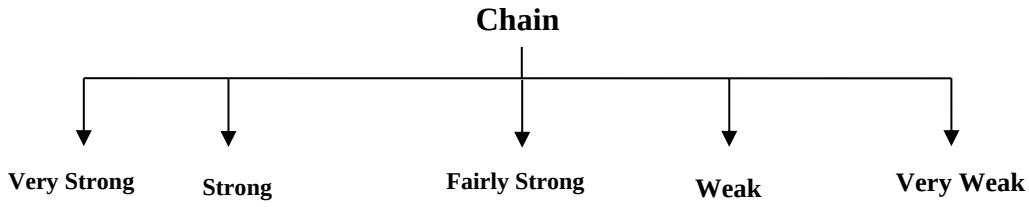
Based on the perception of humans over the years, it is an obvious fact that a chain is only as strong as its weakest link. However, the conversion of the notion into figurative phrase is found in expression, "In every chain of reasoning the evidence of the last conclusion can be no greater than that of the weakest link of the chain, whatever maybe the strength of the rest" Ried (1786) and "A weak part or member will affect the success or effectiveness of the whole" (Manser,2007).

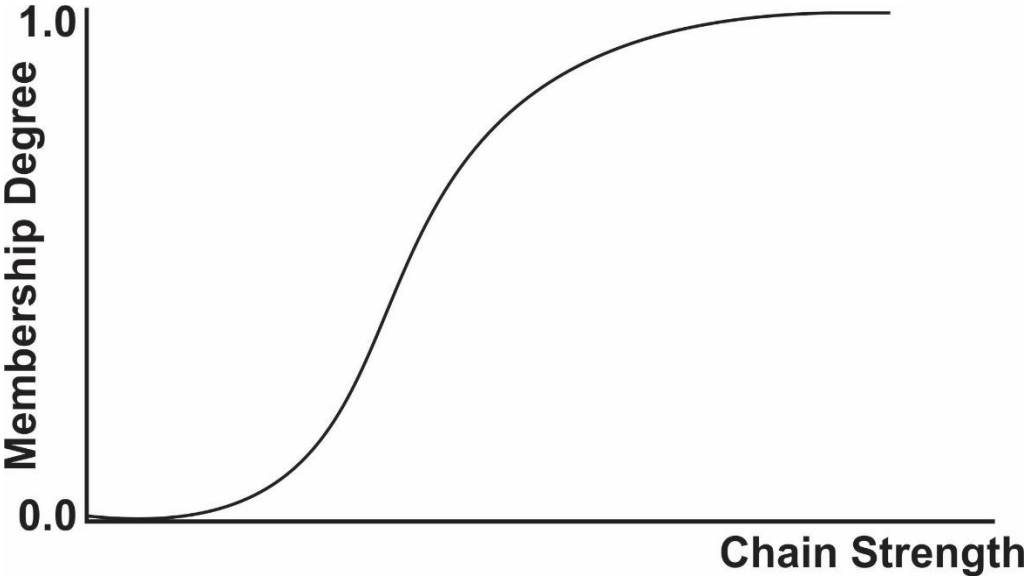
For the purpose of this research and in order to drive the mathematics of our logic, we will consider a five-link chain, valued in words as, very strong, strong, fairly strong, weak and very weak.

Classical Logic



Fuzzy Logic





Notice, in classical logic, there is no provision for the weakest link, for we judge only by strong or weak, i.e., 1 or 0, hence the logic will not be able to defer to the phrase and draw logical conclusions. However, with fuzzy logic our subdivision continues depending on the level of accuracy we desire. In this research, we have restricted ourselves to five.

2.0 Logical Fundamentals

Taking into cognizance the various figurative expressions of the original phrase, we will attempt to consider each chain link as a criterion for a decision resulting in the strength of the entire chain. Hence in trying to logically compute all chain links, we make use of the "anding" logical connective, Yager (2015) to give a decision function $D_{\bar{s}}(l)$, representing the membership degree of the entire chain with respect to strength, since they are working together for a common purpose. Thus, alphabetically we are going to:

a. Represent the strength of each link (l_i), i = 1, 2, 3…5 that make up the chain as basically a criterion for the strength of the entire chain. Hence, we define;
$C_{\bar{s}}(l_i) \in [0, 1]$, i = 1, 2, 3… 5 as the membership degrees of the links, l_i, i = 1, 2, 3…5, representing, very strong, strong, fairly strong, weak and very weak chain link respectively with respect to strength, S.

b. We construct a Decision function $D_{\bar{s}}(l)$, $D_{\bar{s}}(l) \in [0,1]$ on the basis of linguistic formulation of relationship with respect to the link l_i, i=1,2, 3…5 using set theoretic operators or logical connectives.

c. Determine the logical computation of
$C_{\bar{s}}(l_i)$, i=1, 2, 3...5
$C_{\bar{s}}(l_i) \in [0\ 1]$, which gives the function $D_{\bar{s}}(l) \in [0\ 1]$ representing the membership function of the entire chain with respect to strength, S.

2.1 Defining the Fuzzy Sets of Chain Links

Based on this research work, which is a result of our subjective perception for the various chain links, we define the following:

Very Strong Chain Link = $\mathbf{C_{\bar{s}}(l_1)} = 0.95$
Strong Chain Link = $\mathbf{C_{\bar{s}}(l_2)} = 0.75$
Fairly Strong Chain Link = $\mathbf{C_{\bar{s}}(l_3)} = 0.53$
Weak Chain Link = $\mathbf{C_{\bar{s}}(l_4)} = 0.30$
Very Weak Chain Link = $\mathbf{C_{\bar{s}}(l_5)} = 0.15$

2.2 Constructing Decision Function for Chain Strength.

Considering each chain link as a criterion for a decision resulting in chain strength we have:

$$D_{\bar{s}}(l) = C_{\bar{s}}(l_1) \cap C_{\bar{s}}(l_2) \cap C_{\bar{s}}(l_3) \cap C_{\bar{s}}(l_4) \cap C_{\bar{s}}(l_5)$$
$$= 0.95 \cap 0.75 \cap 0.53 \cap 0.30 \cap 0.15$$
$$= 0.15$$

This implies $D_{\bar{s}}(l) = 0.15$

2.3 Chain Strength

Based on our computation, which gives the value of $D_{\bar{s}}(l) = 0.15$, it implies the entire strength of the chain, irrespective of what each link contributes to the computation amounts to $D_{\bar{s}}(l) = 0.15$. Hence the entire chain is said to be very weak.

3.0 Conclusion

The perception by humans over the years, that a chain is only as strong as its weakest link, with figurative expression, Ried (1786) and Manser (2007), can be established using fuzzy logic.

In the fight against COVID-19 Pandemic, the strength of this logic, availed the World Health Organization, the fundamentals of tackling the disease. Hence the success of its control and eradication world-wide is hinge on ensuring the weakest of individual is not vulnerable. This same logic found its way in resolving the choice of a life partner for marital satisfaction (Onyekonwu, 2020).

To inspire further research on this logic, we humbly state that, what legal professionals consider "a judgment beyond every reasonable doubt" points to the fact that, evidence considered for judgment, irrespective of evidential strength, depends on the least performing evidence put forward for the delivery of justice.

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