A MONTE CARLO SIMULATION APPROACH TO MODELLING THE EFFECT OF RESIDUAL CORRUPTION OF INCORRUPT PERSONS ON THE TIME TO EXTINCTION OF CORRUPT INTERACTIONS IN A SYSTEM

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Abstract
Various agencies established by the government of Nigeria over the years in combating corrupt practices has not been completely successful. In this work, an Anti-Corruption Simulation Model (ACSIM) using the Sterile Insect Technique (SIT) approach is developed to mitigate corrupt practices. This was done with the intent of investigating the effect of the residual corruption of the incorrupt person on the time to extinction of the corrupt interactions in a system. The model successfully determined the number of incorrupt persons that will completely prevent the flow of corrupt interactions arising from the corrupt persons in the system. The time to extinction of corrupt interaction in a system with 0.1 proportion of incorrupt interaction, 0.0 and 0.1 residual corruption rates account for 492 and 547 time units to extinction of corrupt interactions respectively. Showing that 0.1 residual corruption rate accounts for additional 55 units of time to extinction. In general, the result showed that, the competing ability notwithstanding, residual corruption rate as low as 0.1 elongate the extinction time of corrupt interactions. Hence the residual corruption of incorrupt persons in a system is directly proportional to the time to extinction of corrupt interactions.

Keyword: *Residual Corruption, Corrupt interaction, incorrupt interaction, total number of interactions, time to extinctions of corrupt interactions.*

1.0 INTRODUCTION

The biological means of insect control that is free of environmental pollution is the Sterile Insect Technique (SIT). It was introduced by Knipling (1955) with the sole aim of diluting the reproductive potential of insect's population. This is done via the introduction of sterile males into the insect population in order for mating to take place. The wild females that mate with these sterile males (fully sterilised) lay eggs that contain dominant lethal mutations. The development of this eggs are hampered as the result of this dominant lethal mutations, hence reducing the next generation of this insects (Walker, 2012).

Klassen (2009) reported this technique as a good control strategy for Islands and small isolated patches that are not under a constant threat of immigration. SIT has recorded a lot of successes over the years in eliminating insect population (Knipling, 1955; Kebede *et al.*, 2015). This work successfully applied the SIT to curb corruption in a system of carrying capacity (K) while considering the effect of residual corruption on the time to extinction of corrupt interactions. This was inferred from the concept of residual fertility of insect as pointed out by Barclay (2001). According to Knipling (1955), sterile male insect need compete with the wild males before mating with the wild females. In combating corruption in a system, the incorrupt persons have to compete favourably with the corrupt persons before they can block corrupt interactions successfully.

The objective of this work is to determine the optimal number of incorrupt persons in a system of carrying capacity (K) (i.e. the sterile males), that will perform the function of blocking corrupt interactions (i.e. corrupt practices) until it goes extinct over time. This was showed to depend on the residual corruption rate across competing abilities of the incorrupt persons as well as the proportion of incorrupt interactions. The major focus in this work is on assessing the negative effect of the residual corruption on the time to extinction of corrupt interactions in a system. This was implemented via Monte Carlo Simulation Modelling Technique

This technique is adopted because most deterministic models of SIT do not capture the random perturbations that take place before mating and hence the need for a stochastic modelling approach that will capture all the perturbed interactions in the system (Bogyo, 1974).

This work extends the work of Peter *et al.* (2019) on a simulation study of the negative effect of the competing ability of incorrupt persons on the time to extinction of corrupt interactions in a system by incorporating the Residual Corruption of the incorrupt person. This is because some proportions of the incorrupt person might yield corrupt interactions and hence the need to study its effect on the time to extinction of corrupt interactions in a system.

The rest of this paper is organised as follows; method, results, discussion, conclusion and recommendation.

2.0 Method

This section present the application of the working principle of the SIT with the effect of residual fertility of the sterile males in curbing corruption in a system as well as the flow chart for the simulation model.

2.1 Population Growth Model

Tsoularis (2001) present a simple population growth model as given in equation (2.1) as

$\frac{dN}{dt} = rN, N(0) = N_0$ (2.1)

Equation (2.2) is the recurrence relation of equation (2.1) according to Barclay *et al. (2016)*

$N_{g+1} = rN_g, 0 < r \leq 1$ (2.2)

where r is the intrinsic rate of growth, N_g is the population of insect at time g and N_{g+1} is the population of insect at time $g + 1$. The population in (2.2) will keep growing exponentially if no control measure is incorporated.

To supress the generational growth, sterile males are introduced in order to bring the population of insect in (2.2) under control. This is expressed as the ratio of wild male insects to the total male insects (sterile males plus wild males). This is captured in the equation (2.3)

$N_{g+1} = rN_g \left(\frac{M_g}{M_g+S}\right)$ (Knipling, 1955) (2.3)

where N is the population size, S is the sterile male M is the number of wild male and r is the intrinsic rate of growth. The quantity $\frac{M_g}{M_g+S}$ is the supressing ability (the ratio of wild males to the total males). Its effect largely depends on the sterile male release (S) with full competing ability.

The population growth model for corruption as inferred from (2.2) above can be re-written as follows

$N_{g+1}^c = rN_g^c, 0 < r \leq 1$ (2.4)

where r is the intrinsic rate of growth of the number of corrupt interactions in the system, N_g^c is the number of corrupt interactions at time c and N_{g+1}^c is the number of corrupt interactions at time g+1. Without control measure, the number of corrupt interactions in (2.4) grows exponentially.

To curtail this number of corrupt interactions $\left(N_{g+1}^c\right)$ in equation (2.4) from growing indefinitely, we borrow a leaf from Knipling (1955) and re-write equation (2.4) as follows;

$N_{g+1}^c = rN_g^c \left(\frac{N_g^c}{N_g^c+\alpha}\right), 0 < r \leq 1.$ (2.5)

The introduction of the ratio term $\left(\frac{N_g^c}{N_g^c+\alpha}\right)$ brings the population under control. Where N_g^c remain as earlier defined, α is the number of incorrupt interactions in a system. Phuc *et al.* (2007) explained the effect of competing ability of sterile male on the time to extinction of insect population in their work on insect sterile method for reducing fertility in female insects. Equation (2.3) becomes

$N_{g+1} = rN_g \left(\frac{M_g}{M_g+\rho S}\right), 0 \leq \rho \leq 1$ (2.6) where ρ is the competing coefficient or competing ability and other variables are as earlier defined.

For our corruption case, we re-write (2.6) as

$$N_{g+1}^{c} = rN_{g}^{c}\left(\frac{N_{g}^{c}}{N_{g}^{c}+\rho\alpha}\right), 0 \leq \rho \leq 1 \qquad (2.7)$$

where ρ is the competing ability of the incorrupt persons, it is a proportion that take values between 0 and 1, (0 and 1 inclusive). The value '0' indicates no competing ability and the value 1 indicates perfect competing ability.

Solving for S in (2.6) with perfect competing ability, the threshold release value of the sterile male insect becomes

$$S^{*} = (r-1)M \qquad (2.8)$$

where M is the steady state population and r is the intrinsic rate of growth.

In some cases, treated male insects are not completely sterilized, which means the sterilised insect has some proportion that are fertile. In such a case, control with SIT become more complicated since the supposed sterile that are expected to render the fertile female infertile end up fertilizing them. Barclay (2001) constructed a simple model to handle residual fertility of the sterile. When there is incomplete sterilization of the released insects, a fraction called γ, of males remains fertile as pointed out by modified Knipling's model as in equation (2.9). The critical sterile release rate is then finite if and only if $\gamma < \frac{1}{r}$ else the population is not controllable by sterile male releases alone. The model for residual fertility according to the Knipling (1955) is,

$$N_{g+1} = rN_{g}\left(\frac{M_{g}+\gamma\rho S}{M_{g}+\rho S}\right), 0 \leq \rho \leq 1,\ \ 0 \leq \gamma < 1 \qquad (2.9)$$

where N_g is the previous generation, N_{g+1} is the current generation, r is the intrinsic rate of growth, M_g is the fertile or wild males, S is the sterile males, γ the proportion of sterile that remain fertile and ρ is the competing ability of the sterile males.

The threshold release for residual fertility model in equation (3.9) at the steady state becomes

$$S^{*} = \frac{M(r-1)}{1-r\gamma} \qquad (2.10)$$

where M is the steady state population of the wild males insects, r is the intrinsic rate of growth and γ is residual fertility proportion.

We re-write the equation (2.9) for residual corruption as follow;

$$N_{g+1}^{c} = rN_{g}^{c}\left(\frac{N_{g}^{c}+\gamma\rho\alpha}{N_{g}^{c}+\rho\alpha}\right), 0 \leq \rho \leq 1,\ \ 0 \leq \gamma < 1 \qquad (2.11)$$

where γ is the proportion of incorrupt interactions that became corrupt, N_g^c, N_{g+1}^c, r, α and ρ are as earlier defined. Equations (2.6) and (2.10) can be summarize mathematically as follow;

$$N_g \rightarrow \{0, if \; S > S^* \; \infty, if \; S < S^* \; M, if \; S = S^* \quad (2.12)$$

For the corrupt case, we have;

$$N_g^c \rightarrow \{0, \text{if } \alpha > S^* \; \infty, \text{if } \alpha < S^* \; M, \text{if } \alpha = S^* \quad (2.13)$$

where S^* is redefined as the threshold value of incorrupt interactions and M is now defined as the steady state value of the number of corrupt interactions.

2.3 Simulation Model

The simulation technique employed in this work is the Monte Carlo Simulation. This was used in sampling the number of interactions (N) from the Poisson distribution with mean approximated to be $\lambda = \binom{K}{2}$.

Though several interactions exist, we assumed in this work that on the average, an effective interaction is between pairs of persons. Using the principle of graph theory, we let the edges represent the interactions while the vertices represent persons. The Poisson distribution is used to sample the total number of interactions (N). The model uses this number of interactions as the number of trials serving as input into a Binomial distribution. This is done to allow for sampling the number of incorrupt interactions (α) with a given proportion of success rate (p) while failure rate (q). We employed binomial distribution because the number of interactions (N) is the combination of corrupt and incorrupt interactions; the incorrupt is treated as success and the corrupt as failure.

The followings are the parameters used in this work; (i) the intrinsic rate of growth (r), (ii) the competing ability of the incorrupt person (ρ), (iii) proportion of incorrupt interactions (p), (iv) residual corruption of incorrupt person (γ) and (v) the carrying capacity (K) of the system.

Given a system of carrying capacity (K) or staff strength, the number of possible ways two persons can interact is $\binom{K}{2}$.

$$\binom{K}{2} = \frac{K(K-1)}{2}. \quad (2.14)$$

Note that

$$K = n_i + n_c \quad (2.15)$$

$$\binom{K}{2} = \binom{n_i}{2} + \binom{n_c}{2} + n_i n_c \quad (2.16)$$

$$\alpha = \binom{n_i}{2} + n_c n_i \quad (2.17)$$

$$N_g^c = \binom{n_c}{2} \quad (2.18)$$

$$n_c^2 - n_c - 2N_g^c = 0 \quad (2.19)$$

Considering the positive root of $\mathrm{n_c}$ from equation (2.19) u*sing quadratic formula, we have*

$$\mathrm{n_i} = \mathrm{K} - \left(\frac{1+\sqrt{1+8\mathrm{N_g^c}}}{2}\right). \qquad (2.20)$$

where K *is the system carrying capacity,* $\mathrm{n_i}$ *is the number of incorrupt persons and* $\mathrm{n_c}$ *is the number of corrupt persons. In equation (2.17) above,* $\binom{\mathrm{n_i}}{2}$ *is the number of ways two incorrupt persons interact,* $\binom{\mathrm{n_c}}{2}$ *is the number of ways two corrupt persons interact and* $\mathrm{n_i n_c}$ *computes the number of incorrupt interactions arising from the interactions between an incorrupt person and a corrupt person. The number of corrupt and incorrupt persons are respectively determined from equations (2.15) and (2.20).*

2.3.1 Model assumptions

i) The intrinsic rate of growth (r) is equal to 1.

ii) The incorrupt persons have the ability to prevent a flow of corruption after interaction with the corrupt persons and may or may not be negatively influenced.

iii) There are no staff transfer and new employment in the period under consideration.

iv) Effective interactions is in pairs (between two persons).

2.3.2 Model algorithm

The algorithm for the simulation model according to Peter et al. (2019) is as given below. The proportion of incorrupt interactions denoted by p takes values between 0 and 1. While the proportion of corrupt interactions is

$$\mathrm{q} = 1 - \mathrm{p}$$

Step 1:

Sample number of interaction N *about* $\binom{\mathrm{K}}{2}$ *mean interactions from the Poisson distribution*

Step 2:

Given this N *number of interactions, sample number of incorrupt interaction* (α) *from the binomial distribution with* p *success rate and* q *failure rate.*

Step 3:

Compute the number of corrupt interactions $(\mathrm{N_g^c})$ as $\mathrm{N} - \alpha$.

Step 4:

Determine (g) *the time to extinction from the growth model as follows*

$$N_{g+1}^c = \mathrm{r} N_g^c \left(\frac{N_g^c}{N_g^c+\alpha}\right);$$

Continue to apply the growth model above while $\mathrm{N_{g+1}^c} > 0$ *and track or note the* $'\mathrm{c}'$ *when* $\mathrm{N}_{g+1}^{\mathrm{c}} = 0$ *(time to extinction) as well as* $\mathrm{N_g^c}$ and α.

Step 5:

Repeat steps 1 to 4 for the same value of p using 1000 different random number streams and compute the means, $\bar{c}$, $\overline{N_g^c}$ *and* $\bar{\alpha}$.

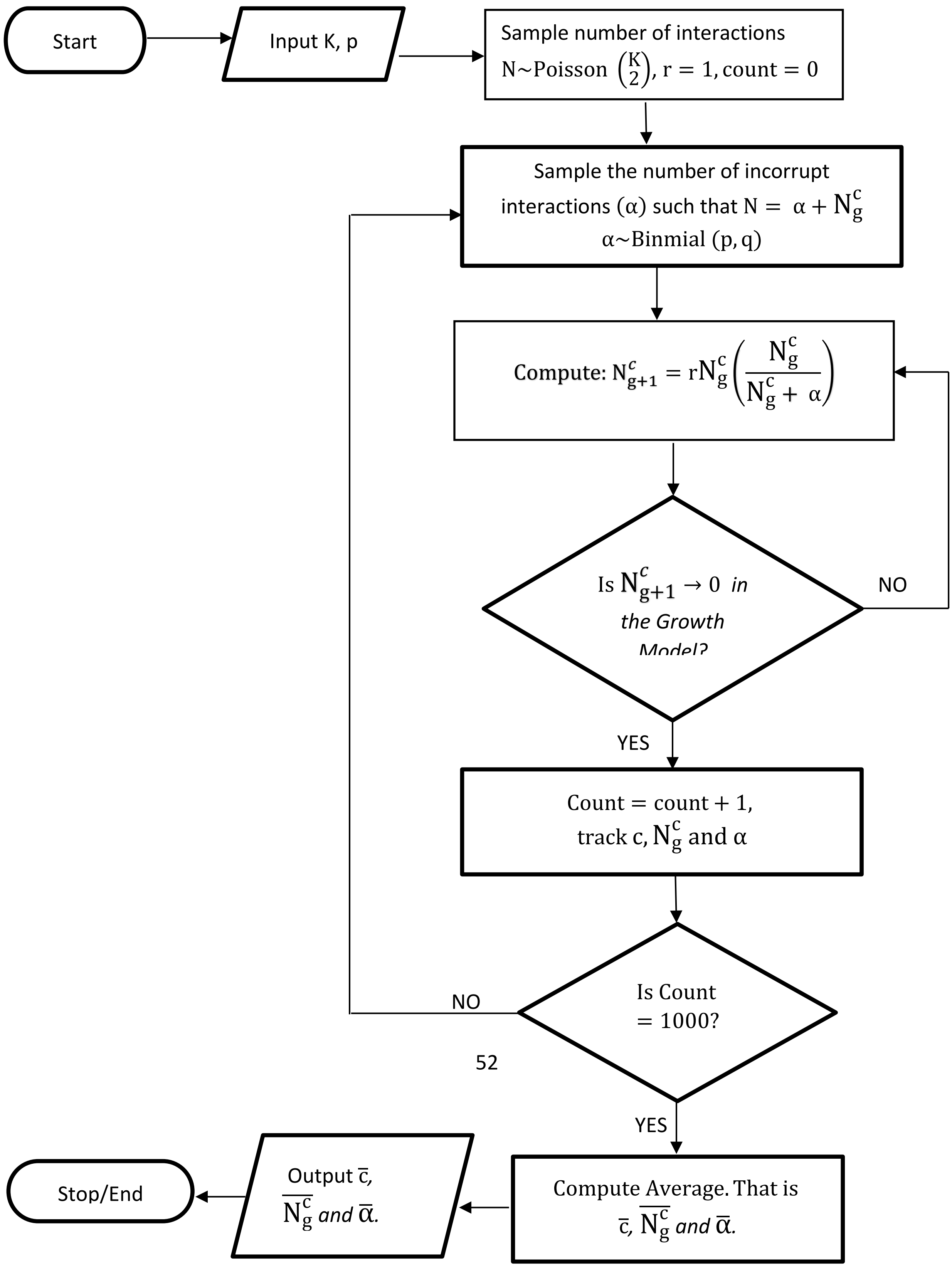


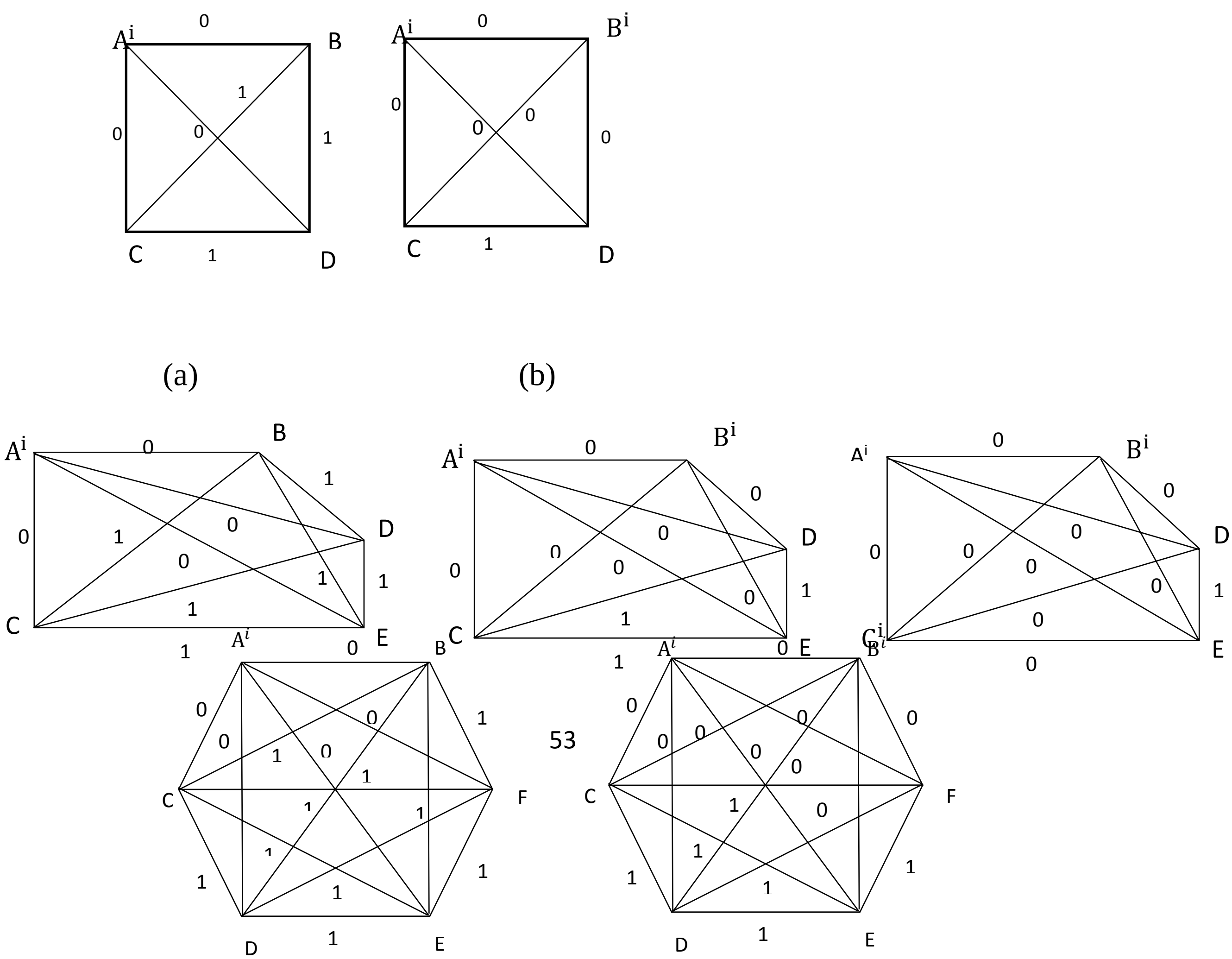
Figure 1: The flow chart of the Simulation Model (Peter et al., 2019).

2.3.3 Verification

The correctness of this model is checked by running the program in segments using stops. Furthermore, the syntax as well as the semantics of the Pascal programming language was also checked.

2.3.4 Validation

The major challenge faced in simulation is validation. This is due to lack of data. Peter *et al.* (2019) surmount this challenge by creating virtual image of real life systems and this is used in this work. The virtual image created is called the 'virtual system'. In these virtual systems created using the knowledge of graph theory, the edge of the graph represent the interactions while vertices represent the persons. For ease of manual handling, four (4), five (5) and six (6) vertex graphs were selected to represent systems with 4, 5 and 6 carrying capacities as show in Figure 2 below.



(f)

(g)

(h) (i)

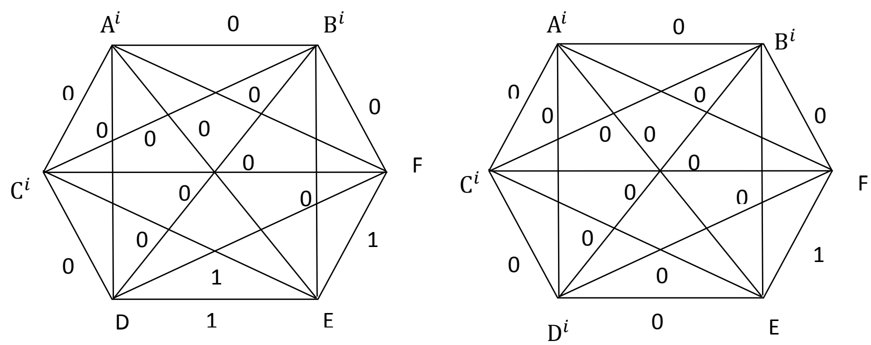


Figure 2: Virtual systems with four (4), five (5) and six (6) carrying capacities (a) One incorrupt person and three corrupt persons (b) Two incorrupt persons and two corrupt persons (c) One incorrupt persons and four corrupt persons (d) Two incorrupt persons and three corrupt persons (e) Three incorrupt person and Two corrupt persons (f) One incorrupt persons and five corrupt persons (g) Two incorrupt persons and four corrupt persons (h) Three incorrupt persons and three corrupt persons (i) Four incorrupt persons and two corrupt person (Peter *et al.*, 2019)

The virtual system for carrying capacities of four (4), five (5) and six (6) are as shown in Figure 2. Any vertex (persons) with superscript i denote the incorrupt person(s) while corrupt person(s) are those without a superscript. Figures 2a and 2b has total number of six interactions (edges) with a carrying capacity of four (4). It consist of one incorrupt person and three corrupt persons as well as two incorrupt and two corrupt persons respectively. The edge is the total of both the zeros (0's) and the ones (1's) where the 0's represent the incorrupt interactions and 1's represent the corrupt interactions. The ratio of zeros (0's) to the total interactions (0's plus 1's) is used to compute the proportion of incorrupt interactions.

Figures 2c-2e is the virtual systems with a carrying capacity of five (5), it has the total number of ten (10) possible interactions. Figure 2c has one incorrupt and four corrupt persons, Figure 2d has two incorrupt and three corrupt persons while Figure 2e has three incorrupt and two corrupt persons. Finally, Figures 2f-2i represent a virtual systems with a carrying capacity of six (6) and a total of fifteen possible interactions. Figure 2f contain one incorrupt and five corrupt persons, Figure 2g has two incorrupt and four corrupt persons, Figure 2h contain three incorrupt and three corrupt persons and Figure 2i has four incorrupt and two corrupt persons.

Using equation (2.14), the total number of possible interactions in each virtual system were determined. This includes the number of corrupt and incorrupt interactions in the system. We make used of the weak law of large number since the number of incorrupt interaction $\left(N_g^c\right)$ in each virtual system created represent an average ($\overline{N_g^c}$) *in the simulated system after 1000 iterations.*

Carrying capacity (K) and proportion of incorrupt interaction $(\mathrm{p_i})$, in each virtual system were used to run the simulated system, where;

$$\mathrm{p_i} = \frac{\alpha}{\binom{K}{2}} \tag{2.20}$$

Correlation between the upper bound of the simulated number of incorrupt interactions and the number of incorrupt interactions from the virtual system were checked to see if they correlate with each other. The two sets of data were used to compute correlation coefficient and thereafter to determine if the correlation between them is strong or not. The number of incorrupt interactions in the virtual systems are extreme (maximum number) value and hence they were compared with the approximate upper bound of the number of incorrupt interactions in the simulated systems.

The simulated system is said to have successfully mirrored the virtual system if there is a strong and significant positive correlation between the two. It therefore follows that the simulation model is validated for use in modelling anticorruption in a system.

3.0 Results

In this section, we present model verification, validation and the interpretation of results.

3.1 Discussion on Model Verification

Validation of this model was done using Tables 1 and 3. The response of the model to the changes in model parameters such as competing ability, residual corruption rate and proportion of incorrupt interactions were observed. Observe from Table 1 and Table 3 that, with a given proportion of incorrupt interaction in the model, the corrupt practices of the corrupt person goes extinct at different time points. The extinctions' time depends on the competing ability of the incorrupt person as well as the residual corruption of the incorrupt person. As the residual corruption rates increases, the average time to extinction also increases. Whereas as the competing ability of the incorrupt person increases, the average time to extinction of corrupt interactions reduces (Figures 6 and 8). From the above, it become obvious that the logic of the simulation model is correct.

3.2 Validation of Model for Use

We created a virtual systems for validation purposes as mentioned above. Three virtual systems were created which includes four (with 1 and 2 incorrupt persons), five (with 1, 2 and 3 incorrupt persons) and six (with 1, 2, 3 and 4 incorrupt persons) carrying capacities. From the virtual system, the following sets of proportions of incorrupt interactions were obtained across the number of incorrupt persons; $\left(\frac{3}{6}, \frac{5}{6}\right)$, $\left(\frac{4}{10}, \frac{7}{10} \text{ and } \frac{9}{10}\right)$ and $\left(\frac{5}{15}, \frac{9}{15}, \frac{12}{15} \text{ and } \frac{14}{15}\right)$ respectively (Table 2).

Table 1: A Distribution of Mean Time to Extinction of corrupt Interactions across selected Competing Abilities for a System of Carrying Capacity of Twenty (20) and a Growth Rate of One (1)

Proportion of Incorrupt Interactions	Competing Ability											
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	n_i	n_c
	0.0	1.0										
0.10	+∞	492	248	166	125	101	85	73	64	57	52	1	19
0.15	+∞	306	155	104	79	64	54	46	41	37	33	2	18
0.20	+∞	214	108	73	56	45	38	33	29	26	24	2	18
0.25	+∞	160	81	55	42	34	29	25	23	20	19	3	17
0.30	+∞	124	64	43	33	27	23	20	18	16	15	3	17
0.35	+∞	97	50	34	26	22	19	16	15	13	12	4	16
0.40	+∞	78	41	28	22	18	15	13	12	11	10	4	16
0.45	+∞	63	33	23	18	15	13	11	10	9	9	5	15
0.50	+∞	51	27	19	15	12	11	10	9	8	7	6	14
0.55	+∞	41	22	15	12	10	9	8	7	7	6	6	14
0.60	+∞	33	18	13	10	9	8	7	6	6	5	7	13
0.65	+∞	26	14	10	8	7	6	6	5	5	5	8	12
0.70	+∞	21	12	8	7	6	5	5	4	4	4	9	11
0.75	+∞	16	9	7	6	5	4	4	4	4	3	10	10
0.80	+∞	12	7	5	4	4	4	3	4	3	3	11	9
0.85	+∞	8	5	4	3	3	3	3	3	2	2	12	8
0.90	+∞	5	3	3	2	2	2	2	2	2	2	13	7
0.95	+∞	3	2	2	2	1	1	1	1	1	1	15	5
1.0	+∞	1	1	1	1	1	1	1	1	1	1	19	1

n_i = Number of incorrupt persons, n_c = Number of corrupt persons

Table 2: A Distribution of Number of incorrupt Interactions in the Simulated and Virtual Systems for a Growth Rate of One (1)

Cases of incorrupt interactions	Proportions of Incorrupt person	Carrying capacity (K)	Simulated				Virtual
			Average number of incorrupt interaction ($\bar{\alpha}$)	95% Confidence Interval ($\bar{\alpha}$) Lower	Upper	*Approximate Upper bound of the C.I for ($\bar{\alpha}$)	Maximum number of incorrupt interactions
1	0.50	4	2.73	2.66	2.80	3.00	3.00
2	0.83	4	3.83	3.75	3.91	4.00	5.00
3	0.40	5	3.00	3.71	3.90	4.00	4.00
4	0.70	5	5.00	5.71	5.93	6.00	7.00
5	0.90	5	3.80	6.97	7.20	8.00	9.00
6	0.33	6	5.82	4.66	4.87	5.00	5.00
7	0.60	6	7.08	7.60	7.87	8.00	9.00
8	0.80	6	4.00	9.95	10.23	11.00	12.00
9	0.93	6	7.00	11.41	11.69	12.00	14.00

***Decimal fractions are treated as full interaction**
****Correlation coefficient between the number of interactions in * and that of the virtual systems $\approx \mathbf{0.99}$**

Table 3: A Distribution of Mean Time to Extinction of corrupt Interactions across selected residual Corruption Rates for a System of Carrying Capacity of Twenty (20) with a perfect Competing Ability and a Growth Rate of One

Proportion of Incorrupt Interactions	Residual Corruption 0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	n_i	n_c
0.10	52	58	65	75	88	106	133	176	264	547	1	19
0.15	33	37	42	49	57	69	87	116	175	353	2	18
0.20	24	27	30	35	42	50	63	85	129	261	2	18
0.25	19	21	24	27	32	39	50	67	101	206	3	17

0.30	15	17	19	22	26	32	41	55	83	169	3	17
0.35	12	14	16	18	22	27	34	46	69	141	4	16
0.40	10	12	13	16	18	23	29	39	59	121	4	16
0.45	9	10	11	13	16	19	25	34	52	105	5	15
0.50	7	8	10	11	14	17	22	30	45	92	6	14
0.55	6	7	8	10	12	15	19	26	40	81	6	14
0.60	5	6	7	9	10	13	17	23	35	72	7	13
0.65	5	5	6	8	9	11	15	20	31	64	8	12
0.70	4	5	6	7	8	10	13	18	28	57	9	11
0.75	3	4	5	6	7	9	12	16	25	51	10	10
0.80	3	3	4	5	6	8	10	14	22	45	11	9
0.85	2	3	3	4	5	7	9	12	19	39	12	8
0.90	2	2	3	3	4	5	7	10	15	32	13	7
0.95	1	2	2	3	3	4	5	7	11	23	15	5
1.00	1	1	1	1	1	1	1	1	1	1	19	1

n_i = Number of incorrupt persons, n_c = Number of corrupt persons

Table 4: A Distribution of Mean Time to Extinction of corrupt Interactions across selected Competing Abilities for a System of Carrying Capacity of Twenty (20) and 0.1 residual Corruption and a Growth Rate of One (1)

Proportion of Incorrupt Interactions	Competing Ability 0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	n_i	n_c
0.10	+∞	547	273	185	139	112	94	81	71	64	52	1	19
0.15	+∞	338	172	116	88	71	60	52	46	41	33	2	18
0.20	+∞	237	121	82	62	50	43	37	33	29	24	2	18
0.25	+∞	177	91	62	47	38	33	28	25	23	19	3	17
0.30	+∞	138	71	48	37	30	26	23	20	18	15	3	17
0.35	+∞	109	56	38	30	24	21	18	16	15	12	4	16
0.40	+∞	87	45	31	24	20	17	15	14	13	10	4	16
0.45	+∞	70	37	26	20	17	14	13	12	11	9	5	15
0.50	+∞	57	30	21	17	14	12	11	10	9	7	6	14
0.55	+∞	46	24	17	14	12	10	9	8	8	6	6	14

0.60	+∞	37	20	14	11	10	9	8	7	7	5	7	13
0.65	+∞	30	16	12	10	8	7	7	7	6	5	8	12
0.70	+∞	23	13	10	8	7	6	6	5	5	4	9	11
0.75	+∞	18	10	8	6	6	5	5	4	4	3	10	10
0.80	+∞	13	8	6	5	5	4	4	4	4	3	11	9
0.85	+∞	9	6	5	4	4	3	3	3	3	2	12	8
0.90	+∞	6	4	3	3	3	3	2	2	2	2	13	7
0.95	+∞	3	2	2	2	2	2	2	2	2	1	15	5
1.00	+∞	1	1	1	1	1	1	1	1	1	1	19	1

n_i = Number of incorrupt persons, n_c = Number of corrupt persons

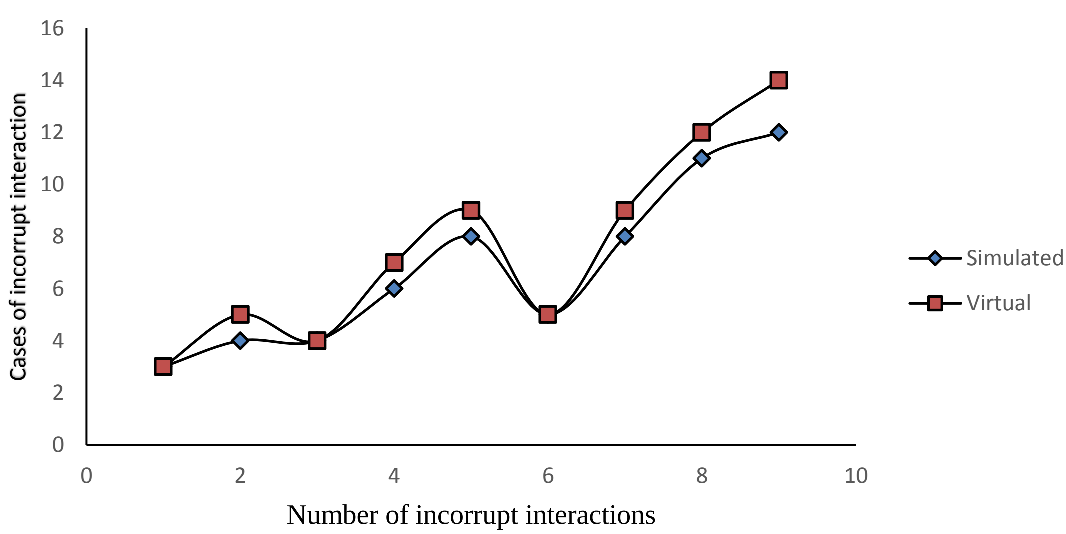
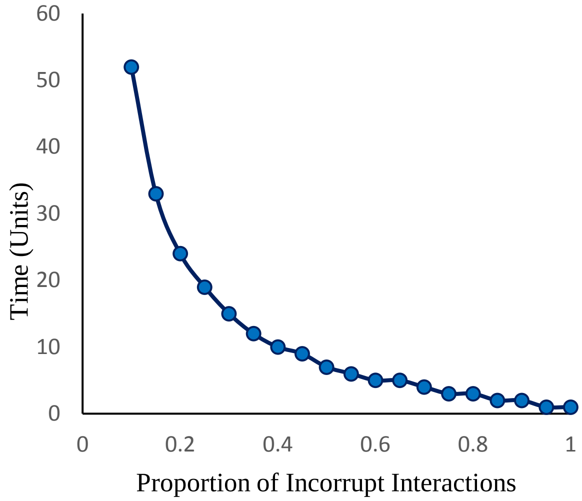
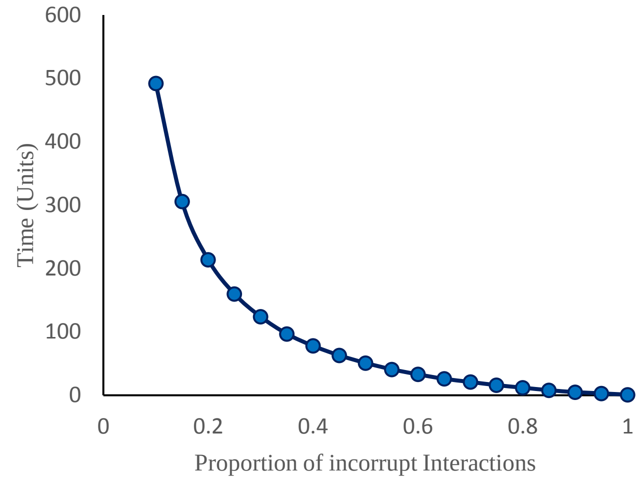


Figure 3:

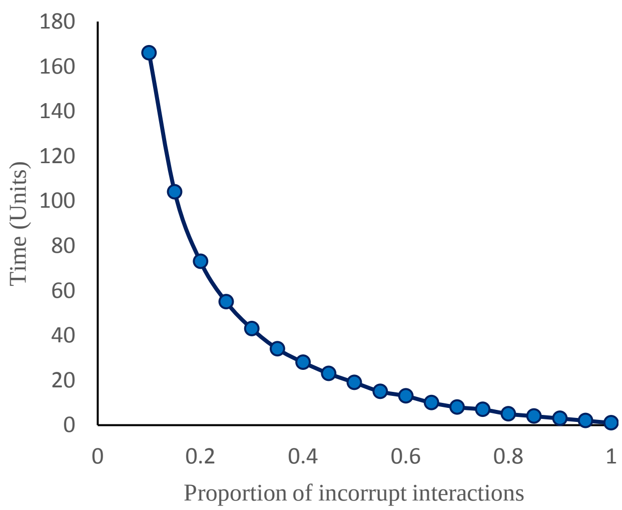
Relationship between the Simulated Systems and the Virtual Systems.



(a) **Relationship between Proportion of incorrupt Interactions and Time to Extinction of corrupt Interactions for the perfect Competing Ability.**

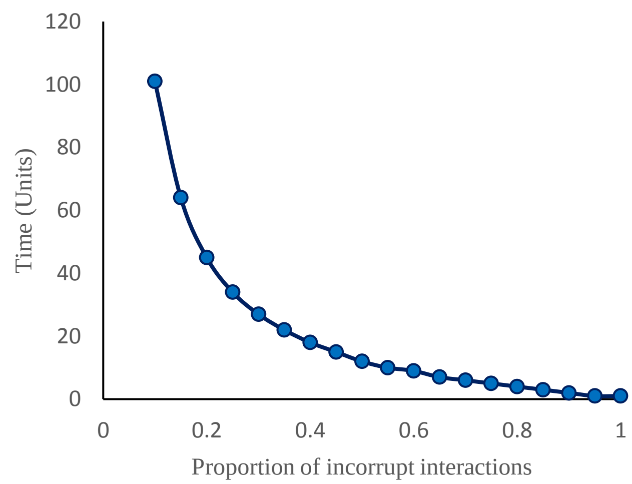


(b) **Relationship between Proportion of incorrupt Interactions and Time to Extinction of corrupt Interactions for the 0.1 Competing Ability.**



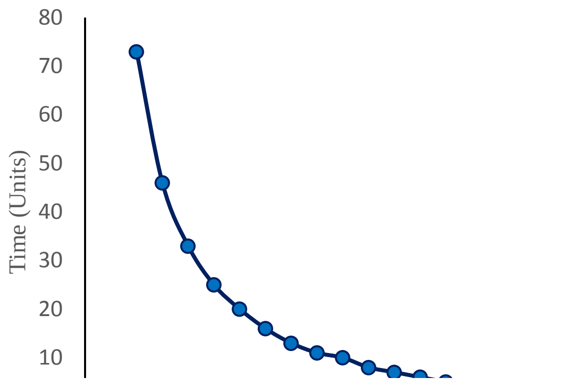
(c)

Interactions for the 0.[illegible] Competing Ability.

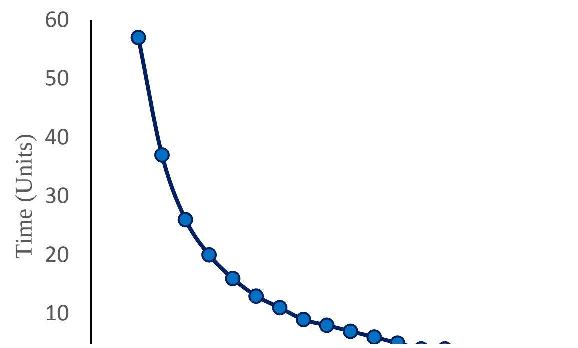


Interactions for the 0.[illegible] Competing Ability.

Figure 4: Relationship between Proportion of incorrupt Interactions and Time to Extinction of corrupt Interactions in a System of Twenty (20) Carrying Capacity with Competing Abilities of 1.0, 0.1, 0.3 and 0.5.



(a) **Relationship between Proportion of incorrupt Interactions and Time to Extinction of corrupt Interactions for the 0.7 Competing Ability.**



(b) **Relationship between Proportion of incorrupt Interactions and Time to Extinction of corrupt Interactions for the 0.9 Competing Ability.**

...of incorrupt Interactions and Time to Extinction of corrupt Interactions in a System of Twenty (20) Carrying Capacity with Competing Abilities of 0.7 and 0.9.

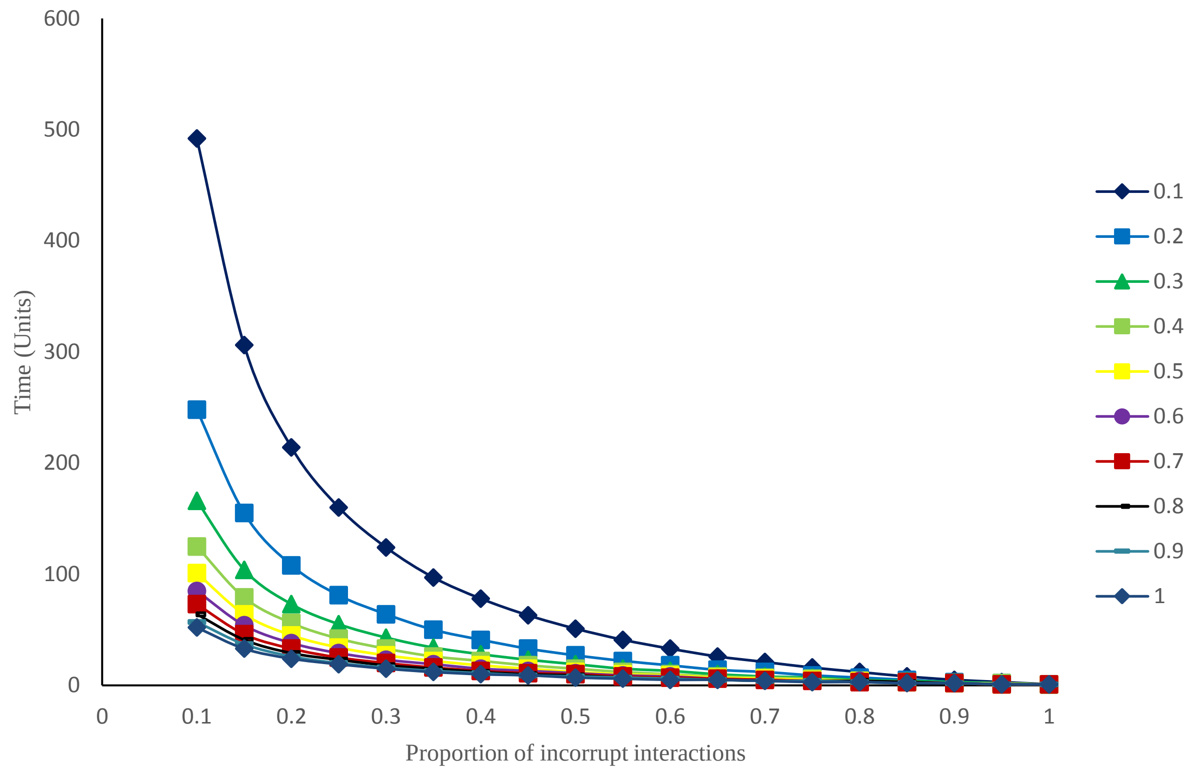
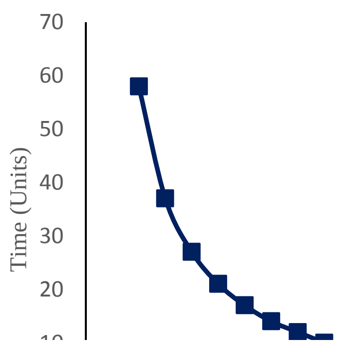
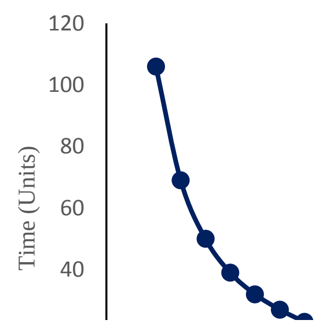


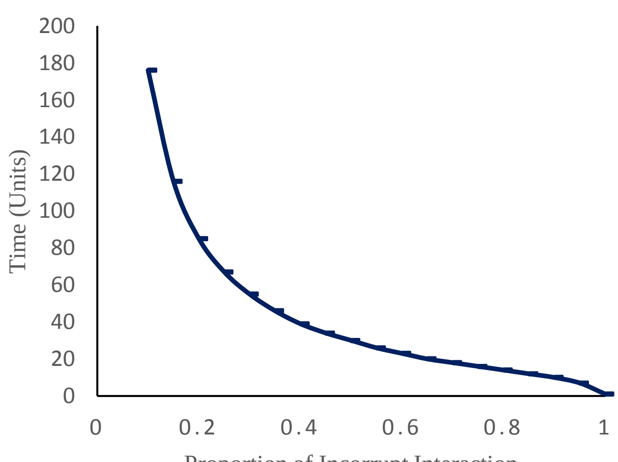
Figure 6: Relationship between proportion of incorrupt Interactions and Time to Extinction across various Competing Abilities.

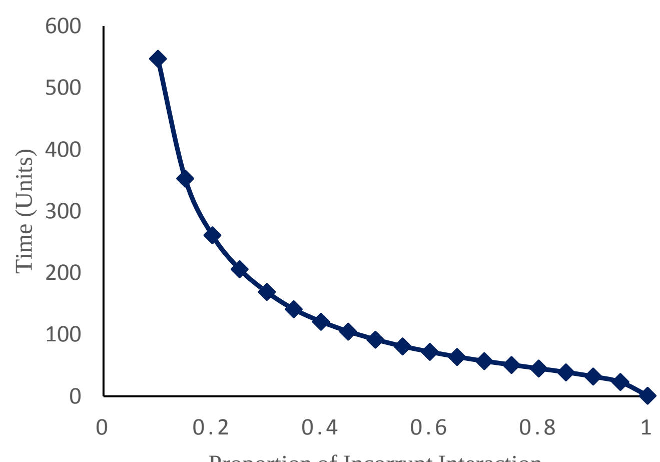


(a) **Relationship between Proportion of incorrupt Interactions and Time to Extinction of corrupt Interactions for 0.1 residual Corruption and perfect Competing Ability.**



(b) **Relationship between Proportion of incorrupt Interactions and Time to Extinction of corrupt Interactions for 0.5 residual Corruption and perfect Competing Ability.**

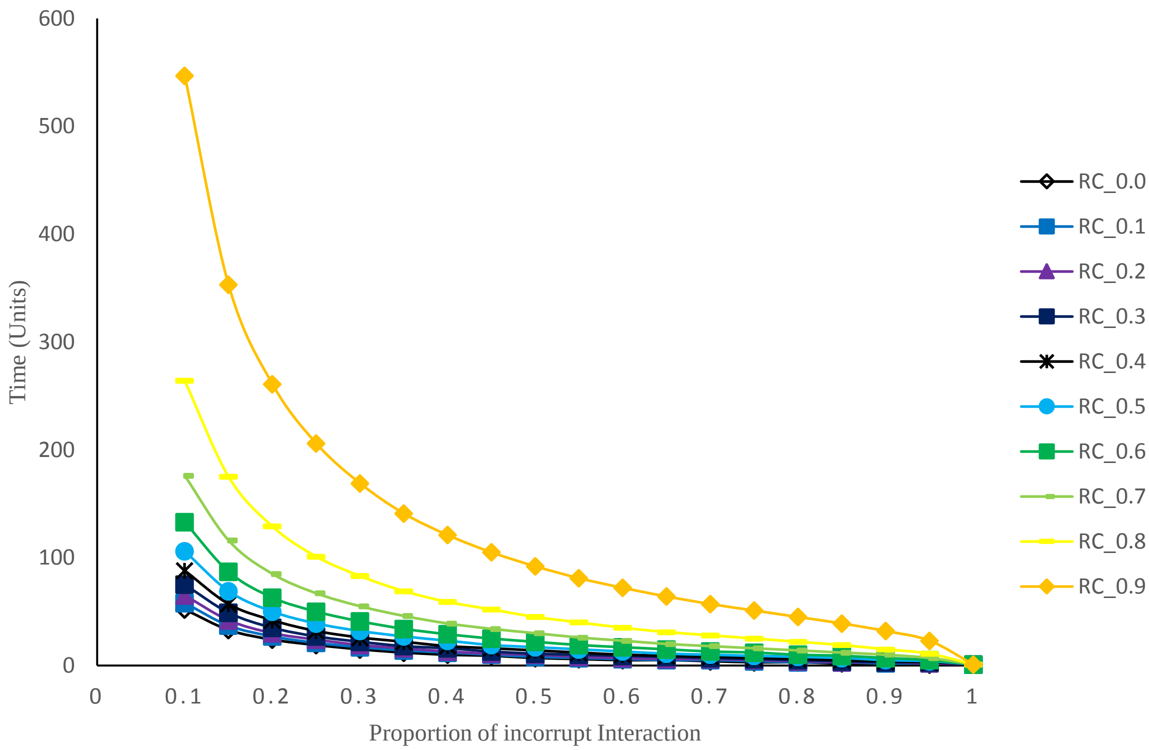




(c) **Relationship between Proportion of incorrupt Interactions and Time to Extinction of corrupt Interactions for 0.71 residual Corruption and perfect Competing Ability.**

(d) **Relationship between Proportion of incorrupt Interactions and Time to Extinction of corrupt Interactions for 0.9 residual Corruption and perfect Competing Ability.**

Figure 7: **Relationship between proportion of incorrupt interactions and the time to extinction of corrupt interactions with 0.1, 0.5, 0.7 and 0.9 residual corruptions and a perfect competing ability of the incorrupt person in a system of twenty (20) carrying.**



to

4.0 Discussion

The simulation model was ran for each virtual system using the aforementioned parameters (i.e. the carrying capacity, residual corruption of the incorrupt person, the proportion of incorrupt interaction and the 1.0 intrinsic growth rate) as shown in Table 2. The number of incorrupt interactions required to block the corrupt interactions in the system is the simulation output of interest.

The result in Table 2 shows that the simulated number of incorrupt interactions is approximately the same as the actual number of incorrupt interactions in the virtual systems created (Figure 2). The graph in Figure 3 and the strong correlation coefficient of 0.99 between the simulated number of incorrupt interactions and the actual number of incorrupt interactions from the virtual systems (Table 2) confirmed the above assertion.

Tables 1 and 3 contain the proportions of incorrupt interaction with 0.05 step size starting from 0.1 all through to 1.0. The various residual corruptions and competing abilities ranging from 0.1 to 1 with the step size of 0.1 were considered. Some of the major features of the simulation model result observed are as follows; as the residual corruption increases from 0.0 to 1.0, the time to extinction also increases from 52 time units to 547 time units. Residual corruption of 0.0 means the incorrupt persons do not display any residual corruption. 0.1 residual corruption rate means the incorrupt person display 10percent tendency of yielding to corrupt practices in the system while 0.2 residual corruption rate means, 20percent tendency of yielding to corrupt practices.

From the above, the time to extinction of corrupt interactions in a system is directly proportional to the residual corruption rate (Table 3 and Figure 8). The time to extinction of corrupt interaction is inversely proportional to the competing abilities of the incorrupt persons in the system (Table 1 and Figure 6). With 0.1 proportion of incorrupt interactions, 0.1 competing ability, it takes up to 492 time units for corruption to go extinct in a system with zero residual corruption (Figure 4b) as compared to 52 time units when the competing ability is one (1) (Figure 4a). More so, a residual corruption of 0.1 is introduced into the system with 0.1 competing ability the time to extinction of corrupt practices become 547 time units as against 492 time units. This imply that 0.1 residual corruption rate account for additional 55 time units to extinction (Table 4, Figure 8).

The result in Table 4 revealed the negative effect of residual corruption on the time to extinction of corrupt interactions irrespective of the competing ability and the proportion of incorrupt interactions.

The graphs of various competing abilities of incorrupt persons are as shown in Figures 4-5. The time to extinctions of corrupt interaction for our system with perfect competing ability is 52 time unit since it is assumed that all the interactions from the incorrupt person cannot be negatively influenced (Figure 4a). The time to extinction on Figure 4a is less compare with that on Figures 4b-5b and this show that, as the competing ability reduces from 1.0 down to 0.0, the time to extinction increases. Hence, the time to extinction of corrupt interaction is inversely proportional to the competing ability of the incorrupt person in the system. For example, in a system with a perfect competing ability of incorrupt persons, when the proportion of incorrupt interactions is 0.45, the time to extinction of corrupt practice is 9 time units, in a system of carrying capacity of twenty (20) with 5 incorrupt and 15 corrupt persons (Figure 4a and Table 1).

In most cases, the competing ability of incorrupt person might not be perfect, this means that the incorrupt persons might not be able to compete perfectly with the corrupt persons. Whenever the assumption of perfect competing ability is violated, equation (2.7) come into play to handle this scenario and hence Figures 4b-5b.

Figures 4b-5b presents the graphs with values of competing abilities 0.1, 0.3, 0.5, 0.7 and 0.9. To demonstrate the effect of competing ability on the time to extinction of corrupt interactions, let make use of 0.45 proportion of incorrupt interactions and a competing ability of 0.2 (Figure 4b and Table 1). The time to extinction of corrupt practices is 33 time units whereas, it is 9 time units for a perfect competing ability with the same number of corrupt and incorrupt interactions. This follows for other competing abilities (Figures 4c-4d, 5a-5b and 6).

The effect of residual corruption on the time to extinction can be seen in Table 4 and Figures 7-8. Irrespective of the competing ability of the incorrupt persons, increase in residual corruption rate will elongate the time to extinction of corrupt interactions across the proportion of incorrupt interactions.

In summary, it is observed from the analysis that, as the residual corruption of the incorrupt persons increases, the time to extinction of corrupt interactions also increases. This means that residual corruption of the incorrupt person is directly proportional to the time to extinction of the corrupt interactions in a system. Whereas, as the competing ability of the incorrupt person reduces, the time to extinction increases. That is, the closer the competing ability is to 1, the smaller the time to extinction of the corrupt practices in the system becomes.

5.0 Conclusion and Recommendation

The following conclusions were drawn from the study;

(i) The residual corruption of incorrupt persons in a system is directly proportional to the time to extinction of corrupt interactions.

(ii) The competing ability of incorrupt persons in a system is inversely proportional to the time to extinction of corrupt interactions.

(iii) The proportion of incorrupt interactions is directly proportional to the time to extinction of corrupt interactions.

We recommend that, this model should be applied in curbing corruption in a system within the stated carrying capacity.

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