A COUPLED FIXED POINT THEOREM FOR MAPPINGS HAVING THE MIXED MONOTONE TYPE IN PARTIALLY ORDERED S-METRIC SPACE

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Abstract
In this paper we use the notion of *S-metric space to prove a coupled fixed theorem for mapping having the mixed monotone property in partially ordered S-metric space. Our result generalize an existing result in the settings of S-metric space.*

Keywords*: Coupled fixed point, mixed monotone property, partially ordered set, S-metric space.*

1. Introduction

Fixed point theory is an exciting branch of mathematics. It is a mixture of analysis, topology and geometry. The space X is said to have the fixed point property for a map T: $X \rightarrow X$ if there exist $x \in X$ such that $Tx = x$. Over the last 50 years or so, the theory of fixed point has been revealed as a very important tool in the study of nonlinear phenomena (Afshari, 2014). Banach's contraction principle ensures under appropriate conditions the existence and uniqueness of a fixed point. It is the simplest and one of the most important results in fixed point theory.

Some authors for example, Ran & Reurings (2004), Bhaskar & Lakshmikantham (2006) and Lakshmikantham and Ciric (2009) proved some well-known results of fixed point in partially ordered metric space. Especially Bhaskar and Lakshmikantham (2006) introduced the notion of coupled fixed point and proved some coupled fixed point results in partially ordered metric space.

Because of its importance in mathematics and applied sciences some authors have tried to give generalizations of metric spaces in several ways. These generalizations were also used then to extend the scope of the study of fixed point theory. For example,
In 1963, Gähler introduced the notion of a 2-metric space as follows:

Definition 1.1 Let X be a nonempty set. A function $d: X^3 \to \mathbb{R}$ is said to be a 2-metric on X if for all $x, y, z, a \in X$, the following condition hold:
(d1) For any distinct point $x, y \in X$ there exist $z \in X$, $d(x, y, z) \neq 0$
(d2) $d(x, y, z) = 0$ if any of the two elements of the set $\{x, y, z\}$ in X are equal.
(d3) $d(x, y, z) = d(x, z, y) = d(y, x, z) = d(z, x, y) = d(y, z, x) = d(z, y, x)$
(d4) $d(, y, z) \leq d(x, y, a) + d(x, a, z) + d(a, y, z)$
The pair (X, d) is called a 2-metric space.

In 1984, Dhage in his Ph.D thesis identified condition (d2) as a weakness in Gahler's theory of 2-metric space. To overcome these problems, in 1992 he then introduced the concept of a D-metric space.

Definition 1.2 Let X be a nonempty set. A function $D: X^3 \to \mathbb{R}$ is called a D-metric on X if for all $x, y, z, a \in X$, the following condition hold:
(D1) $D(x, y, z) \geq 0$ for all x, y, z, a in X and $D(x, y, z) = 0$ if and only if $x = y = z$
(D2) $D(x, y, z) = D(x, z, y) = D(y, x, z) = D(z, x, y) = D(y, z, x) = D(z, y, x)$
(D3) $D(x, y, z) \leq D(x, y, a) + D(x, a, z) + D(a, y, z)$
The pair (X, D) is called a D-metric space.
Mustafa and Sims (2006), introduced the notion of G-metric space and suggested an important generalization of metric space as follows.

Definition 1.3 Let X be a nonempty set. A function $G: X^3 \to \mathbb{R}^+$ is called a G-metric on X if it satisfies the following condition: For all $x, y, z, a \in X$,
(G1) $G(x, y, z) = 0$ if and only if $x = y = z$
(G2) $0 \leq G(x, y, y)$ with $x \neq y$
(G3) $G(x, x, y) \leq G(x, y, z)$ with $z \neq y$
(G4) $G(x, y, z) = G(x, z, y) = G(y, x, z) = G(z, x, y) = G(y, z, x) = G(z, y, x)$
(G5) $G(x, y, z) \leq G(x, x, a) + G(a, y, z)$
The pair (X, G) is called a G-metric space.

Thereafter Mustafa et al. (2008) proved some fixed point thereoms in G-metric space.
Sedghi, Shobe & Zhou (2007) introduced the notion of D^*-metric space as follows.

Definition 1.4 Let X be a nonempty set. A function $D^*: X^3 \to \mathbb{R}^+$ is called a D^*-metric on X if it satisfies the following condition: For all $x, y, z, a \in X$,

(D*1) $D^*(x,y,z) \geq 0$
(D*2) $D^*(x,y,z)$ =0 if and only if $x=y=z$
(D*3) $D^*(x,y,z) = D^*(x,z,y) = D^*(y,x,z) = D^*(z,x,y) = D^*(y,z,x) = D^*(z,y,x)$
(D*4) $D^*(x,y,z) \leq D^*(x,y,a) + D^*(a,z,z)$
The pair(X, D^*) is called a D^*-metric space.

The aim of this paper is to generalize the result of Mehta and Joshi (2010) in the setting of ***S***-metric space.

2. Definitions and Preliminaries:

Sedghi, Shobe & Aliouche (2012) introduced a new generalize metric space called S-metric space.

Definition 2.1 Let X be a nonempty set. An S-metric space on X is a function $S : X^3 \to [0,\infty)$ that satisfies the following conditions. For each $x, y, z, a \in X$

(S1) $S(x,y,z) \geq 0$ for all $x,y,z \in X$
(S2) $S(x,y,z) = 0$ if and only if $x = y = z$
(S3) $S(x,y,z) \leq S(x,x,a) + S(y,y,a) + S(z,z,a)$
The pair $(X;S)$ is called an S-metric space.
The following examples, definitions and lemmas are given by Sedghi et al, (2012).
Example 2.1 Let $\mathbb{R}$ be the real line. Then $S(x,y,z) = |x - z| + |y - z|$ for all $x, y, z \in \mathbb{R}$ is an S-metric on. This S-metric is called the usual S-metric on $\mathbb{R}$.
Example 2.2 Let $X = \mathbb{R}^2$ and d an ordinary metric on X. Then $S(x,y,z) = d(x,y) + d(x,z) + d(y,z)$ is an S-metric on X.

Example 2.3 Let $X = \mathbb{R}^n$ and $\|.\|$ a norm on X. Then $S(x,y,z) = \| y + z - 2x \| + \| y - z \|$ is a S-metric on X.
Definition 2.2 Let (X,S) be a S-metric space.

(1) A sequence $\{x_n\}, x \in X$ converges to x if and only if $S(x_n,x_n,x) \to 0$ as $n \to \infty$.
That is for each $\epsilon > 0$ there exist $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, $S(x_n,x_n,x) < \epsilon$ and we denote this by $\lim_{n\to\infty} x_n = x$.
(2) A sequence x_n is called a Cauchy sequence if for each $\epsilon > 0$, there exist $n_0 \in \mathbb{N} \in$ such that for each $n,m \geq n_0$, $S(x_n,x_n,x_m) < \epsilon$

(3) The S-metric space (X, S) is said to be complete if every Cauchy sequence converges.

Lemma 2.1 Let (X, S) be an S-metric space. Then $S(x, x, y) = S(y, y, x)$
Lemma 2.2 Let (X, S) be an S-metric space. Then $S(x, x, z) \leq 2S(x, x, y) + S(y, y, z)$
Lemma 2.3 Let (X, S) be an S-metric space. If the sequence $\{x_n\} \in X$ converges to x, then x is unique.
Lemma 2.4 Let (X, S) be an S-metric space. If the sequence $\{x_n\} \in X$ converges to x, then $\{x_n\}$ is a Cauchy sequence.
Lemma 2.5 Let (X, S) be a S-metric space. If there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$, then $\lim_{n\to\infty} S(x_{n,} x_{n,}\ y_n) = S(x, x, y)$

Bhaskar & Lakshmikantham (2006) stated the following definitions and result.

Definition 2.3 (See [3]) Let $(X, \leq)$ be a partially ordered set and d be a metric on X such that (X, d) is a complete metric space. Consider the product space $X \times X$ the following partial order: for $(x, y), (u, v) \in X \times X$,

$$(u, v) \leq (x, y) \Leftrightarrow x \geq u, y \leq v.$$

Definition 2.4 Let $(X, \leq)$ be a partially ordered set and $F: X \times X \to X$. We say that F has the mixed monotone property if $F(x, y)$ is monotone nondecreasing in x and is monotone non increasing in y, that is for any $x, y, z \in X$,

$$x_1, x_2 \in X, x_1 \leq x_2 \Rightarrow F(x_1, y) \leq F(x_2, y)$$

and

$$y_1, y_2 \in X, y_1 \leq y_2 \Rightarrow F(x, y_1) \geq F(x, y_2).$$

Definition 2.5 An element $(x, y) \in X \times X$ is called a coupled fixed point of the mapping $F: X \times X \to X$ if

$$F(x, y) = x \quad \text{and} \quad F(y, x) = y$$

Mehta and Joshi (2010) proved the following result.

Theorem 2.1 Let $(X, \leq)$ be a partially ordered set and (X, d) is a complete metric space. Let $F: X \times X \to X$ be a continuous mapping having the mixed monotone property on X such that there exists elements $x_0, y_0 \in X$ with

$$x_0 \leq F(x_0, y_0) \quad \text{and} \quad y_0 \geq F(y_0, x_0).$$

Suppose that there exist non-negative real numbers p and q with $p + q < 1$ such that

$$d\big(F(x, y), F(u, v)\big) \leq p\min\{d(F(x, y), x), d(F(u, v), , x)\}$$
$$+q \min\{d(F(x, y), u), d(F(u, v), , u)\}$$

$$+\min\{d(F(x,y),,u),d(F(u,v),,x)\}$$

for all $x,y,u,v \in X$ with $x \geq u, y \leq v$. Then F has a coupled fixed point in X.

3. Results and Discussion

In this section, we generalize the result of Mehta and Joshi (2010) in the setting of $\boldsymbol{S}$-metric space.

Theorem 3.1 Let $(X,\leq,S)$ be a partially ordered complete S-metric space and $F: X \times X \to X$ be a continuous mapping having the mixed monotone property on X and let there exists points x_0, y_0 with

$$x_0 \leq F(x_0,y_0),\ \ y_0 \geq\ F(y_0,x_0).$$

Suppose that there exist $p \geq 0, q \geq 0$ with $p+q<1$ such that

$$\begin{aligned} S\big(F(x,y),F(x,y),F(u,v)\big) \leq {} & p\min\{S(F(x,y),F(x,y)x),S(F(u,v),F(u,v),x)\} \\ & +q\ \min\{S(F(x,y),F(x,y),u),S(F(u,v),F(u,v),u)\} \\ & +\min\{S(F(x,y),F(x,y),u),S(F(u,v),F(u,v),x)\} \quad (1) \end{aligned}$$

for all $x,y,u,v \in X$ with $x \geq u, y \leq v$. Then F has a coupled fixed point in X.

Proof.

Let $x_0, y_0 \in X$ with

$$x_0 \leq F(x_0,y_0),\ \ y_0 \geq\ F(y_0,x_0)\ . \qquad (2)$$

Define the sequences $\{x_n\}$, $\{y_n\}$ in X such that for all n =0, 1, 2, …

$$x_{n+1} = F(x_n,y_n) \qquad y_{n+1} = F(y_n,x_n) \qquad (3)$$

We claim that for all n =0, 1, 2, … , $\{x_n\}$ is nondecreasing and $\{y_n\}$ is non-increasing. That is

$$x_n \leq x_{n+1}\ ,\ \ y_n \geq y_{n+1} \qquad (4)$$

From Eqs. (2) and (3), we have for n =0,
$x_0 \leq F(x_0,y_0),\ \ y_0 \geq\ F(y_0,x_0)$ and $x_1 = F(x_0,y_0)$, $y_1 = F(y_0,x_0)$
This implies $x_0 \leq x_1$, $y_0 \geq y_1$.

Thus equation (4) holds for n =0.

Also suppose that (4) holds for some $n \in \mathbb{N}$. That is $x_n \leq x_{n+1}\ ,\ \ y_n \geq y_{n+1}$.

We now show that it is true for $n+1$.

By the mixed monotone property of F, we have

$$x_{n+2} = F(x_{n+1}, y_{n+1}) \geq F(x_n, y_{n+1}) \geq F(x_n, y_n) = x_{n+1}$$
$$y_{n+2} = F(y_{n+1}, x_{n+1}) \leq F(y_n, x_{n+1}) \leq F(y_n, x_n) = y_{n+1}$$

Thus by mathematical induction, equation (4) holds for $n \in \mathbb{N}$.

Therefore,

$$x_0 \leq x_1 \leq x_2 \leq \ldots \leq x_n \leq x_{n+1} \leq \ldots$$
$$y_0 \geq y_1 \geq y_2 \geq \ldots \geq y_n \geq y_{n+1} \geq \ldots$$

As $x_n \geq x_{n-1}$, $y_n \leq y_{n-1}$, we have from Eq. (1)

$$\begin{aligned}
&S\big(F(x_n, y_n), F(x_n, y_n), F(x_{n-1}, y_{n-1})\big) \\
&\leq p\min\{S(F(x_n, y_n), F(x_n, y_n), x_n), S(F(x_{n-1}, y_{n-1}), F(x_{n-1}, y_{n-1}), x_n)\} \\
&+q\min\{S(F(x_n, y_n), F(x_n, y_n), x_{n-1}), S(F(x_{n-1}, y_{n-1}), F(x_{n-1}, y_{n-1}), x_{n-1})\} \\
&+\min\{S(F(x_n, y_n), F(x_n, y_n), x_{n-1}), S(F(x_{n-1}, y_{n-1}), F(x_{n-1}, y_{n-1}), x_n)\}
\end{aligned}$$

Hence

$$\begin{aligned}
&S(x_{n+1}, x_{n+1}, x_n) \\
&\leq p\min\{S(x_{n+1}, x_{n+1}, x_n), S(x_n, x_n, x_n)\} \\
&\quad +q\min\{S(x_{n+1}, x_{n+1}, x_{n-1}), S(x_n, x_n, x_{n-1})\} \\
&\quad +\min\{S(x_{n+1}, x_{n+1}, x_{n-1}), S(x_n, x_n, x_n)\} \\
&= qS(x_n, x_n, x_{n-1})
\end{aligned}$$

Hence $S(x_{n+1}, x_{n+1}, x_n) \leq qS(x_n, x_n, x_{n-1})$ (5)

Again, as $y_n \leq y_{n-1}$, $x_n \geq x_{n-1}$, we have from Eq. (1)

$$S\big(F(y_{n-1}, x_{n-1}), F(y_{n-1}, x_{n-1}), F(y_n, x_n)\big)$$

$$\begin{aligned}
&\leq p\min\{S(F(y_{n-1}, x_{n-1}), F(y_{n-1}, x_{n-1}), y_{n-1}), S(F(y_n, x_n), F(y_n, x_n), y_{n-1})\} \\
&\quad +q\min\{S(F(y_{n-1}, x_{n-1}), F(y_{n-1}, x_{n-1}), y_n), S(F(y_n, x_n), F(y_n, x_n), y_n)\} \\
&\quad +\min\{S(F(y_{n-1}, x_{n-1}), F(y_{n-1}, x_{n-1}), y_n), S(F(y_n, x_n), F(y_n, x_n), y_{n-1})\}
\end{aligned}$$

Hence

$S(y_n, y_n, y_{n+1})$

$\leq p\min\{S(y_n, y_n, y_{n-1}), S(y_{n+1}, y_{n+1}, y_{n-1})\}$
$+q\min\{S(y_n, y_n, y_n), S(y_{n+1}, y_{n+1}, y_n)\}$
$+\min\{S(y_n, y_n, y_n), S(y_{n+1}, y_{n+1}, y_{n-1})\}$
$= pS(y_n, y_n, y_{n-1})$

Hence $S(y_n, y_n, y_{n+1}) \leq pS(y_n, y_n, y_{n-1})$

(6)

Adding (3.5) and (3.6) , we get

$$S(x_{n+1}, x_{n+1}, x_n) + S(y_{n+1}, y_{n+1}, y_{n-1})$$
$$\leq qS(x_n, x_n, x_{n-1}) + pS(y_n, y_n, y_{n-1})$$
$$\leq (p+q)S(x_n, x_n, x_{n-1}) + (p+q)S(y_n, y_n, y_{n-1})$$
$$= (p+q)[\, S(x_n, x_n, x_{n-1}) + S(y_n, y_n, y_{n-1})].$$

Let $A = p + q < 1$

$$S(x_{n+1}, x_{n+1}, x_n) + S(y_{n+1}, y_{n+1}, y_{n-1})$$
$$\leq A[S(x_n, x_n, x_{n-1}) + S(y_n, y_n, y_{n-1})]$$
$$\leq A^2[S(x_{n-1}, x_{n-1}, x_{n-2}) + S(y_{n-1}, y_{n-1}, y_{n-2})]$$

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$$\leq A^n[S(x_1, x_1, x_0) + S(y_1, y_1, y_0)]$$

Moreover by lemma 2.2, we have for all $n \leq m$

$$S(x_n, x_n, x_m) + S(y_n, y_n, y_m)$$
$$\leq (2S(x_n, x_n, x_{n+1}) + 2S(y_n, y_n, y_{n+1}))$$
$$+(S(x_{n+1}, x_{n+1}, x_n) + S(y_{n+1}, y_{n+1}, y_n))$$
$$\leq (2S(x_n, x_n, x_{n+1}) + 2S(y_n, y_n, y_{n+1}))$$
$$+(2S(x_{n+1}, x_{n+1}, x_n) + 2S(y_{n+1}, y_{n+1}, y_n))$$
$$+ \ldots +(2S(x_{m-2}, x_{m-2}, x_{m-1}) + 2S(y_{m-2}, y_{m-2}, y_{m-1}))$$
$$+(S(x_{m-1}, x_{m-1}, x_m) + S(y_{m-1}, y_{m-1}, y_m))$$
$$\leq (2S(x_n, x_n, x_{n+1}) + 2S(y_n, y_n, y_{n+1}))$$
$$+ \ldots +(2S(x_{m-1}, x_{m-1}, x_m) + 2S(y_{m-1}, y_{m-1}, y_m))$$

$$\leq 2[(A^n + A^{n+1} + ... + A^{m-1})(S(x_1, x_1, x_0) + S(y_1, y_1, y_0))]$$

$$\leq \frac{2A^n}{1-A} (S(x_1, x_1, x_0) + S(y_1, y_1, y_0))$$

Since $A < 1$, taking limit as $n, m \to \infty$, we get

$$\lim_{n,m\to\infty} \{ S(x_n, x_n, x_m) + S(y_n, y_n, y_m)\} = 0.$$

It implies that

$$\lim_{n,m\to\infty} S(x_n, x_n, x_m) = \lim_{n,m\to\infty} S(y_n, y_n, y_m) = 0.$$

Therefore $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in X.
Since X is complete, there exist $x, y \in X$ such that as $n \to \infty$, $x_n \to x$ and $y_n \to y$.
Hence by taking limit as $n \to \infty$ and using Eq. (3), we get

$$x = \lim_{n\to\infty} x_n = \lim_{n\to\infty} F(x_{n-1}, y_{n-1}) = F\left(\lim_{n\to\infty} x_{n-1.} \lim_{n\to\infty} y_{n-1}\right) = F(x, y)$$

$$y = \lim_{n\to\infty} y_n = \lim_{n\to\infty} F(y_{n-1}, x_{n-1}) = F\left(\lim_{n\to\infty} y_{n-1.} \lim_{n\to\infty} x_{n-1}\right) = F(y, x)$$

Thus we have $F(x, y) = x$, $\quad F(y, x) = y$ implying that F has a coupled fixed point

We now give an example as shown in Mehta & Joshi (2010) to support our result in the settings of S-metric space.

Example 3.1 Let $X = [0,1]$ $x \leq y \Leftrightarrow x, y \in [0,1]$ with the usual order $\leq$. Let $(X, \leq, S)$ be a partially ordered complete S-metric with the usual S-metric defined as in example 1.1. That is
$S(x, y, z) = |x - z| + |y - z|$. So,
$S(x, x, y) = |x - y| + |x - y| = 2|x - y| = 2d(x, y)$.

Define $F: X \times X \to X$ by

$$F(x, y) = \begin{cases} \dfrac{x - y}{2}, & if\ x \geq y \\ 0, & otherwise \end{cases}$$

Then F is continuous and has the mixed monotone property.
Let there exist $x_0 = y_0 = 0$ such that
$x_0 = 0 \leq F(0,0) = F(x_0, y_0)$ and $y_0 = 0 \geq F(0,0) = F(y_0, x_0)$.
Next we show that the mapping F satisfies Eq. (1) with $p = \frac{1}{2}, q = \frac{1}{4}$
For $x = 1, y = 0, u = 0, v = 0$ we have the following possibilities for values of (x, y) and (u, v) such that $x \geq u$ and $y \leq v$.

Case I: If $(x, y) = (u, v) = (0,0)$ or $(1,0)$ or $(0,1)$ or $(1,1)$.
Then $S\big(F(x, y), F(x, y), F(u, v)\big) = 2d\big(F(x, y), F(u, v)\big) = 0$, which shows that Eq. (1) holds.
Case II: If $(x, y) = (0,0),\ (u, v) = (0,1)$ or $(x, y) = (1,1), (u, v) = (0,1)$.
Then $S\big(F(x, y), F(x, y), F(u, v)\big) = 2d\big(F(x, y), F(u, v)\big) = 0$, which shows that Eq. (1) holds.
Case III: If $(x, y) = (1,0),\ (u, v) = (0,0)$ or $(x, y) = (1,0), (u, v) = (0,1)$.
Then L.H.S of Eq. (1)

$$S\big(F(x, y), F(x, y), F(u, v)\big) = 2d\big(F(x, y), F(u, v)\big) = 2d\big(F(1,0), F(0,0)\big) = 2d\left(\frac{1}{2}, 0\right) = 1$$

and R.H.S of Eq. (1)

$$\frac{1}{2}\min\{2d\left(\frac{1}{2}, 1\right), 2d(0,1)\} + \frac{1}{4}\min\left\{2d\left(\frac{1}{2}, 0\right), 2d(0,0)\right\} + \min\left\{2d\left(\frac{1}{2}, 0\right), 2d(0,1)\right\} = \frac{3}{2}$$

Thus Eq. (1) holds.
Case IV: If $(x, y) = (1,0),\ (u, v) = (1,1)$.
Then L.H.S of Eq. (1)

$$S\big(F(x, y), F(x, y), F(u, v)\big) = 2d\big(F(x, y), F(u, v)\big) = 2d\big(F(1,0), F(1,1)\big) = 2d\left(\frac{1}{2}, 0\right) = 1$$

and R.H.S of Eq. (1)

$$\frac{1}{2}\min\{2d\left(\frac{1}{2}, 1\right), 2d(0,1)\} + \frac{1}{4}\min\left\{2d\left(\frac{1}{2}, 1\right), 2d(0,1)\right\} + \min\left\{2d\left(\frac{1}{2}, 1\right), 2d(0,1)\right\} = \frac{7}{4}$$

Thus Eq. (1) holds.
Hence all the conditions of Theorem 3.1 are satisfied and therefore F has a coupled fixed point.

4. Conclusion

Theorem 2.1 is a result already proved in the settings of metric space. This result also hold in the setting of *S*-metric space as shown in this paper (Theorem 3.1). Thus we conclude that *S*-metric is a generalization of metric space.

5. Recommendation

Various results for fixed point theorem in metric space has been established. Some of these results have not been (or cannot be) generalized in the setting of S-metric space.

We therefore recommend that the existence of coupled fixed point for different mapping in the setting of S-metric space be investigated.

Also the mixed monotone type mapping may be extended to a more general setting of multivalued mixed weakly monotone type mapping.

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